Analysis of clearance effect on the chaotic vibration of a gear set

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ABSTRACT

In this study the dynamic model of a gear system has been extracted considering the effect of clearance between teeth and transmission error. Due to the fact that static transmission errors occur for a variety of reasons, including manufacturing errors and tooth deformation, and have an intermittent nature, a Fourier series has been used to approximate it. The clearance is due to the installation and gear wear, also the need for better lubrication and reduced interaction between the teeth, requires that there be some clearance between the gears. Therefore, a function is used to model the effect of clearance on the dynamic behavior of system. The excitation torque is considered sinusoidal and the load torque is constant. After modeling the above mentioned, the governing equations have been extracted and numerically solved to obtain the vibration response of the gear system. In order to analyze the effect of clearance on gear system vibrations, the response was obtained for different values of clearance. The time response, frequency spectrum, and phase plane are plotted to investigate these effects. The results show that the vibration response is chaotic and the value of clearance has a significant effect on this chaotic behavior.

Keywords: Gear, chaos, vibration, transmission error, clearance

1. INTRODUCTION

Gears are one of the most important components in industrial rotating machinery and power transmission systems and therefore their dynamic modeling and vibration analysis are important research topics. For a long time, the dynamics of rotating systems were examined using linear models [1]. However, more detailed studies have shown that a nonlinear model is needed to observe and analyze the phenomena that occur in response to the vibration of the gears. Various factors cause nonlinearity of the gear dynamic model, such as backlash, transmission error, teeth friction, variable stiffness and so on. These nonlinear factors can cause different nonlinear phenomena such as sub-harmonics and super-harmonics, quasi-periodic and chaotic. A lot of research were done in this subject which some of them are reviewed in this section.

Clearance is considered as one of the nonlinearity factors in gears [2]. In one research [3], a two-degree-of-freedom model has been used to investigate the effect of clearance. Using numerical and approximate methods, they have obtained periodic and chaotic responses. In another study [4], variable stiffness and clearance were considered as nonlinearity factors and were obtained bifurcation and chaos in the vibration response. Dynamic responses and stability of the rotor-gear-bearing system have also been investigated and many a periodic responses have been obtained [5]. The motor-gear system with clearance has also been studied where periodic and chaotic responses have been observed [6]. In this research, a gear set with clearance and transmission error is modeled and the effect of clearance variations on the chaotic vibration is studied.

2. Dynamic modeling of the gear system

The dynamical model of a pair of gears is shown in Fig. 1. The gears are modeled as two disks indicating their inertia, and the gears are modeled as springs and dampers. In this model, only the torsional vibrations of the gears are considered and the transverse vibration of shaft is ignored. Applied torque to gears is T_1 and T_2, which depending on the type of drive, the excitation torque T_1 can be periodic. Accordingly, the excitation torque T_1 is considered as follows:

$$T_1 = T_{1m} + T_{1a}cos(\omega_T t) \quad (1)$$

Where T_{1m} and T_{1a} are the mean and the amplitude, respectively. ω_T is the frequency of excitation torque oscillations which assumed equal to the gear mesh frequency ω_m. The value of load torque T_2 assumed constant and obtained from the relation $T_2 = T_{1m}\frac{R_2}{R_1}$.

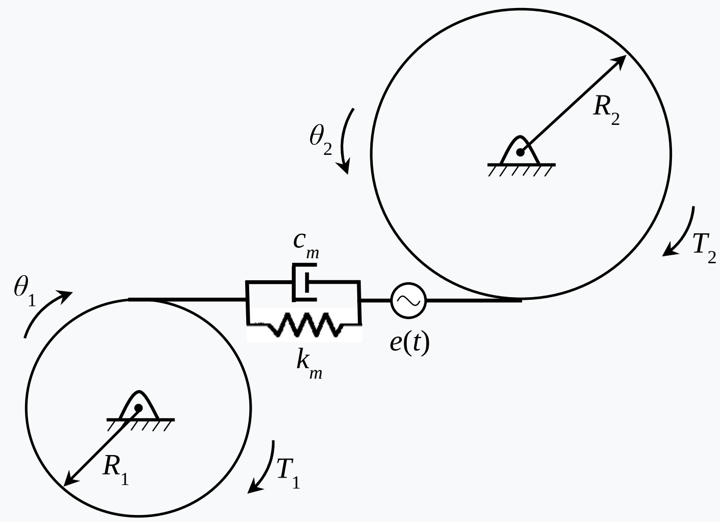

Figure 1. Dynamic model of the gear set

Static transmission error $e(t)$ occurs for different reasons such as manufacturing and tooth deformation errors, and is one of the important factors in vibration generation in gears. Due to the periodic change of the mating teeth, the static transmission error has periodic nature, so it can be modeled using the Fourier series. Therefore, the static transmission error is considered as follows:

$$e(t) = \sum_j E_j \cos(j\omega_m t - \varphi_j) \quad (2)$$

In the above equation $\omega_m = N_1\omega_1 = N_2\omega_2$ is the gear mesh frequency.$E_j$and$\varphi_j$are also the static transmission error amplitudes and phases obtained by applying Fourier transform on the error signal. The values of E_j and φ_j for the first four harmonics are extract from the analysis in [7] and are given in Table 1.

Table 1. Amplitude and phase ratio for the first four harmonics static transmission error

j	E_j	$\varphi_j\ (rad)$
1	10.73	0.935
2	2.92	1.99
3	2.73	2.52
4	2.50	2.88

The dynamic transmission error is also defined as $e_d(t) = R_1\theta_1 - R_2\theta_2$ [8] and so the actual transmission error can be written as follows:

$$u = e_d(t) - e(t) = R_1\theta_1 - R_2\theta_2 - e(t) \tag{3}$$

The installation and gear wears are some of reasons which clearance occurs, also it is necessary forbetter lubrication and reduce teeth interactions. The function$f(u)$ has been used as follows for modeling the clearance [8].

$$f(u) = \begin{cases} u - b & u > b \\ 0 & -b \le u \le b \\ u + b & u < -b \end{cases} \tag{4}$$

Therefore, the governing equations for the gear system shown in Figure 1, are as follows:

$$I_1\ddot{\theta}_1 + R_1 c_m \dot{u} + R_1 k_m f(u) = T_1 \tag{5}$$

$$I_2\ddot{\theta}_2 - R_2 c_m \dot{u} - R_2 k_m f(u) = -T_2$$

By using the Equation (3), Equations (5) are simplified as follows:

$$m\ddot{u} + c_m\dot{u} + k_m f(u) = F_e + F_T \tag{6}$$

The parameters of theEquation (6) are as follows:

$$m = \frac{I_1 I_2}{I_1 R_2^2 + I_2 R_1^2}\,, \quad F_e = -m\ddot{e}\,, \quad F_T = m\left(\frac{T_1 R_1}{I_1} + \frac{T_2 R_2}{I_2}\right) \tag{7}$$

To solve the differential equation governing the rotational vibrations of the gear system, it is necessary to specify the values of the constant parameters of the equation. As a case study, the specifications of the gear system are assumed as in Table 2.

Table 2. Gear system specifications

Excitation parameters	Mesh parameters	Number of teeth	Base radius (mm)	Inertia(kg.mm^2)	Mass (kg)
$\omega_1 = 1500$ rpm	km=900 N/mm	N1= 43	R1= 60.61	I1= 4800	m1=1.98
$T_{1m} = 20$ N.m	cm=500 N.s/m	N2= 28	R2= 39.47	I2= 800	m2= 0.72

3. Results

The Equation (6) has solved numerically and the vibration response of gear system has obtained. To analysis the effect of clearance on the chaotic vibration in gear system, two cases are considered. In both cases, different values of clearance from zero to 150 micrometers are considered. In case 1 and 2, the ratio of $\frac{T_{1a}}{T_{1m}}$ are assumed to be 2 and 4, respectively.

3.1. Case 1: $\frac{T_{1a}}{T_{1m}} = 2$

In this case, the ratio of the excitation torque amplitude to its average torque range is assumed to be 2. The vibration response for different values of clearance, including clearance of zero, 50 and 150 micrometers is obtained and plotted. Figure 2 shows the time response, frequency spectrum and phase plane for zero clearance. Because the excitation frequency ω_T is considered equal to the gear mesh frequency ω_m, the vibration response only includes harmonics of ω_m and the first harmonic has a greater share in the response. On the phase plane, it is obvious that the ratio of signal frequencies isinteger. In this case, the dynamic behavior of the system is linear, but the presence of different frequencies in the response is only due to internal excitation by the static transmission error with harmonics of gear mesh frequency. Figures3 and 4 show the time response, frequency spectrum and phase plane for clearance values of 50 and 150 micrometers. The time response in the presence of a completely non-linear behavior has shown that this behavior can also be a chaotic behavior. This can be seen in the frequency spectrum with multiple frequencies and in the phase plane with strange attractors and multiple paths.

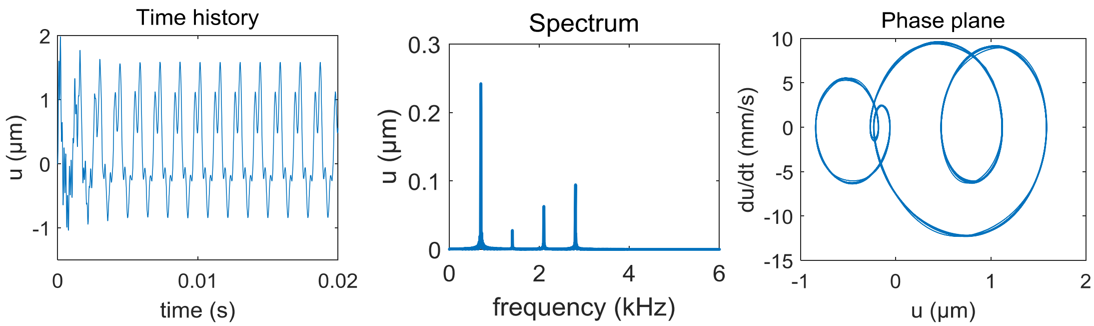

Figure 2. Time response, frequency spectrum and phase plane for b = 0 µm

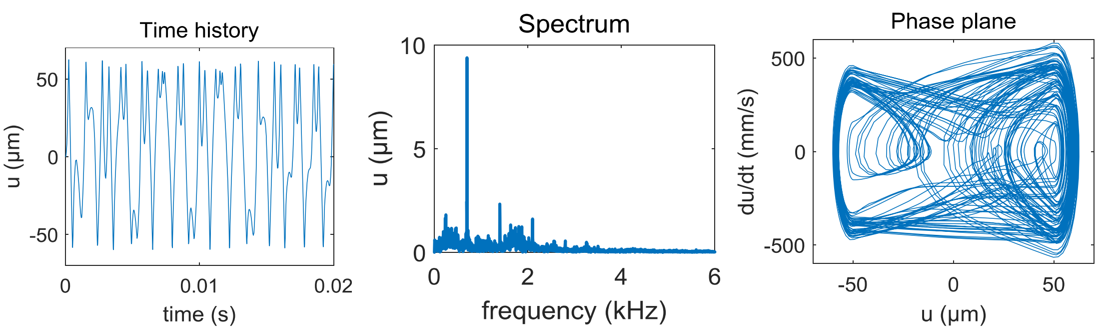

Figure 3. Time response, frequency spectrum and phase plate for b = 50 µm

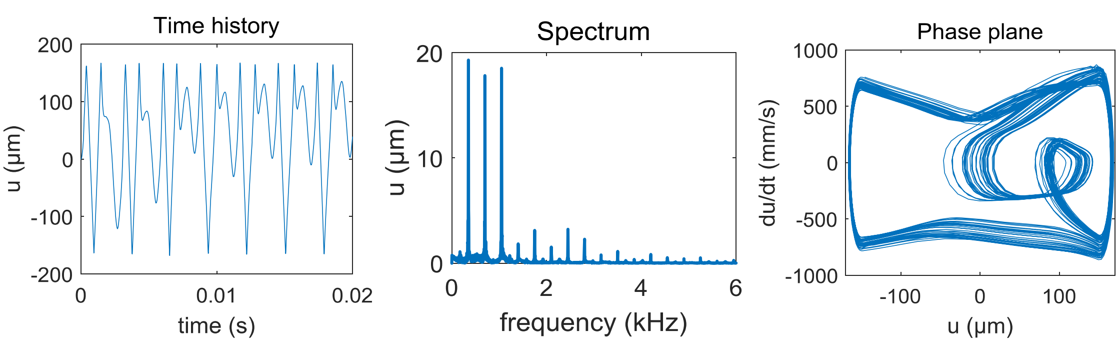

Figure 4. Time response, frequency spectrum and phase plane for b = 150 µm

3.2. Case 2: $\frac{T_{1a}}{T_{1m}} = 4$

In this section, to investigate the effect of the excitation torque amplitude, its value is considered to be 4 times of the excitation torque mean. The excitation torque frequency $\omega_T$is also assumed to be equal to the gearmesh frequency ω_m. Increasing the amplitude of the excitation torque has increased the influence of the mesh frequency in vibration response. Thisinfluence can be seen in the frequency spectrum and phase plane plots. Also, in the case of zero clearance, the response only includes the mesh frequency and its harmonics, but by increasing the clearance value, the system behavior becomes nonlinear and other frequencies and side bands also appear. Nonlinear behavior can also be seen in the phase plane for 50 and 150 micrometers clearance. Time responses in these two irregular behaviors can confirm the presence of chaos in the vibration response of the system.

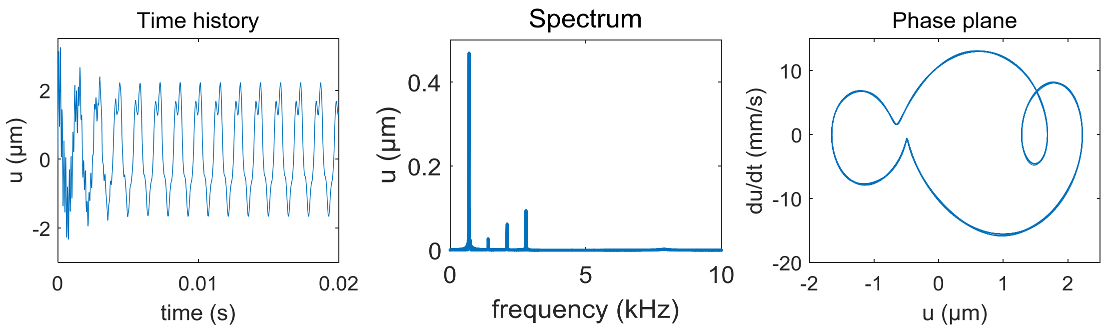

Figure 5. Time response, frequency spectrum and phase plane for b = 0 μm

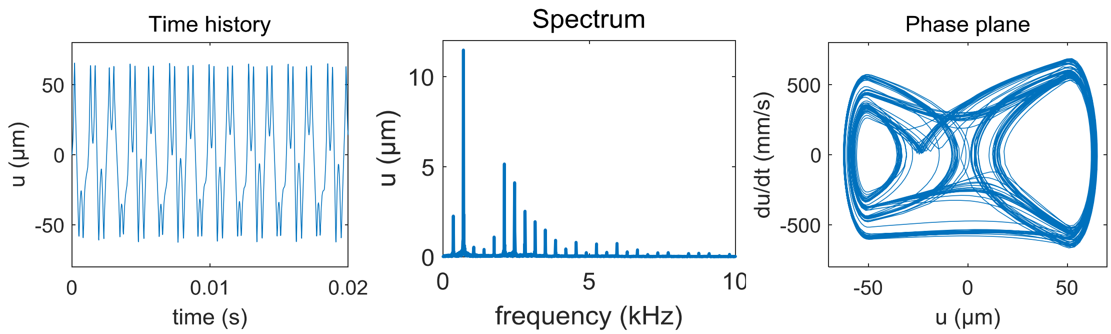

Figure 6. Time response, frequency spectrum and phase plane for b = 50 μm

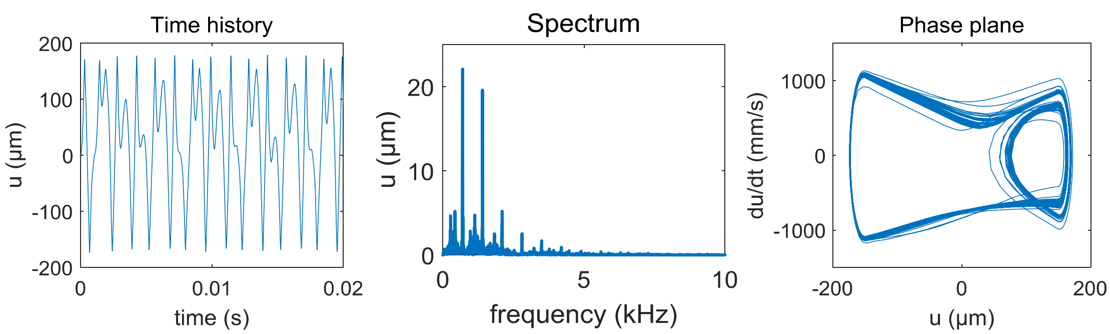

Figure 7. Time response, frequency spectrum and phase plane for b = 150 μm

4. Conclusion

In this paper, dynamic modeling and vibration response analysis of a gear system are investigated. The gear system includes of two spur gears which the equation of torsional vibration is extracted and solved. The teeth clearance as nonlinearity source is modeled by a function and different values of clearance are considered. The transmission error was also modeled using the Fourier series. Also, the excitation torque was assumed to be periodic but the load torque was considered to be constant. The vibration behavior of gear system was analyzed by the time response, frequency spectrum and phase plane. Results show that the zero clearance was lead to the linear behavior and periodic vibration.

But by increasing the clearance value, quasi-periodic and chaotic behavior were appear. In frequency spectrums, a lot of frequency and side bands confirms the chaotic behavior. In phase planes, the presence of strange attractors and many trajectories without any instability confirms the chaotic behavior. The study of chaotic vibration in gear system can be useful for system design and fault detection by vibration analysis.

5. References

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