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Improvement of energy dissipative particle dynamics method to increase accuracy

Marzie Borhani1 · Somaye Yaghoubi1

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Abstract

In this article, dissipative particle dynamics with energy conservation eDPD is used for simulating hydrodynamic behavior and heat transfer of DPD particles in a two-dimensional channel with parallel planes. To this end, a Fortran programming code is developed and the results are presented as dimensionless velocity and temperature profles on the cross section perpendicular to the fow direction inside the channel. For the indented geometry, thermal and dynamic boundary conditions have been considered. The dynamic boundary condition of solution domain in the fow's direction is periodic, and for modeling the walls, freezing layers of DPD particles with Bounce-Back refection has been used. For the thermal boundary condition, it is assumed that the wall temperature is constant and the temperature of each DPD particle in contact with the wall is the same as the wall temperature. In this article, for the frst time, for modeling the walls four frozen layers with Bounce-Back refection are used and the efect of particle exit on two and three-layers confgurations is investigated. Furthermore, for the frst time, modifed velocity Verlet integration algorithm is improved by adding heat transfer equations. And considering λ = 0.65 in the algorithm; the indented geometry is well simulated. In order to validate the results, first, the effect of regular and random initial distribution is compared. Furthermore, the results of wall alignment are compared with those obtained from CFD approach. In this paper, in addition to studying the efect of wall alignment and initial particle arrangement, the infuence of the size of cells for averaging and the time steps in the output results are investigated.

Keywords Dissipative particle dynamics · Heat transfer · Boundary conditions · No-slip · Meso-scale

Introduction

Various attempts have been made for the solution of fuid dynamics and heat transfer problems using methods such as computational fuid dynamics (CFD) and atomic techniques such as molecular dynamics (MD). Study and simulation of geometries in meso-scale has always been one of the favorite issues of researchers. Currently, several approaches are used in meso-scale, the most popular of which include Brownian dynamics (BD), Lattice Boltzmann method (LBM), dissipative particle dynamics (DPD) and molecular dynamics (MD). For example, molecular dynamics (MD) approach and Lattice Boltzmann method (LBM) have been used for simulating of nanofluids $[1-3]$ $[1-3]$, slip flows $[3, 4]$ $[3, 4]$ $[3, 4]$, nanotubes and microchannel [[5–](#page-13-1)[8\]](#page-13-2), porous media [[9,](#page-13-3) [10\]](#page-13-4); for example,

 \boxtimes Somaye Yaghoubi s.yaghoubi@pmc.iaun.ac.ir Karimipour et.al. studied the electric feld and microchannel type efects on H2O/Fe3O4 nanofuid dynamical manner using molecular dynamics [\[11\]](#page-13-5). Or they develop MD Method to Simulate the fow and thermal domains of H2O/ Cu nanofuid in a nanochannel afected [[12\]](#page-13-6). However, these approaches are not suitable for studying of mesostructured in some problems. As a result, dissipative particle dynamics (DPD) was frst introduced as a particle-based approach by Hoogerbrugge and Koelman for solving hydrodynamic problems [\[13](#page-13-7)]. This method is very similar to the MD approach with some diferences. MD method sometimes added a large number of unnecessary details to the solution [[13](#page-13-7), [14](#page-13-8)]. The equilibrium conditions in the fuctuation–dissipation theory were proposed by Español, and Warren in 1995 and 1997 and added to the DPD equations [\[15](#page-13-9), [16](#page-13-10)]. In the recent decades, DPD methods have been used for various applications such as hydrodynamic simulation of particles suspended in a liquid $[17-19]$ $[17-19]$ $[17-19]$, multiphase flow $[20-22]$ $[20-22]$, polymers $[23-26]$ $[23-26]$, droplets $[27-29]$ $[27-29]$ $[27-29]$ and biological systems $[30, 31]$ $[30, 31]$ $[30, 31]$ $[30, 31]$ $[30, 31]$, flow around spheres and cylinders $[32-35]$ $[32-35]$ $[32-35]$, microchannels,

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nanochannels and nanoparticles [[36–](#page-13-23)[39](#page-14-0)]. For example, Yaghoubi, et.al. introduced a new weight function in this method and used it to simulate fows with values close to the real Schmitt number [\[40\]](#page-14-1). In another study, they showed that using DPD particles with intrinsic size can help reduce the number of particles required for the simulation. This decrease in the number of particles in turn results in a more feasible simulation [[41\]](#page-14-2). The energy conservation model for simulation of one-dimensional conduction heat transfer was introduced by Ripoll et.al. and Ripoll and Español in 1998 and 2000. Along with satisfying the Fourier law, this model also correctly depicted the equilibrium variations [[42\]](#page-14-3). Dissipative particle dynamics method with energy conservation is known as eDPD. Qiao and He [\[35\]](#page-13-22) used eDPD for thermal conductivity of nanocomposites and heat transfer in nanofuids [[43,](#page-14-4) [44](#page-14-5)]. Borhani and Yaghoubi investigated the behaviors of diferent weight functions on the wall of a two-dimensional channel and presented the results as dimensionless velocity and temperature profles. They showed that weight function with higher dissipative viscosity is more suitable for accurate estimation of wall characteristics and the profles [\[45](#page-14-6)]. Abu-dana, used eDPD in order to apply various thermal boundary conditions in 2D heat transfer problems. These boundaries included constant temperature, constant heat fux and adiabatic conditions [[46\]](#page-14-7). He also simulated 2D thermal conductivity in another study and correctly modeled Derlikke and Neumann boundary conditions and compared them with the analytical results [\[47\]](#page-14-8). In 1999, Mackie et.al. used eDPD for two conduction and convection heat transfer scenarios [[48](#page-14-9)]. Yamada et.al. used eDPD for modeling of forced convection for constant temperature and constant heat fux boundary conditions. They then calculated the Nusselt number for both conditions and compared the results with the analytical solution. The error was estimated to be 2.3% compared with the analytical solution [\[49\]](#page-14-10).

One of the most important problem in this method is wall modeling and the efect of fuid particles entering the wall layers; therefore, due to the importance of the particle behavior near the wall, for the frst time, in the current study four frozen layers are used along with Bounce-Back refection and the efect of particle output on two and three-layers confgurations is investigated. In addition, integration algorithms have been used to solve the conservation equations and determining the location, velocity and temperature of each DPD particle. Also these integration algorithms have been used to determine the force and net heat fux. Furthermore, in this study, for the frst time, modifed velocity Verlet integration algorithm was improved by adding heat transfer equations with relaxation factor of λ = 0.65. In order to validate the created code, it is investigated whether initial DPD particles distribution affect the behavior of the particles and fnal results in equilibrium. Also, the results were compared with those obtained from CFD approach.

In this article, ["Theory](#page-3-0)" section presents the equations in DPD approach before explaining the hydrodynamic and thermal boundary conditions of the problem. Then, the integration and search algorithms used in this study are introduced. In the next section, the simulated results are presented, but at frst, the efect of initial regular and random confguration on the equilibrium is investigated. Then, the efect of particle's exits in dimensionless temperature and velocity profles in two and three-layers confgurations on the walls are presented and compare with CFD approach for validation. Also, the relative error values are reported. Finally, the effect of cell size in the averaging on the solution domain and time steps in the output results is investigated. The ["Results and discussion"](#page-6-0) section presents a fnal conclusion based on the results and discussions.

Theory

Governing equations

This section first introduces equations governing DPD before expanding these equations by assigning a temperature to each particle and presenting conservation (mass, momentum and energy) equations. The forces in DPD approach can be divided into three categories of conservative forces \vec{F}_{ij}^C ; Dissipative forces \vec{F}_{ij}^D ; and random forces \vec{F}_{ij}^R . These forces transfer between in pairs for each two particles interacting with each other. The DPD forces have a short natural range. This means that each particle can only interact with its neighboring particles. This neighborhood is determined using a distance called cutoff radius or r_c [\[49\]](#page-14-10):

$$
\vec{F}_{ij}^C = F_{ij}^C(r_{ij}) \vec{e}_{ij}
$$
 (1)

$$
\vec{F}_{ij}^{\text{D}} = -\gamma_{ij} w^{\text{D}}(r_{ij}) (\vec{e}_{ij} \cdot \vec{v}_{ij}) \vec{e}_{ij}
$$
 (2)

$$
\vec{F}_{ij}^{\text{R}} = \sigma_{ij} w^{\text{R}}(r_{ij}) \xi_{ij} \Delta t^{-1/2} \vec{e}_{ij}
$$
 (3)

In which $\vec{e}_{ij} = \vec{r}_{ij}/r_{ji}$ is the unit vector in the direction of the line connecting particles i and j. Furthermore, $\vec{r}_{ij} = \vec{r}_i - \vec{r}_j$ and r_{ij} is the length of vector \vec{r}_{ij} . F_{ij}^C is defined as weight function of conservative force; $w^D(r_{ij})$ and $w^R(r_{ij})$ are dissipative and random weight functions, respectively, and γ ⁱ and σ ⁱ are the domain of dissipative and random forces, respectively.

The dissipative force F^D is added for decrease in relative velocity between DPD particles. This force simulates the efects of viscosity in the system and is a function of relative velocity and distance between particles. The random or fuctuating force, *F*^R, is added to the system for creating the necessary degrees of freedom. In fact, when large granulation

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technique is used, the real degrees of freedom in the system decrease. As a result of decrease in degrees of freedom, Brownian motion is not observed in the particles. In order to compensate this efect, it is necessary to add a term for interaction forces. This force is the term which results in increase in system temperature with increase in particles' movement.

Based on fuctuation–dissipation theory, random and dissipative weight functions follow Eq. [4](#page-4-0). Furthermore, the domain of random and dissipative forces is defned according to Eq. [5](#page-4-1) [\[50](#page-14-11)]:

$$
\omega^{\mathcal{D}}(r) = \left(\omega^{\mathcal{R}}(r)\right)^2\tag{4}
$$

$$
\gamma = \frac{\sigma^2}{2k_\text{B}T} \tag{5}
$$

Equation [4](#page-4-0) is used in classic DPD, and Eq. [5](#page-4-1) presents the necessary condition for equilibrium between random and dissipative forces in order to reach an isothermal system. In the eDPD approach, each DPD particle is assigned a temperature along with a velocity value and DPD equations are revised.

$$
C_{\rm v} \frac{\mathrm{d}T_{\rm i}}{\mathrm{d}t} = q_{\rm i} \tag{9}
$$

In which, q_i is the heat flux between particle and C_v is the heat capacity of DPD particles at constant volume. Usually, the heat capacity of DPD particles is used in its dimensionless format or $\overline{C}_v = \frac{C_v}{k_B}$, where k_B is the Boltzmann constant. Therefore, heat flux q_i is defined as follows [[49\]](#page-14-10):

$$
q_{i} = \sum_{\substack{i \neq j \\ j=1}}^{N} q_{ij} = \sum_{\substack{i \neq j \\ j=1}}^{N} \left(q_{ij}^{\text{visc}} + q_{ij}^{\text{cond}} + q_{ij}^{\text{R}} \right)
$$
(10)

In which the frst term shows the viscose heat fux, the second term shows the convection heat fux and the last term belongs to random heat fux. It is worth mentioning that the sums used in conservation laws should be applied on particles located inside the cutoff radius r_C . Replacing these heat fux equations results in the following energy conservation equation:

$$
q_{i} = \sum_{\substack{i \neq j \\ j=1}}^{N} \left[\left(\frac{1}{2C_{v}} \left(w^{D} \left(r_{ij} \right) \left[r_{ij} \left(\vec{e}_{ij} \cdot \vec{v}_{ij} \right)^{2} - \frac{\sigma_{ij}^{2}}{m_{i}} \right] - \sigma_{ij} w^{R} \left(r_{ij} \right) \left(\vec{e}_{ij} \cdot \vec{v}_{ij} \right) \xi_{ij} \right) \right) + \left(\kappa_{ij} w^{R2} \left(r_{ij} \right) \left(\frac{1}{T_{i}} - \frac{1}{T_{j}} \right) \right) + \left(\alpha_{ij} w^{R} \left(r_{ij} \right) \Delta t^{-\frac{1}{2}} \xi_{ij}^{e} \right) \right] \tag{11}
$$

In the eDPD approach, conservation laws govern particles' movements. The movement equation for ith particle is defned as follows [[49](#page-14-10)]:

$$
\frac{\mathrm{d}\vec{r}_{i}}{\mathrm{d}t} = \vec{v}_{i} \tag{6}
$$

$$
m_i \frac{\mathrm{d}\vec{v}_i}{\mathrm{d}t} = \vec{f}_i = \vec{f}_i^{\text{int}} + \vec{f}_i^{\text{ext}} \tag{7}
$$

where \vec{r}_i and \vec{v}_i are the location and velocity of the ith particle; m_i is the mass of that and \vec{f}_i^{ext} is the applied external forces such as gravity, while \vec{f}^{int}_i is defined based on Eq. [8](#page-4-2) [\[49\]](#page-14-10):

$$
\vec{f}_{i}^{\text{int}} = \sum_{\substack{i \neq j \\ j=1}}^{N} \vec{F}_{ij} = \sum_{\substack{i \neq j \\ j=1}}^{N} \left(\vec{F}_{ij}^{C} + \vec{F}_{ij}^{D} + \vec{F}_{ij}^{R} \right)
$$
\n(8)

In this equation, N is the total number of particles, F_{ij} is the force applied to particle i by particle j, so that $\vec{F}_{ii} = \vec{F}_{ii}$ because all forces in this approach have the dual action–reaction nature. Therefore, the conservation of linear momentum is maintained. On the other hand, due to constant number of particles in the system during simulation, the conservation of mass is satisfed. In the eDPD approach, due to assignment of temperature T_i to particle i, it is necessary to satisfy energy conservation law. This can be written as Eq. [9](#page-4-3) [\[49\]](#page-14-10):

In which, $\kappa_{\rm ii}$ and $\alpha_{\rm ii}$ are the powers of convection and random heat fluxes, T_i and T_j are the temperature of particle i and j, respectively, and Δt is the time step. Furthermore, ξ_{ij}^e is a random number with mean of zero and variance of 1. Each pair of particles in the equilibrium results in a ξ_{ij}^e value so that $\xi_{ij}^e = -\xi_{ji}^e$. The negative sign is placed in order to satisfy energy conservation equations.

In the eDPD approach, due to addition of temperature to the particles, it is necessary to use particles' temperature instead of the T in Eq. [5.](#page-4-1) In fact, T in Eq. [5](#page-4-1) is the harmonic average of the temperature of i and j particles.

$$
T = \frac{2T_i T_j}{T_i + T_j} \tag{12}
$$

By replacing Eq. [12](#page-4-4) in 5, the modifed random dissipative equation for eDPD systems is created:

$$
\gamma = \frac{\sigma^2 (T_i + T_j)}{4k_\text{B} T_i T_j} \tag{13}
$$

where κ_{ij} and α_{ij} are the convection and random heat fluxes similar to Eq. [11](#page-4-5) and defned as follows [\[49](#page-14-10)]:

$$
\kappa_{ij} = \frac{C_v^2 k_0 (T_i + T_j)^2}{4k_B} \tag{14}
$$

$$
\alpha_{ij} = \sqrt{2k_B\kappa_{ij}}\tag{15}
$$

In this equation, k_B is the Boltzmann constant and k_0 is a control parameter of thermal conductivity of eDPD particles. The value of k_0 is empirical and is determined by comparing the results of this solution with the fnite volume method for the same geometry.

Furthermore, in this article, the classic weight function [\[51\]](#page-14-12) is used:

$$
\omega^{\mathcal{D}}(r) = (\omega^{\mathcal{R}}(r))^{2} = \begin{cases} 1 - \frac{r_{ij}}{r_{\mathcal{C}}} & r_{ij} \le r_{\mathcal{C}} \\ 0 & r_{ij} > r_{\mathcal{C}} \end{cases}
$$
(16)

Integration algorithms

Abu-dana [[52\]](#page-14-13) in 2014 expanded the Grout–Warren algorithm by adding energy conservation equations. According to this algorithm, the V-V used in eDPD approach is as follows:

Step 1:

$$
r_{i}(t + \Delta t) = r_{i}(t) + \Delta t v_{i}(t) + \frac{1}{2}(\Delta t)^{2} f_{i}(t)
$$

Step 2:

$$
\widetilde{v}_{i}(t + \Delta t) = v_{i}(t) + \lambda \Delta t f_{i}(t)
$$

$$
\widetilde{T}_i(t + \Delta t) = T_i(t) + \frac{\lambda}{C_v} \Delta t f_i(t)
$$

Step 3:

$$
f_{i}(t + \Delta t) = f_{i}(r_{i}(t + \Delta t), \widetilde{v}_{i}(t + \Delta t))
$$

$$
q_i(t + \Delta t) = q_i \bigg(r_i(t + \Delta t), \widetilde{v}_i(t + \Delta t), \widetilde{T}_i(t + \Delta t) \bigg)
$$

Step 4:

$$
v_{i}(t + \Delta t) = v_{i}(t) + \frac{1}{2}\Delta t (f_{i}(t) + f_{i}(t + \Delta t))
$$

$$
T_{\rm i}(t + \Delta t) = T_{\rm i}(t) + \frac{1}{2C_{\rm v}}\Delta t (q_{\rm i}(t) + q_{\rm i}(t + \Delta t))
$$
\n(17)

In the second step, along with estimated velocity, an estimated temperature is also assigned to the particles which is the result of temperature and heat fux calculated in the previous time step. In the third step, after updating the forces, the heat fux is also updated. The important part is the presence of estimated velocity $\tilde{v}_i(t + \Delta t)$ which is required in the viscose heat fux. In the fnal step, along with modifying the velocity, particles' temperatures are also updated. In the current article, for the frst time a combination of this algorithm with modifed velocity Verlet is used for modeling heat transfer using DPD approach with relaxation factor of λ =0.65. Choosing a larger value for λ means that the time steps can be larger. However, this value should be optimized because a very large relaxation factor results in high integration error. Although this algorithm does not satisfy time reversibility condition, but it is an accurate method for integration of conservation equations and has a reasonable calculation cost [[53](#page-14-14)].

Boundary conditions

The current study uses particle freezing and Bounce-Back refection approach together for modeling of channel walls. In this refection, it is assumed that after impacting the wall, the particles return to the fuid in the opposite direction of their initial path. This means that tangent and normal components of the momentum are inversed.

$$
(V_n)_{\text{After Collision}} = -V_n \tag{18}
$$

$$
(V_{\rm t})_{\rm After~Collision} = -V_{\rm t}
$$

In order to simulate constant temperature boundary condition, it is assumed that each particle with an initial temperature (diferent or equal to the wall temperature) adsorbs heat from the wall after impacting the wall with constant temperature and returns to the fuid.

The dimensionless temperature for temperature boundary condition on the wall is showed as follows:

$$
\theta = \frac{T - T_{\rm w}}{T_{\rm m} - T_{\rm w}}\tag{19}
$$

In which, T_w is the wall temperature, T is the local fluid temperature and T_m is the mean temperature in the fluid cross section. It is worth noting that the dimensionless temperature gradient is not dependent on x, therefore, thermal expansion condition of this parameter is shown as $\frac{\partial \theta}{\partial x} = 0$.

The mean temperature is calculated using the following equation:

$$
T_{\rm m} - T_{\rm w} = \frac{f_{\rm Ac} (T - T_{\rm w}) u \rm{d}z}{f_{\rm Ac} u \rm{d}z} \tag{20}
$$

In some cases, it is necessary to calculate fuid properties in infnite domains. Using periodic boundary condition, the calculation domain will gain a periodic nature and the repeat of the images in this domain continues to infnity. The mechanism of this boundary means that around the solution domain, images of the solution are created in various directions and these images fll the entire calculation space.

This is despite the fact that the problem is only solved for that range and its answers are generalized to infnity. The periodic boundary condition is a general condition: in the current article, since in the channel, z direction is perpendicular to the fow direction, periodic boundary condition is applied in the x direction. Therefore, particles exit the solution domain and reenter the domain with similar velocity and position. For the isotherm temperature boundary condition, the dimensionless temperature should be determined using

$$
(x, z) = \theta(x + l, z) = \theta(x + 2l, z) = \dots
$$
 (21)

the following equation [[49\]](#page-14-10):

Therefore, temperature feld in the fully developed region repeats at certain time intervals. The diferentiating the dimensionless temperature diference versus the time results in the following equation:

$$
\frac{\mathrm{d}\theta}{\mathrm{d}t} = \frac{1}{T_{\rm m} - T_{\rm w}} \frac{\mathrm{d}T}{\mathrm{d}t} + \frac{T - T_{\rm w}}{\left(T_{\rm m} - T_{\rm w}\right)^2} \frac{\mathrm{d}T_{\rm m}}{\mathrm{d}t} \tag{22}
$$

In the fully developed flow regime, $\frac{dT_m}{dt} = 0$ and therefore:

$$
\frac{dT}{dt} = (T_m - T_w) \frac{d\theta}{dt}
$$
\n(23)

By combining Eqs. (23) (23) (23) and (7) , the modified equation for constant temperature condition of the wall is as follows [\[49\]](#page-14-10):

$$
\frac{\mathrm{d}\theta}{\mathrm{d}t} = \frac{1}{C_{\rm v}(T_{\rm m} - T_{\rm w})} \left(q_{\rm ij}^{\rm visc} + q_{\rm ij}^{\rm cond} + q_{\rm ij}^{\rm R} \right) \tag{24}
$$

Search Algorithms

Since forces and heat fluxes have short-range nature in eDPD, it is possible to signifcantly reduce the calculation volume necessary for searching for particles. Therefore, in order to improve the calculations, it is necessary to present a rapid solution. The general idea behind search algorithms is that in order to calculate the force and heat fux of a particle, it is sufficient to investigate its neighboring particles in a cutoff radius r_C in each step. This means that in search algorithms, the goal is fnding particles present in this neighborhood. In this article, a cellular search algorithm is used. In this algorithm, the domain is divided into $M \times M \times M$ cells. The dimension of each cell in this network is defned as $l = L/M$ and should be determined so that it is slightly larger than the cutoff radius. Therefore, it can be said that particles in each cell can only interact with the particles in the same cell and its neighboring cells and it is not necessary to investigate other cells. This results in a signifcant reduction in calculation volume. In fact, in each search, it is only

Fig. 1 The schematic of the investigate geometry

Table 1 eDPD parameters

| ρ | a_{ii} | ₹ ïi | $\mathcal{L}_{\mathcal{C}}$ | U 17 | |
|----------------|----------|------|-----------------------------|---------------------|-----------------------|
| $\overline{4}$ | 18.75 | | 1.0 | 1.0×10^{5} | 1.26×10^{-4} |

necessary to calculate the cells on the right and top and add them to the reaction values previously measured for cells on the left and bottom.

Forced Convection Heat Transfer Problems

Figure [1](#page-6-2) shows a schematic of laminar flow in this channel. As can be seen, the fow enters the channel with the force of $F_{\text{ext}} = 0.01$ which satisfies all constant temperature conditions in the walls. The channel's dimensions in the x and z directions are $L=5$ and a=40 and the walls have no-slip condition. Furthermore, the fow has fully developed conditions. The temperature condition on the wall is constant temperature in which the temperature values of upper and lower walls are equal to each other and $Tw = 1$. in DPD unit. The temperature of input fuid is equal to 1.0 in DPD unit.

The constant parameters used in eDPD solution are pre-sented in Table [1](#page-6-3) [[49\]](#page-14-10):

Results and discussion

Investigating regular and random confgurations

In order to ensure the correctitude of the computer coding, regular and random confgurations were investigated in the frst step. In ordered confguration, fuid particles are placed in ordered layers in the channel at the beginning of the simulation. These particles have the temperature of 1.0 in DPD unit and random velocities with Gaussian distribution.

In the random confguration, particles are placed randomly at the initial time. For the velocity and temperature, the values are the same as in ordered case. These two confgurations at the initial time are shown in Fig. [2](#page-7-0).

Fig. 3 Effect of initial configuration on dimensionless velocity profile in equilibrium

Figures [3](#page-7-1) and [4](#page-7-2) show the velocity and the dimensionless temperature diference profle for regular and random initial distribution. As can be seen, the initial configuration has no efects on velocity and temperature profles in equilibrium. In fact, due to the presence of interacting forces and heat fux between eDPD particles in the initial time step, ordered confguration is destroyed and eDPD particles move to the positions determined through movement equations.

Fig. 4 Effect of initial configuration on dimensionless temperature profle in equilibrium

The number of frozen layers required for wall modeling

In this section, the efect of number of frozen layers on the wall modeling is investigated. In order to prevent the particles from escaping the solution domain, wall particle freezing is used. This adds the wall's characteristics to the calculation domain while also acting as a barrier preventing the exiting of fuid particles. Furthermore, the density of the wall and fuid is a constant number presented in Table [1](#page-6-3). The reason for using equal density in these two environments is

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to prevent density variations near the wall. When the fuid particles reach wall particles in their cutoff radius, they will be affected by an imaginary gravitational force in case of diference in density which results in increased error. In previous studies [[54–](#page-14-15)[56\]](#page-14-16), two frozen layers are used to this end. Using the same approach, the geometry investigated in the current study also initially used two frozen layers as the wall. However, after reaching the steady state and evaluating the velocity profles, unacceptable variations were observed near the wall. A more careful investigation of the wall showed that fuid particles have exited the channel as shown in Fig. [5.](#page-8-0) The three-layer confguration is also shown in this fg.

The velocity profiles for two and three-layers configuration wall are presented in Fig. [6.](#page-8-1) As can be seen, in the fuid area and specially in the estimation of maximum velocity at the center of the channel, both confgurations show acceptable compatibility with CFD solution results. The estimated maximum velocity in two and three-layer confgurations was equal to V_{2L} =1.480 and V_{3L} =1.491, which, when compared to the CFD value of $V_{\text{CFD}} = 1.50$, show 1.3% and 0.6% error, respectively. However, close to the wall, due to the diffusion of fuid particles into the solid area, a large amount of variation is observed. These variations are not dependent on the number of difused particles and instead depend on the velocity of these particles. These variations have resulted in an error of 40% for two-layer confguration and an error of 14.6% for the three-layer confguration in the dimensionless velocity profle.

Fig. 5 Particles exiting through the wall in **a.** two-layer confguration **b.** three-layer configuration

Fig. 6 The velocity profle in diferent wall confgurations

The dimensionless temperature diference profles for diferent number of frozen layers are shown in Fig. [7](#page-8-2). As can be seen, difusion of particles in the wall has resulted in signifcant variations in the temperature profle. These variations are higher in two-layer confguration such that this confguration has an error of 33.7% on the upper wall.

Based on the observations in this section, a four-layer wall confguration was used which presents in more accurate results compared to two and three-layer confgurations has smaller amount of particle difusion. The result of this simulation is presented in the following sections.

Fig. 7 The dimensionless temperature diference profle at diferent wall configurations

Fig. 8 The effect of time step on the velocity profile

Fig. 9 The effect of time step on dimensionless temperature difference profle (constant wall temperature)c

Efect of time step

One of the most important factors in numerical calculations is the selection of proper time steps. Time steps should be selected in a way that maintain the details of the calculations while preventing unfeasibly large calculation costs. A large time step results in rounding error, while small time steps increase the number of repeated calculations and therefore the time necessary for reaching steady state. In the current study, the effect of time steps on output results was investigated. To this end, four different time steps of $dt = 0.1$; 0.05; 0.005 and 0.0005 s were selected.

Figures [8](#page-9-0) and [9](#page-9-1) show the velocity and temperature profiles. At $dt = 0.1$ s; due to the large time step, a large number of particles have difused through the wall and the problem has failed to correctly simulate the interactions between the fuid and the walls. This system is also unable to estimate mean velocity and maximum velocity profles which has resulted in divergence of the problem. In the time step of $dt = 0.05$ s, similar to the previous time step, a large number of particles have difused through the wall and the velocity profle still shows a signifcant diference with CFD results. The maximum velocity in this case as an error of 20% compared to CFD results and in the dimensionless temperature graph for constant temperature boundary condition, a constant error of 9.5% is observed. Therefore, it is necessary to reduce the time step. At $dt = 0.005$ s, the highest errors in velocity profle near the wall and at the center of channel are 2.8% and 0.66%, respectively, while dimensionless temperature profle has an error of 1.1% and 0.31% at these two positions. Now, it is necessary to determine whether further decrease in time step can lead to signifcant improvement in the results.

At $dt = 0.0005$ s, the velocity and temperature profiles are almost identical to the results obtained at $dt = 0.005$ s, while the variations near the walls are more severe. This is due to the increase in discretization errors which is one of the disadvantages of this time step along with increased calculation cost. Furthermore, no signifcant improvement in the results is observed compared to the time step of $dt = 0.005$ s, while the calculation cost in $dt = 0.0005$ is ten times higher than the calculation cost at $dt = 0.005$. This can be a significant disadvantage in large systems with a large number of particles. Based on these results, the optimum time step for this study was selected to be $dt = 0.005$ s.

Efect of number of cells in the vertical direction of the channel

In the eDPD approach, statistical averaging is used for the output results. To this end, the solution domain is divided into cells in the desired direction and the hydrodynamic and thermal properties are investigated separately in each cell. Each rectangular division is known as a single cell, and its side is called rbin. Each cell has an average velocity and temperature which are the average of the velocity and temperature for all the particles in that cell. Therefore, the number of cells can be an important parameter for increasing the accuracy of the calculations.

Due to the statistical nature of the eDPD method, the selection of rbin can greatly afect the fnal answers and their accuracy, which is evident in Figs. [10](#page-10-0) and [11](#page-11-0). The number of particles in a small rbin is low and during an averaging, one

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Fig. 10 The efect of averaging cell size in the z-axis direction on velocity profle **a.** rbin=1.0, **b.** rbin=0.5, **c.** rbin=0.05, **d.** rbin=0.005 and **e.** rbin=0.005

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Fig. 11 The efect of averaging cell size in the z-axis direction on dimensionless temperature profle **a.** rbin=1.0, **b.** rbin=0.5, **c.** rbin=0.05, **d.** rbin=0.005 and **e.** rbin=0.005

or two particles may be removed from the cell which has a great efect on statistical calculations and then the averaging operation may encounter an error. It is even possible to fnd a cell that does not contain any particles. In addition, in small rbin, due to the expensive calculations, achieving equilibrium is time consuming. In large rbin, the results are not accurate enough due to the large number of particles in each cell. Then, it is important to fnd an optimal size for the rbins.

Figures [10](#page-10-0) and [11](#page-11-0) show the efect of the number of cells in the direction of z-axis for velocity profle and dimensionless temperature profle. As can be seen, for rbin=1.0, the velocity profle has a correct shape but is jagged. Furthermore, this profile fails to present sufficient details near the wall. For better understanding of this fact, consider a cell near the wall. Due to the large size of the cell, a large number of wall particles are placed in a single cell and their zero velocity signifcantly afects the average fuid velocity in that cell. Since all particles, including wall and fuid particles, in a single cell are recognized by their average velocity, this means that not only the average velocity is lower than the actual value but that an average velocity is also assigned to the wall particles, both of which result in error close to the wall. This problem is also observed in the dimensionless temperature profles. In the velocity graph, the maximum velocity at the center of channel is estimated to be 1.46 which has 2.7% error compared to CFD results. This error has been somewhat fixed for $rbin=0.5$, resulting in an error of 1.3%. However, the problem close to the wall is still observed in this case, resulting in a calculated velocity of 0.1 for wall particles. Similar to the velocity, temperature graphs also have a jagged look, while both graphs estimate correct maximum values.

For $r \text{bin} = 0.05$, the velocity profile has an error of less than 0.05% at its maximum, while the velocity close to the walls is also correctly predicted. Smaller rbin values result in smaller errors but reducing the cell size to $rbin=0.005$ and $rbin = 0.0005$ results in variations in the graph. This is due to the fact that the statistical calculation used results in a set amount of error for each cell. If the number of cells exceeds an optimum amount, this error increases and results in variation in graphs and undesirable results. Furthermore, the size of cells should be selected in a way that at least one eDPD particle is present in each cell. If the cells are smaller than a certain amount, it will be possible for cells with no particles to exist, resulting in zero velocity and temperature values and calculation error.

Conclusions

In this paper, a two-dimensional channel with parallel planes was investigated using the EDPD method. By considering a temperature for each DPD particle, the heat fux term of energy conservation equations was correctly simulated. For this goal, a Fortran programming code was developed. Also, for the frst time in this method, modifed velocity Verlet integration algorithm was improved by adding heat transfer equations with relaxation factor of λ = 0.65. In order to properly simulate the wall confgurations and preventing the exit of fuid particles from the solution domain and implementation of no-slip conditions on the walls, for the frst time a four-layer frozen particle confguration and bounce-back refection rules were employed. Also two approaches were considered to validate the written code: frst, the efect of regular and random confgurations was investigated in the frst step and the results indicated that the type of confguration has no efects on the stability and the output results. Second, the results of this method in two and three-layer wall arrangement were compared with the results of CFD approach, which had an acceptable consistency in estimating the maximum values in the dimensionless velocity and temperature profles. In the next step, the efect of time steps on the output results and stability was investigated. The size of time step had a direct efect on the output results, with particles difusing into the wall at large time steps resulting in divergent solution, while small time steps resulted in variations in the velocity and temperature profle graphs due to the increase in calculation error. Therefore, the optimum time step of $dt = 0.005$ s was selected. Finally, the effect of averaging cell sizes in the x-axis direction of the channel was investigated. If the number of averaging cells was smaller than a certain value, details of the solution were lost. Signifcant increase in the number of averaging cells resulted in variations in the entire graphs which were more signifcant near the walls. Based on these results, the width of averaging cell was set to rbin $= 0.05$.

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