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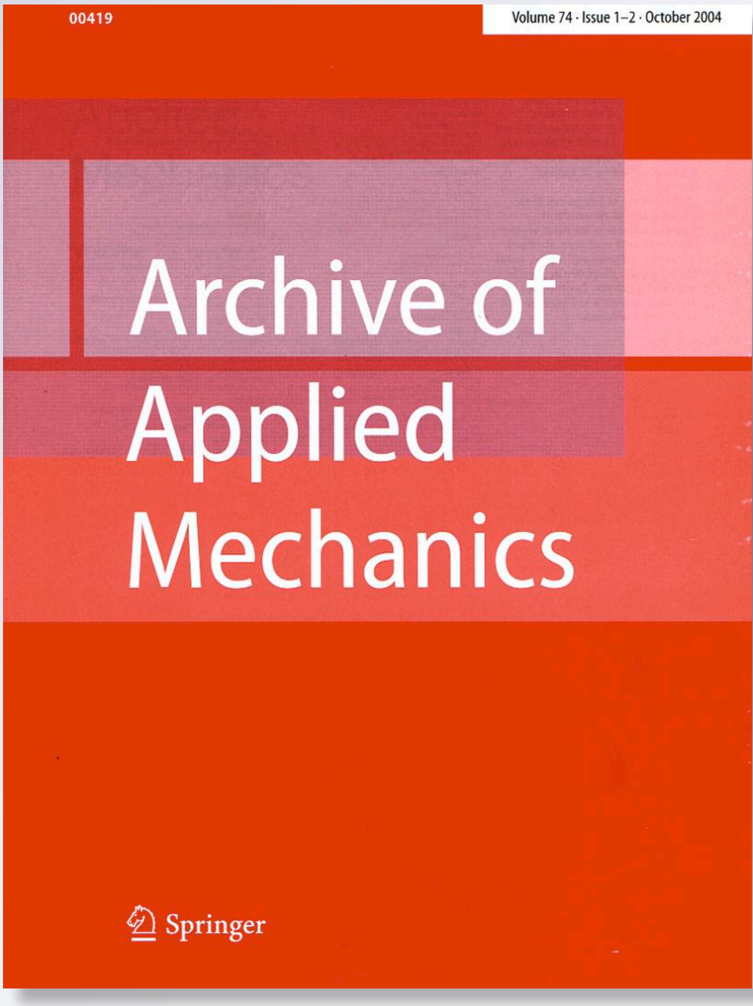

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ORIGINAL

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Abstract In this paper, the nonlinear vibration of a thin circular functionally graded material plates is studied. The plate thickness is constant, and the material properties of the plate are assumed to vary continuously through the thickness. The governing equations and boundary conditions are extracted. The assumed-time-mode method is used to analyze these equations. The time variable is eliminated by assuming a harmonic response for nonlinear vibration and using Kantorovich time averaging technique. Utilizing shooting and Runge–Kutta methods, the set of first-order nonlinear differential equations are solved. The effect of volume fraction index in free and forced vibration response and jump phenomenon is studied. The results show that jump phenomenon occur according to volume fraction index and uniform temperature in the special frequencies of forced vibration response.

Keywords Jump phenomenon · Nonlinear vibration · Functionally graded plates

1 Introduction

When structure elements have large deflection, a significant geometrical nonlinearity is induced. While the plate deflection is of the same order of its thickness, the membrane effect is considerable in analyzing plate behavior. Metals have been used in the engineering field for many years because of their excellent strength and toughness. However, the strength of a metal is reduced after it has been in a high-temperature environment for a period of time. To resist this phenomenon, the surfaces of metals are usually coated with a heat-proof material. In recent years, functionally graded materials (FGMs) were introduced for high-temperature environment. FGMs are inhomogeneous composite materials that are made from ceramics and metals, which have both the heat resistance of ceramics and the toughness of metals. FGM properties are continually varied from one point to another; in fact, the surface of plate in high-temperature is ceramic, and the other surface is metal. This is achieved by gradually varying volume fraction of constituent materials. FGMs have different applications, especially in aircrafts, space vehicles, automobile, defense industries, electronics, biomedical engineering and many engineering structures [1–7].

Several studies on vibration and control of functionally graded plates are available in the literature. Praveen and Reddy [1] conducted the nonlinear transient response of functionally graded plates subjected to lateral dynamic loading using finite element method. Yang and Shen [7] investigated the vibration characteristics and transient response of shear-deformable FGM plates made of temperature-dependent materials in thermal environments. Differential quadratic technique and Galerkian approach were used to determine the transient

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response of the plate subjected to lateral dynamic loads. Huang and Sandman [5] and Huang and Al-Khatat [8] investigated the nonlinear free and forced vibrations of circular plates by solving a two-point boundary value problem. Sobhani Aragh and Yas [9] investigated on elasticity solution for free vibrations analysis of functionally graded fiber orientation and volume fraction cylindrical panel using differential quadrature method. They showed that the normalized natural frequency of the functionally graded fiber orientation cylindrical panel is smaller than that of a discrete laminate composite panel and close to that of a 4 layer. Woo and Meguid [10] studied the nonlinear analysis of functionally graded plates and shells. They provided an analytical solution based on Fourier series for large deflection of plates and shells subjected to mechanical load and temperature field. Dong [11] investigated on three-dimensional free vibration of functionally graded annular plates with different boundary conditions using the Chebyshev–Ritz method. He showed that for various material gradient indices, the eigenfrequency increases with the increase in annular plate thickness. For functionally annular plates with the same thickness, the eigenfrequency gradually approaches to the constant value with the increase in the material gradient index. Huang and Shen [12] discussed the nonlinear vibration and dynamic response of functionally graded rectangular plates in a thermal environment using shear deformation theory. They assumed that the thermal field has uniform distribution throughout the plate surface, although it varies through thickness. The material properties are temperature-dependent and vary along thickness. The results revealed that the temperature field and volume fraction index have significant effect on the nonlinear vibration and dynamic response of the simply supported rectangular plate with no in-plane displacements. Kitipornchai et al. [13] studied the random vibration of functionally graded rectangular laminates in a temperature field. They used the third-order shear deformation plate theory.

Allahverdizadeh et al. [14] developed a semi-analytical approach for nonlinear free and forced axisymmetric vibration of a thin circular functionally graded plate. They show that the free vibration frequencies are dependent on vibration amplitudes and that the volume fraction index has a significant influence on the nonlinear response characteristics of the plate. In this paper, the jump phenomenon is analyzed in the nonlinear vibration of a thin circular functionally graded plates. For harmonic vibration, the time variable is eliminated by employing Kantorovich averaging method. The shooting and Runge–Kutta integration methods utilized to numerically solve the nonlinear boundary value problem at uniform temperature.

2 Material properties

Consider a circular FGM plate of constant thickness h and radius a made of ceramic and metal in cylindrical coordinates r, θ and z located in its initially undeformed configuration. Suppose that the upper surface ($z = \frac{1}{2}h$) is pure metal. Material properties P of the functionally graded plate are assumed to vary along thickness of the plate. These properties can be expressed with the following equation

$$P = P_c V_c + P_m V_m \tag{1}$$

where c and m refer to ceramic and metal, respectively. V_c and V_m are the volume fraction of the ceramic and metal that relate together as:

$$V_c + V_m = 1 \tag{2}$$

P_c and P_m are the temperature-dependent properties of lower and upper surfaces, respectively, and may be represented as a function of temperature,

$$P = P_0(P_{-1}T^{-1} + 1 + P_1 T + P_2 T^2 + P_3 T^3) \tag{3}$$

where P_{-1}, P_0, P_1, P_2 and P_3 are temperature coefficients. Temperature-dependent properties of Si_3N_4(Silicon Nitride) and *SUS304* (Stainless Steel) are [7,15]:

$$Si_3N_4: \quad E_c = 348.43\left(1 - 3.070e - 4T + 2.160e - 7T^2 - 8.946e - 11T^3\right) \quad (Gpa) \tag{4a}$$
$$\rho_c = 2370 \quad \left(\mathrm{kg/m^3}\right)$$

$$SUS304: \quad E_m = 201.04\left(1 + 3.079e - 4T - 6.534e - 7T^2\right) \quad (Gpa) \tag{4b}$$
$$\rho_m = 8166 \quad \left(\mathrm{kg/m^3}\right)$$

For a plate with a uniform thickness h and a reference surface at its middle plane, the volume fraction V_c can be written as

$$V_c(z) = \left(\frac{z}{h} + \frac{1}{2}\right)^n \tag{5}$$

The volume fraction index n dictates the material variation profile across the plate thickness. It is assumed that the effective Young's modulus E is temperature-dependent, where the mass density ρ is independent of the temperature. Poisson's ratio υ is assumed constant. From Eqs. (1) to (5), we have [3,16] :

$$P(z) = (P_c - P_m)\left(\frac{z}{h} + \frac{1}{2}\right)^n + P_c \tag{6}$$

Figure 1 illustrates the variation of dimensionless Young's modulus $E(z)/E_m$ through dimensionless plate thickness z/h with different values of volume fraction index n and different uniform temperature. It is obvious from Eq. (5) that, $n = 0$ indicates pure ceramic plate, and by increasing n, the volume fraction of metal increases. Therefore, for large amounts of n, the significant part of plate thickness is metal. It is observed that in a same z/h, the dimensionless Young's modulus decreases by increasing n and uniform temperature. Figure 2 illustrates the variation of dimensionless mass density $\rho(z)/\rho_m$ with the dimensionless thickness. By increasing n at the same z/h, $\rho(z)/\rho_m$ increases, and also at the same n, by increasing z/h, the amount of $\rho(z)/\rho_m$ decreases.

3 Equilibrium equations

Using Kirchhoff plate theory, the strains at any level z from the neutral plane are:

$$\varepsilon_r = u_{,r} + \frac{1}{2}(w_{,r})^2 - zw_{,rr} \tag{7}$$

$$\varepsilon_\theta = \frac{u}{r} - \frac{zw_{,r}}{r} \tag{8}$$

where ε_r and ε_θ are the normal strains along the r and θ directions. In terms of Hooke's law, the radial and circumferential stresses are given by:

$$\sigma_r = \frac{E(z,T)}{1-\nu^2}(\varepsilon_r + \nu\varepsilon_\theta) \tag{9}$$

$$\sigma_\theta = \frac{E(z,T)}{1-\nu^2}(\varepsilon_\theta + \nu\varepsilon_r) \tag{10}$$

$$E(z,T) = E_{cm}(T)\left[\frac{2z+h}{2h}\right]^n + E_m(T) \tag{11}$$

The equilibrium equation for forces in the radial direction, neglecting the longitudinal inertia, is as:

$$N_{r,r} + \frac{N_r - N_\theta}{r} = 0 \tag{12}$$

Since the principal vibrations take place in the direction perpendicular to the middle plane, it is reasonable to neglect the longitudinal inertia. The membrane forces N_r and N_θ are obtained from

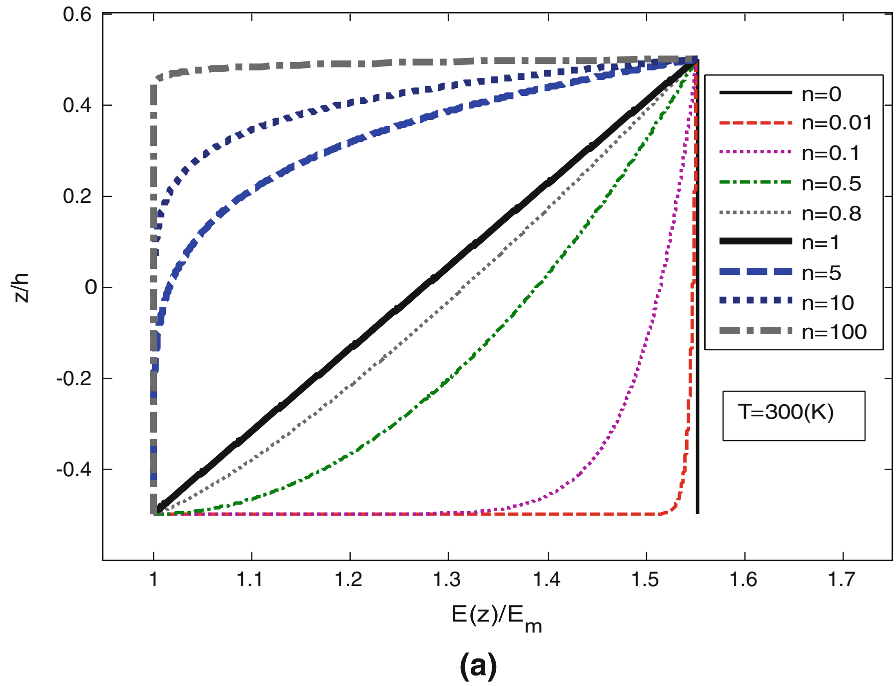
$$(N_r, N_\theta) = \int_{-\frac{h}{2}}^{\frac{h}{2}} (\sigma_r, \sigma_\theta)\,\mathrm{d}z \tag{13}$$

where

$$N_r = \frac{h}{1-\nu^2}\left\{A\left[u_{,r} + \frac{1}{2}(w_{,r})^2 + \nu\frac{u}{r}\right] - Bh\left[w_{,rr} + \nu\frac{1}{r}w_{,r}\right]\right\} \tag{14a}$$

$$N_\theta = \frac{h}{1-\nu^2}\left\{A\left[\frac{u}{r} + \nu(u_{,r} + \frac{1}{2}(w_{,r})^2)\right] - Bh\left[\frac{1}{r}w_{,r} + \nu w_{,rr}\right]\right\} \tag{14b}$$

and A and B are as follows,



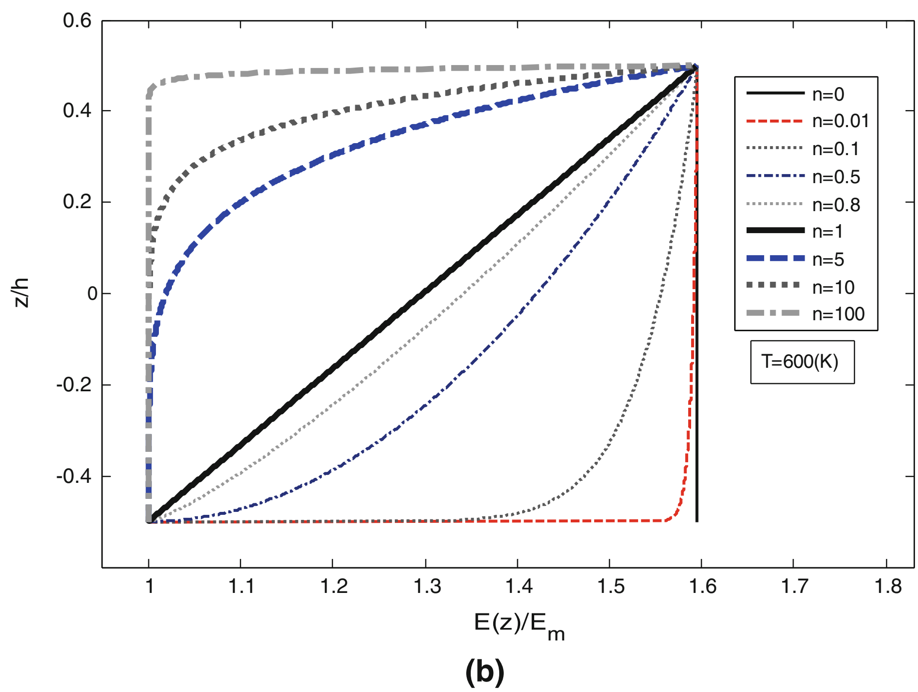


Fig. 1 Variation of the dimensionless Young modulus, dimensionless plate thickness with volume fraction index n : **a** $T = 300$ K, **b** $T = 600$ K

$$A = E_m + \frac{E_{cm}}{n+1}, \quad B = \frac{nE_{cm}}{2(n+1)(n+2)}, \quad E_{cm} = E_c - E_m \tag{15}$$

By eliminating the radial displacement function $u(r, t)$ from Eqs. (14a) to (14b), the compatibility equation is obtained using equation (12).

$$(N_r + N_\theta)_{,r} = \frac{-hA}{2r}(w_{,r})^2 \tag{16}$$

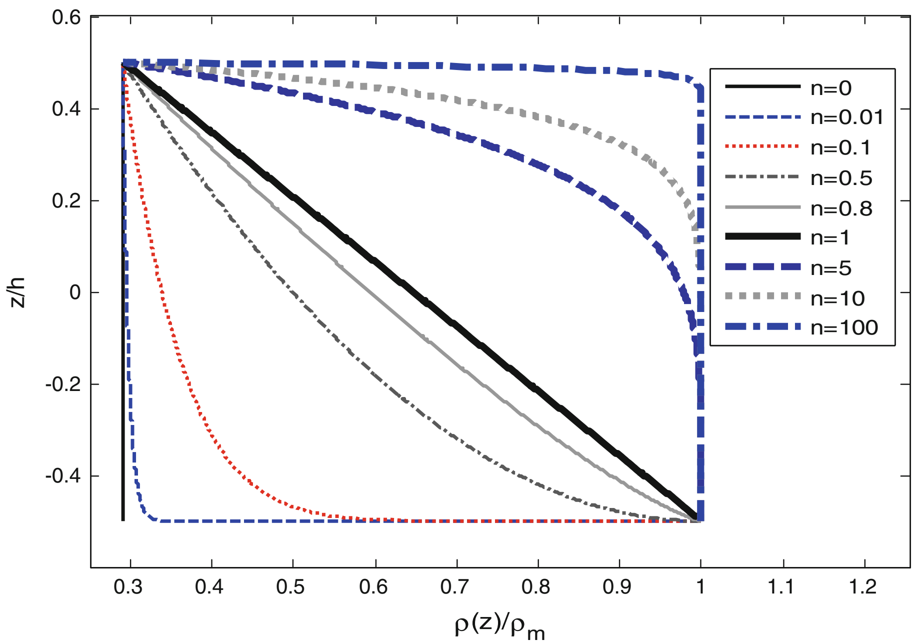


Fig. 2 Variation of the dimensionless mass density according to dimensionless plate thickness

By introducing stress function $\psi\ (r, t)$, and the corresponding relations below:

$$N_r = \frac{\psi}{r}, \quad N_\theta = \psi_{,\,r} \tag{17}$$

into Eq. (16), we obtain

$$\psi_{,\,rr} + \frac{1}{r}\psi_{,\,r} - \frac{\psi}{r^2} = \frac{-hA}{2r}\left(w_{,\,r}\right)^2 \tag{18}$$

The equation of moment equilibrium about a circumferential tangent is

$$M_{r,r} + \frac{M_r - M_\theta}{r} = Q_r \tag{19}$$

where Q_r is the shearing force per unit length and

$$\left(M_r, M_\theta\right) = \int_{-\frac{h}{2}}^{\frac{h}{2}} \left(\sigma_r, \sigma_\theta\right) z\mathrm{d}z \tag{20}$$

and

$$M_r = \frac{h^3}{1-\nu^2}\left\{-R\left[w_{,\,rr} + \nu\frac{1}{r}w_{,\,r}\right] + \frac{B}{h}\left[u_{,\,r} + \frac{1}{2}\left(w_{,\,r}\right)^2 + \nu\frac{u}{r}\right]\right\} \tag{21}$$

$$M_\theta = \frac{h^3}{1-\nu^2}\left\{-R\left[\frac{1}{r}w_{,r} + \nu w_{,rr}\right] + \frac{B}{h}\left[\frac{u}{r} + \nu\left(u_{,r} + \frac{1}{2}\left(w_{,r}\right)^2\right)\right]\right\} \tag{22}$$

$$R = \frac{E_m}{12} + \frac{\left(n^2+n+2\right)}{4\left(n+1\right)\left(n+2\right)\left(n+3\right)}E_{cm} \tag{23}$$

By applying d'Alembert's principle, dynamic equilibrium of the transverse forces which act on an annular element requires that

$$\left(rQ_r\right)_{,\,r} + \left(rN_r w_{,\,r}\right)_{,\,r} + rP\left(r, t\right) = h\left[\frac{\rho_{cm}}{n+1} + \rho_m\right] r w_{,\,tt} \tag{24}$$

where $P(r,t)$ is the lateral loading intensity. By combining Eqs. (19)–(24), one may obtain the following relation as:

$$-\left[\frac{h^3B^2}{A\left(1-\nu^2\right)}-\frac{h^3R}{1-\nu^2}\right]\nabla^4 w-\frac{1}{r}\left(\psi w_{,r}\right)_{,r}+h\left[\frac{\rho_{cm}}{n+1}+\rho_m\right]w_{,tt}$$
$$-\frac{h^2B}{1-\nu^2}\left[\frac{r}{hA}\psi_{,rrrr}+\frac{5-\nu}{hA}\psi_{,rrr}+\frac{3-2\nu}{rhA}\psi_{,rr}+\frac{\nu}{r^2hA}\psi_{,r}-\frac{\nu}{r^3hA}\psi\right]$$
$$-\frac{h^2B}{1-\nu^2}\left[w_{,r}w_{,rrr}+\left(w_{,rr}\right)^2+\frac{2-\nu}{r}w_{,r}w_{,rr}\right]=P(r,t) \quad (25)$$

Eqs. (18) and (25) are the dynamic forms of von-Karman's equations, where the longitudinal and rotary inertias are neglected. Together, they govern the finite amplitude axisymetric vibration of a thin annular or circular plate. The following dimensionless variables are used to obtain the nondimensional form of governing equations.

$$r^*=\frac{r}{a},\quad h^*=\frac{h}{a},u^*=\frac{u}{a},\quad w^*=\frac{w}{a},\phi=\frac{\psi}{a^2E_m},\tau=\frac{1}{a}\sqrt{\frac{E_m}{\rho_m}}t \quad (26)$$
$$\Omega^*=a\sqrt{\frac{\rho_m}{E_m}}\Omega, q\left(r^*,\tau\right)=\frac{P(r,t)}{E_m},\quad A^*=\frac{A}{E_m},B^*=\frac{B}{E_m},R^*=\frac{R}{E_m}$$

4 Boundary conditions

In order to complete the formulation of the problem, the governing Eqs. (18) and (25) should accompany a set of boundary conditions. The boundary conditions for $w^*(r^*,\tau)$ depend upon the degree of transverse constraints, and those for $\phi(r^*,\tau)$ depend on the degree of radial constraint. These conditions at the inner and outer edges should be added for all dimensionless time τ. For a circular plate with a clamped immovable edge, the boundary conditions at the center and outer edges ($r=1$) are, respectively, as follow:

$$w_{,r}=0, u=0 \quad (27)$$

and

$$w=0, w_{,r}=0, u=0 \quad (28)$$

5 Solution of the governing equations

So far, an exact solution of the differential equations (18) and (25), which satisfies the boundary conditions of the form (27) and (28), is unknown. The standard Fourier analysis used in linear vibration problems cannot be applied in an exact sense due to the nonlinear character of the differential equations, which causes a coupling of vibration modes. Consequently, the analysis (solution) of the problem should be accomplished in some approximate manner. In most of the studies carried out at large vibration amplitudes of circular plates, the separation of variables and the function-space techniques have been used to eliminate the assumed-mode from the equations. As a result, the problem has been reduced to a nonlinear ordinary differential equation. In this equation, the time t is the independent variable and the approach is named as assumed space-time solution. To find the solution of Eqs. (18) and (25) with boundary conditions (27) and (28), the averaging procedure has been used. By applying the Kantorovich averaging technique, the time variable is eliminated and the dynamical governing equations are reduced to a nonlinear eigenvalue problem [8,17,18].

The first simplification is imposed by taking the time varying loading intensity to be of the form [5],

$$q\left(r^*,\tau\right)=Q\left(r^*\right)\sin\Omega\tau \quad (29)$$

where the plate is being subjected to an uniformly distributed sinusoidal loading, and it is assumed that the steady-state response can be closely approximate by the expression:

$$w\left(r^*,\tau\right)=G\left(r^*\right)\sin\Omega\tau \quad (30)$$
$$\phi\left(r^*,\tau\right)=F\left(r^*\right)(\sin\Omega\tau)^2 \quad (31)$$

where $F(r)$ and $G(r)$ are undetermined shape functions of the vibration.

The governing equations are:

$$\begin{cases} F_{,rr} + \frac{1}{r}F_{,r} - \frac{F}{r^2} = -\frac{hA}{2r}\left(G_{,r}\right)^2 \\ -\left[\frac{h^3B^2}{A(1-\nu^2)} - \frac{h^3R}{1-\nu^2}\right]\nabla^4 G - \frac{3}{4r}\left(FG_{,r}\right)_{,r} - h\left[\frac{\rho_{cm}}{n+1}+1\right]\Omega^2 G - \frac{h^2B}{1-\nu^2}\left(G_{,rr}\right) = Q\left(r\right) \end{cases} \tag{32}$$

The sign "*" is eliminated in the above equations, and at the special case, Eq. (32) are obtained in accordance with [19]. The governing equations (32) along with the set of boundary conditions form a nonlinear two-point boundary value problem, which describes the harmonic response of a circular plate undergoing finite amplitude oscillations.

Shooting method is used to solve the set of first-order nonlinear differential equations with the associated boundary conditions. In order to do so, the Runge–Kutta and Newton–Raphson methods are utilized [20,21]. Typically, in typical boundary value problems, the known values are given at the boundaries. To solve an nth order differential equation by Runge–Kutta method, we should have n known values at the same point, but they are distributed on the boundaries. Suppose that we have k known values on the first boundary, thus $l = n - k$ values are known on other boundaries. Consequently, the known values on the first boundary are: $c_1, \ldots, c_k$ and the known values at the other boundaries are: $c_{k+1}, \ldots, c_n$. Now by adding l predefined values $u_1, \ldots, u_l$ to the known values on the first boundary and by using shooting method, the predefined boundary values are determined.

A singularity exists in the numerical computation where r tends to zero. To remove this singularity, we suppose a solid circular plate of radius c at the center of the plate, such that at $r = c$, the boundary conditions are

$$G_{,r} = 0, \quad G_{,rrr} = -\frac{1-\nu^2}{h^3R}\left(\frac{1}{2}Qc\right) - \frac{1}{c}\left(G_{,rr}\right), \quad F_{,r} = \frac{\nu}{c}F \tag{33}$$

The edge of circular plate is clamped at $r = l$, so the boundary conditions can be expressed as

$$G = 0, \quad G_{,r} = 0, \quad F_{,r} = \nu F \tag{34}$$

By considering initial value problem corresponding to the boundary value problem (32–34) and introducing new variables as

$$\mathbf{Z}(r) = [G, G_{,r}, G_{,rr}, G_{,rrr}, F, F_{,r}, \Omega^2]^T = [z_1, z_2, z_3, z_4, z_5, z_6, z_7]^T \tag{35}$$

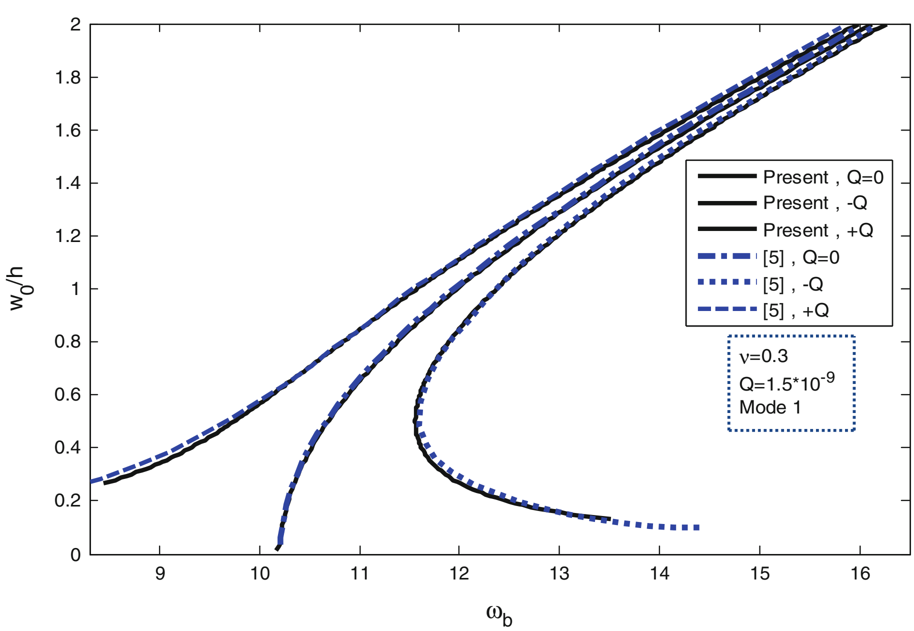


Fig. 3 Free and forced vibration response of clamped circular metallic plate at the first mode

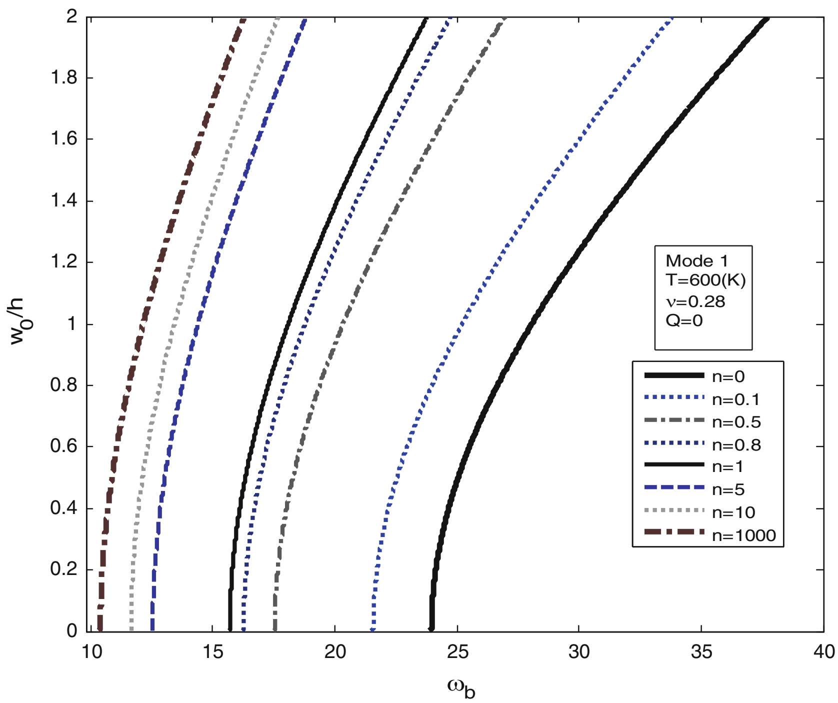


Fig. 4 Dimensionless central amplitudes of axisymmetric vibration versus first nondimensional natural frequencies at $T = 600$ K

The governing equations can be rewritten in the first-order differential equation form as

$$z_{1,r} = z_2 \tag{36}$$

$$z_{2,r} = z_3 \tag{37}$$

$$z_{3,r} = z_4 \tag{38}$$

$$z_{4,r} = -\frac{2}{r}z_4 + \frac{1}{r^2}z_3 - \frac{1}{r^3}z_2 - \left(\frac{1}{\left(\frac{h^3B^2}{A(1-\nu^2)} - \frac{h^3R}{1-\nu^2}\right)}\right) \times \left[\frac{3}{4r}(z_2z_6 + z_3z_5) + h\left(\frac{\rho_{cm}}{n+1} + 1\right)\Omega^2 z_1 + \left(\frac{h^3B}{1-\nu^2}\right)z_3 + Q\right] \tag{39}$$

$$z_{5,r} = z_6 \tag{40}$$

$$z_{6,r} = -\frac{1}{r}z_6 + \frac{1}{r^2}z_5 - \left(\frac{hA}{2r}\right)z_2^2 \tag{41}$$

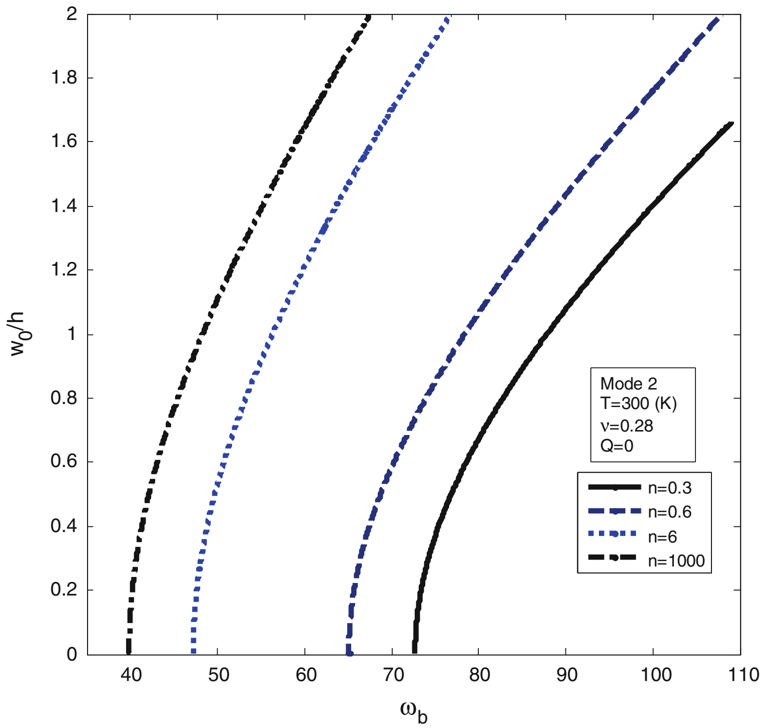
$$\mathbf{Z}(c) = \left[G(c), 0, u_1, -\left(\frac{1-\nu^2}{h^3R}\right)\left(\frac{Q}{2}\right)c - \frac{u_1}{c}, u_2, \frac{\nu u_2}{c}, u_3\right]^T \tag{42}$$

with

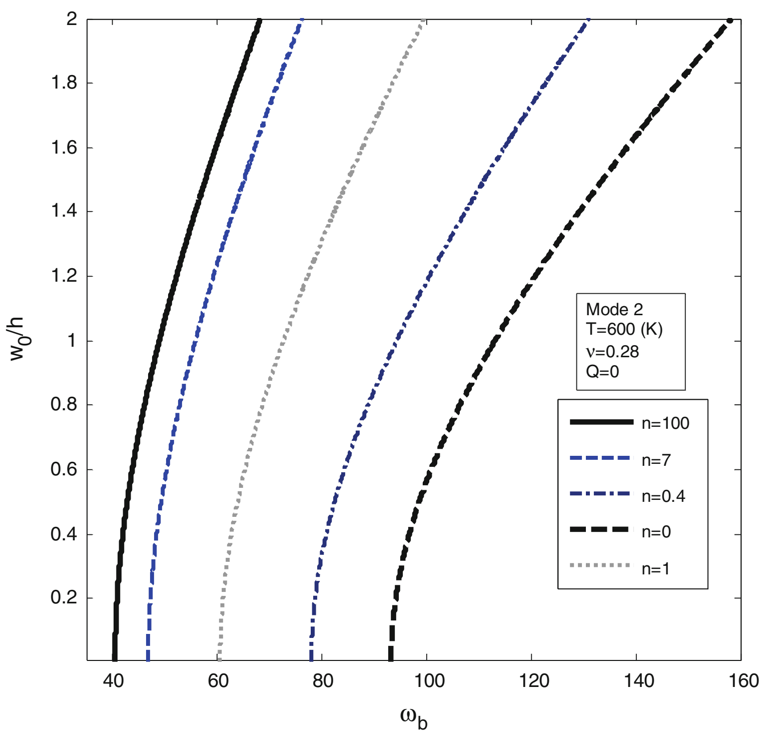
$$\mathbf{U} = \{u_1, u_2, u_3\}^T \tag{43}$$

where $\mathbf{U}$ is an unknown vector, representing the missing initial values at the center of the plate, i.e., ($r = c$).

By employing a fourth-order Runge–Kutta method with variable steps to integrate Eqs. (36)–(41) and, at the same time, by using a Newton–Raphson method to correct the assumed values for $\mathbf{U}$, numerical solutions have been obtained.



(a)



(b)

Fig. 5 Dimensionless central amplitudes of axisymmetric vibration versus second nondimensional natural frequencies at **a** $T = 300$ K, **b** $T = 600$ K

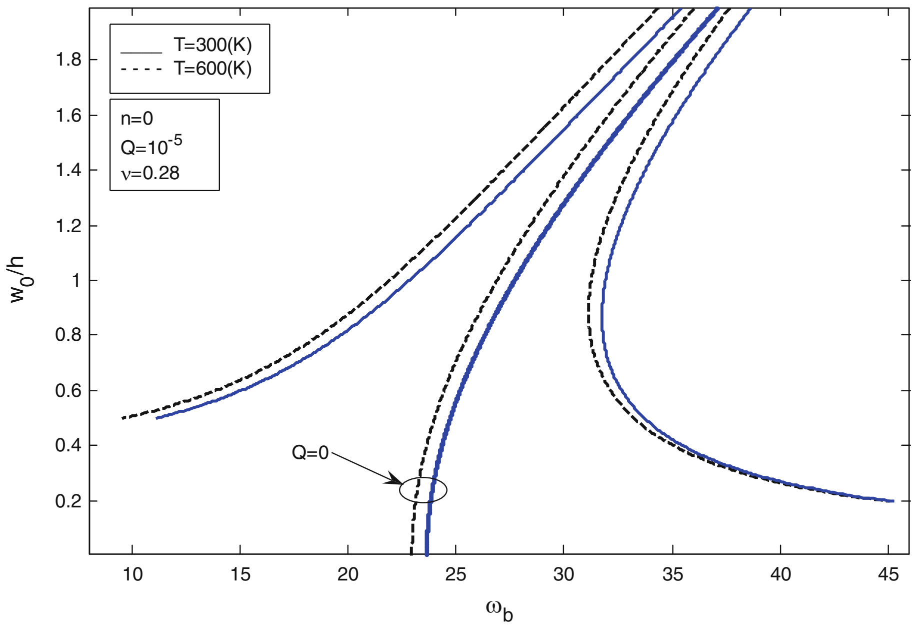


Fig. 6 Response of free and harmonic forced vibration at the first mode for $n = 0$ and different uniform temperature T

6 Results and discussion

The presented results are validated by considering the linear and nonlinear steady-state free and forced vibration of a clamped circular metallic ($n >> 10$) plate for prescribed values of $a/h = 136.59$ and $\upsilon = 0.3$. The dimensionless linear natural frequencies of the clamped circular plate are given by [22]

$$\omega_{bi} = \frac{a^2\Omega_i}{h}\sqrt{12\rho_m\left(1-\nu^2\right)/E_m} \tag{44}$$

According to the vibration mode, the starting point in the amplitude-frequency figures for linear free vibration is ω_{bi}. The volume fraction of metallic phase increases by increasing n. From Eq. (44), and for $n >> 10$, it can be seen (Fig. 3) that the results in the linear and nonlinear case are very close to those of [8,10]. The first linear fundamental frequency is $\omega_{b1} = 10.216$. The figure also shows the jump phenomenon and also the influence of amplitude on the nondimensional linear and nonlinear frequencies around the first mode. The uniformly distributed harmonic load Q is 1.5×10^{-9}.

We suppose that the functionally graded plate has a radius of $0.1m$ and a thickness of $0.004m$. In order to take the place of the solid circular plate, we set $c = 10^{-4}$.

Figures 4 and 5 show the variation of the first and second nondimensional natural frequencies of the clamped circular functionally graded plate at uniform temperature, respectively. The fundamental frequency of the plate increases with the amplitude of vibration. This is due to the fact that the in-plane forces in the plate contribute to the lateral stiffness resulting from nonlinear coupling. A hardening type of nonlinearity is observed in these figures.

Figure 6 illustrates the response of free ($Q = 0$) and harmonic forced vibration ($Q = 10^{-6}$) of the clamped circular functionally graded plate at the first mode and different uniform temperature. It is considered that the frequency is decreased when the uniform temperature is increased and the elasticity modulus is decreased.

Figures 7 and 8 demonstrate the free and harmonic forced ($Q = 10^{-6}$) vibration response of a clamped circular FGM plate at first mode with different values of n and T. According to Eq. (3), the effect of temperature variation at frequencies is obvious. By increasing n, dimensionless Young's modulus decreases (Fig. 1) and the dimensionless mass density increases (Fig. 2), and therefore, the fundamental natural frequency (Ω) decreases.

For several values of volume fraction index or uniform temperature, hardening behavior of the system is indicated in Figs. 6, 7 and 8, and a hysteretic effect arises for increasing and decreasing excitation frequency. If we increase slowly the excitation's frequency, the amplitude follows the upper curve as long as possible.

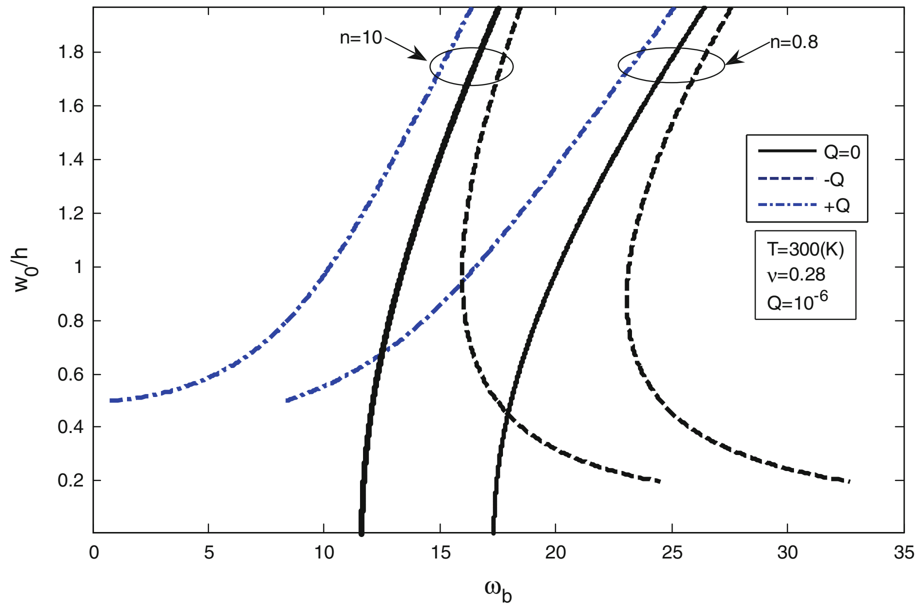


Fig. 7 Response of free and harmonic forced ($Q = 10^{-6}$) vibration at the first mode for $T = 300$ K and different n

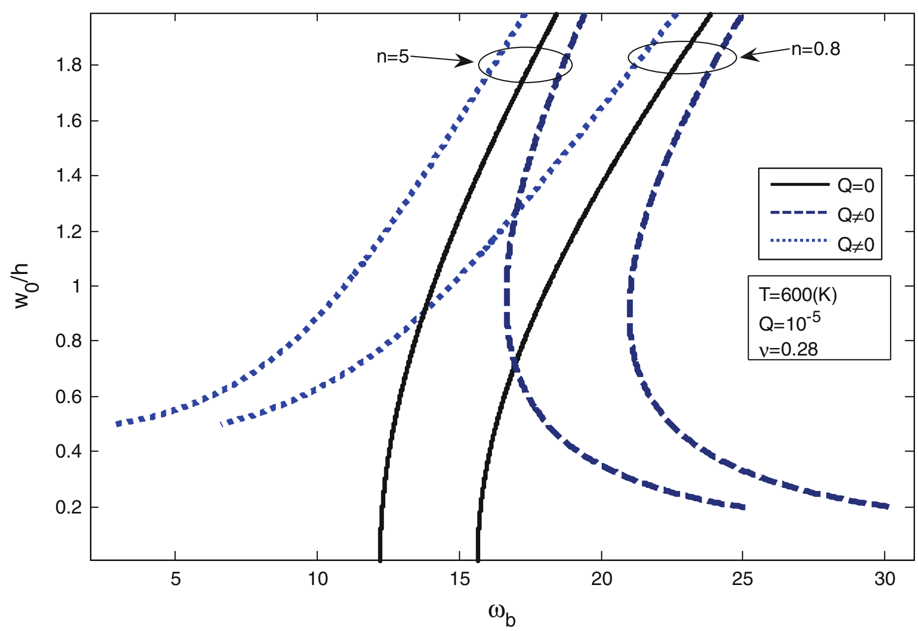


Fig. 8 Response of free and harmonic forced ($Q = 10^{-6}$) vibration at the first mode for $T = 600$ K and different n

Then, it falls down abruptly and follows the lower curve. If we decrease the frequency beginning with a high level, which is over the resonance, then the amplitude follows the lower curve as long as possible and at the point of vertical tangency, jumps up to the upper curve. Thus, an important indicator of the resonance in a non-linear system is the difference between the stationary amplitudes if one increases (or decreases) the excitation frequency. This gives rise to jump phenomenon, commonly found in nonlinear vibratory systems, and in the FGM plate, it also depends on volume fraction index and uniform temperature.

7 Conclusion

In this paper, the free and forced vibration of a clamped circular functionally graded thin plate has been studied and the results show that the variation of n is influential on the natural frequencies. As a result, the frequency is proportional to the square of elasticity modulus to mass density of metal. By increasing n, the elasticity modulus decreases, while the mass density increases. So, by increasing n, the frequency decreases. The nonlinear vibration frequencies are dependent on vibration amplitudes. This is due to the fact that the in-plane forces in the plate contribute to the lateral stiffness resulting from nonlinear coupling. It has been considered that the frequency decreased when the uniform temperature was increased and elasticity modulus was decreased. Moreover, in the special frequencies of forced vibration response, jump phenomenon has been observed according to volume fraction index and uniform temperature.

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