Nonaffine-Nonlinear Adaptive Control of an Aircraft Cabin Pressure System Using Neural Networks

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Abstract: In this paper, an adaptive neural network controller is proposed for a nonaffine-nonlinear aircraft cabin pressure system with unknown parameters. A multilayer neural network is used to represent the controller structure. The ultimate boundedness of the closed-loop system is guaranteed through a Lyapunov stability analysis by introducing a suitably driven adaptive rule. The effectiveness of the proposed adaptive controller is illustrated by considering an aircraft cabin pressure system, and the simulation results verify the merits of the proposed controller. **DOI:** 10.1061/(ASCE)AS.1943-5525.0000262.

Author keywords: Aircraft cabin pressure system; Adaptive control; Nonaffine-nonlinear systems; Neural networks.

Introduction

Because the human ear is very sensitive to even small pressure changes, the cabin pressure regulating system is an important part of the air management system of aircraft. The cabin is pressurized by compressed bleed air directed into the cabin from the aircraft engines and is controlled by the outflow valve rate of airflow out of the cabin. The cabin pressure–regulating system should guarantee the cabin pressure and its rate of change to satisfy the specification requirement throughout the flight envelope. The cabin pressure–regulating system has three development stages, pneumatic type, electronic-pneumatic type, and digital type, and research has recently attracted more attention to the improvement of the pneumatic structure and less attention to digital control (Tang et al. 2005). The digital cabin pressure–regulating system has been widely used in many kinds of aircraft, with the digital controller in its core (Furlong 1971). Fuzzy controllers have been considered as general tools for controlling the cabin pressure system (Kwong et al. 1994; Vascak et al. 2001; Wu and Luo 1995; Mamdani 1974). These results cannot guarantee stability of the closed-loop system.

This work proposes a direct adaptive controller for a nonaffine-nonlinear aircraft cabin pressure system with unknown parameters. The multilayer neural networks (MNNs) are used to compensate for the unknown parameters. The ultimate boundedness of the closed-loop system is guaranteed through a Lyapunov analysis, and the tracking error tends to stay in the neighborhood of zero. Zhu et al. (2009, 2010) have studied a similar problem for an aircraft cabin pressure control system using a fuzzy controller, but their results cannot guarantee stability of the closed loop. This paper is organized as follows. The "Design of Set-Point Signal" section describes the computing of the optimal and adjustable set-point value for cabin pressure. In the "Mathematical Model and Error Dynamic Derivation" section, the problem statement of cabin pressure system is presented. The main results of adaptive controller and stability analysis are presented in the "Proposed Adaptive Controller: Design and Stability Analysis" section. An illustrative example is then used in the "Simulation" section to demonstrate the effectiveness of the adaptive technique, and finally the "Conclusion" section concludes the paper.

Design of Set-Point Signal

In this section, the optimal and adjustable set-point value for cabin pressure is computed throughout the aircraft ascent and descent to provide a reduced cabin pressure rate of change during the nonlinear aircraft ascents and descents. The cabin pressure control system must operate to provide the desired requirements at all the segments of flight. Three of the most important requirements that must be considered for pressurization of the control system design are listed as follows (Scheerer and Willenbrink 2003):

1. The cabin pressure decreases during the ascent so that a minimum human comfort pressure should not exceed approximately 567 mm Hg = 75,600 Pa

$$P_c > 567 \text{ mm Hg} \tag{1}$$

where P_c = cabin pressure.

2. The differential pressure must not exceed a certain threshold because the aircraft fuselage may be damaged or destroyed [Society of Automotive Engineers (SAE) International 2000]

$$P_c - P_{\text{atm}} < 309 \text{ mm Hg} \tag{2}$$

where P_{atm} = atmospheric pressure.

3. Maximum passenger comfort during the flight is achieved by minimizing the cabin pressure rate of change during ascent and descent so the rates may not exceed the equivalent of approximately -0.22 mm Hg/s = -29.3 Pa/s for ascent and 0.13 mm Hg/s = 17.34 Pa/s for descent

$$\begin{aligned} &\text{Ascent: } -0.22 < \dot{P}_c < 0 \text{ mm Hg/s} \\ &\text{Descent: } 0 < \dot{P}_c < 0.13 \text{ mm Hg/s} \end{aligned} \tag{3}$$

Zeinaly et al. (2011) considered the problem of design of a set-point signal so the difference between the cabin pressure and ambient

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Note. This manuscript was submitted on December 31, 2011; approved on May 30, 2012; published online on July 14, 2012. Discussion period open until October 1, 2014; separate discussions must be submitted for individual papers. This paper is part of the ***Journal of Aerospace Engineering***, Vol. 27, No. 3, May 1, 2014. ©ASCE, ISSN 0893-1321/2014/3-597–603/$25.00.

pressure is considered as a function of aircraft altitude during the flight; therefore, the set point of cabin pressure can be calculated. Fig. 1 shows the desired cabin pressure and ambient pressure in a typical flight (set-point signal).

Mathematical Model and Error Dynamic Derivation

This section introduces the problem statement and derivation of the error dynamics. Before setting up the cabin pressure differential equations, the following assumptions are considered from Zhu et al. (2009, 2010):

1. The cabin temperature keeps constant while pressure is adjusted;
2. The cabin volume keeps constant;
3. The air in the cabin can be looked at as an ideal gas, and its pressure, temperature, and volume satisfy the state equation of ideal gas;
4. The leakage area of the cabin is constant; and
5. The leakage flow of the cabin is zero.

Based on the assumptions, the expressed cabin pressure differential equations follow Zhu et al. (2009, 2010)

$$\frac{dP_c}{dt} = \frac{RT_c}{V_c}(W_i - W_o) \tag{4}$$

where, based on the ratio of P_c/P_{atm}, is upper or lower than 1.893; and W_i and W_o = cabin input air and cabin output air W_o, respectively, as described subsequently (Bykov 1965).

If $P_c/P_{\text{atm}} < 1.893$ (subsonic mode)

$$W_o = 0.95\mu_{\text{sub}}A\sqrt{\frac{3}{\gamma RT}P_{\text{atm}}(P_c - P_{\text{atm}})} \tag{5}$$

and if $P_c/P_{\text{atm}} \geq 1.893$ (supersonic mode)

$$W_o = \mu_{\text{super}}AP_c\sqrt{\frac{\gamma}{RT}\left(\frac{2}{\gamma+1}\right)^{(\gamma+1/\gamma-1)}} \tag{6}$$

where A is defined as

$$A = A_0(1 - \cos\theta) \tag{7}$$

where A_0 = exhaust valve maximum flow. By substituting Eqs. (5)–(7) into Eq. (4), the process of cabin pressure is described by the following differential equation:

$$\dot{P}_c = f(P_c, \theta) \tag{8}$$

For the differential equation of the cabin pressure system described in Eq. (8), the general form of this system is defined as

$$\dot{x} = f(x, u) \tag{9}$$

Eq. (9) represents the most general state space of a nonaffine-nonlinear system with one input and one state variable, where x = state variable of the system, u = input signal, and the function $f(x,u)$ is unknown and sufficiently smooth. From Eqs. (8) and (9), P_c (cabin pressure) is obtained as a state variable.

Assumption 1

The desired continuous time trajectory x_d and its time derivative are given and bounded.

Assumption 2

For each subsystem, the positive constants H, f^L, and f^H exist such that

$$0 < f^L \leq \frac{\partial f(x,u)}{\partial u} \leq f^H \tag{10}$$

and

$$\left|\frac{d}{dt}\left[\frac{\partial f(x,u)}{\partial u}\right]\right| \leq H \tag{11}$$

Consider the following state-dependent transformation:

$$\begin{aligned} \dot{x} &= v \\ v &= f(x, u^*) \end{aligned} \tag{12}$$

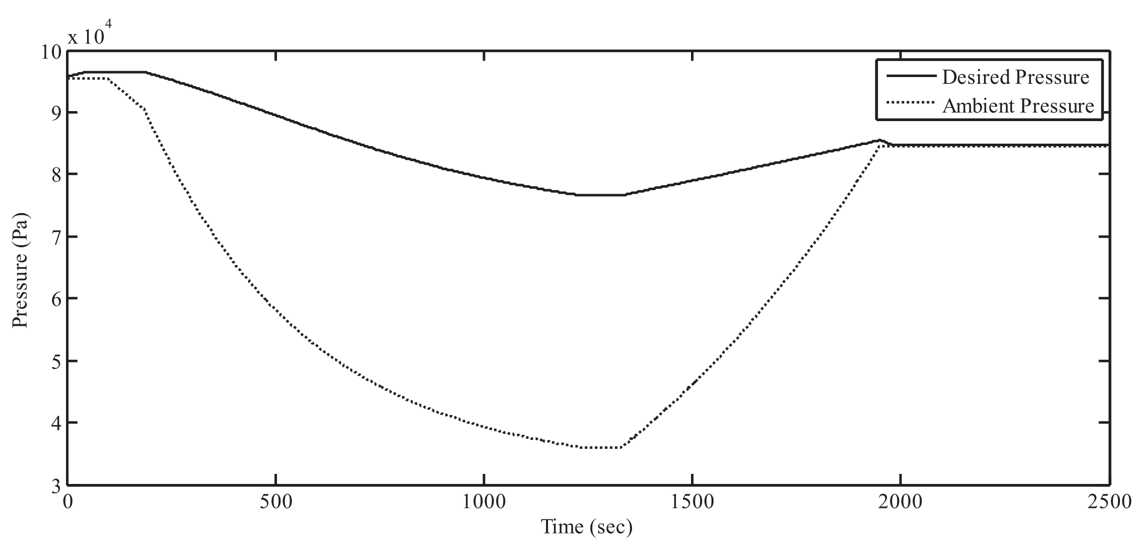

Fig. 1. Desired cabin pressure and ambient pressure in a typical flight

where v = pseudocontrol signal; and u^* = ideal local control function. The pseudocontrol signal v is in general not a function of the control signal u but rather a state-dependent operator, and from Assumption 2

$$\frac{\partial[v - f(x,u^*)]}{\partial u} \neq 0 \tag{13}$$

The implicit function theorem (Bartle 1964) implies that in a neighborhood of every $(x,u) \in \Phi \times R$, an implicit function $\alpha(x,v)$ exists, such as

$$v - f[x, \alpha(x,v)] = 0 \tag{14}$$

Determining $\alpha(x,v)$ denoted by u^* in general is not always possible given that f may not be known. Karimi et al. (2009) and Karimi and Menhaj (2010) have discussed this topic. The mean value theorem implies that $\lambda \in (0,1)$ exists such that (Lang 1983)

$$f(x,u) = f(x,u^*) + (u - u^*)f_u \tag{15}$$

where $f_u = [\partial f(x,u)/\partial u]|_{u=u_\lambda}$, with $u_\lambda = \lambda u + (1-\lambda)u$.

Now x_d is defined as the desired output, and $\bar{\mathbf{x}}_d$ and e are

$$\begin{aligned} \bar{\mathbf{x}}_d &= [x_d, \dot{x}_d]^T \\ e &= x - x_d \end{aligned} \tag{16}$$

Therefore, the error dynamic can be written as

$$\dot{e} = f(x,u) - \dot{x}_d \tag{17}$$

Substituting Eq. (15) into Eq. (17) yields

$$\dot{e} = f(x,u^*) + (u - u^*)f_u - \dot{x}_d \tag{18}$$

Using $v = f(x,u^*)$, Eq. (18) is rewritten as

$$\dot{e} = v + (u - u^*)f_u - \dot{x}_d \tag{19}$$

The pseudocontrol v is designed as

$$v = -k_1 e + \dot{x}_d \tag{20}$$

where $k_1 > 0$. Combining Eqs. (19) and (20), the error dynamic can be written as

$$\dot{e} = -k_1 e + (u - u^*)f_u \tag{21}$$

Proposed Adaptive Controller: Design and Stability Analysis

This section presents a neural network (NN)-based controller for Eq. (21) with unknown parameters. The ideal local control function u^* may be represented by a multilayer feed-forward neural network (MFNN) with three layers (Lewis et al. 1996; Zhang et al. 1999) (or any other approximation structure) such that

$$u^*(z) = W^{*T}\boldsymbol{\psi}\left(V^{*T}\bar{\mathbf{z}}\right) + u_k(\mathbf{z}) + \varepsilon(\mathbf{z}) \tag{22}$$

where $\mathbf{z} = [x, x_d, v, 1]^T \in \mathbb{R}^4$ is the input vector; $V^* = [v_1, v_2, \ldots, v_l]$ is called the weight matrix of the first-to-second layer; l = number of hidden-layer neurons; $W^* = [w_1, w_2, \ldots, w_l]$ is called the weight vector of the second-to-third layer; and $\boldsymbol{\psi} = [\psi_1, \psi_2, \ldots, \psi_l]$ is called the activation vector function. The activation function $\psi(.)$ is chosen as

$$\psi(z_a) = \frac{1}{1 + e^{-z_a}}, \quad z_a \in \mathbb{R} \tag{23}$$

In Eq. (22), the term $\varepsilon(z)$ is called the NN approximation error satisfying $|\varepsilon(z)| \le \varepsilon_l$, where $\varepsilon_l > 0$; and $u_k(z)$ is a prior control term developed based on a prior model (experience) to improve the initial control performance. The ideal constant weights W^*, V^* are defined as

$$(W^*, V^*) = \underset{(W,V)}{\arg\min}\left[\sup_{z\in\Omega_z}\left|W^T\boldsymbol{\psi}\left(V^T\bar{\mathbf{z}}\right) + u_k(\mathbf{z}) - u^*(\mathbf{z})\right|\right] \tag{24}$$

Assumption 3

On the compact set Ω_z, The ideal neural network weights W^*, V^* are bounded by

$$\|W^*\| \le w_m, \quad \|V^*\|_F \le v_m \tag{25}$$

where w_m, v_m = positive constants. In Eq. (25), $\|.\|$ denotes the 2-norm and $\|.\|_F$ is the Frobenius norm.

Let $\hat{W}$ and $\hat{V}$ be the estimates of W^* and V^*, respectively. The estimation errors of the weight matrices are defined as

$$\tilde{W} = \hat{W} - W^*, \quad \tilde{V} = \hat{V} - V^* \tag{26}$$

The Taylor series expansion $\boldsymbol{\psi}(V^{*T}\bar{z})$ for a given $\hat{V}^T\bar{\mathbf{z}}$ may be written as

$$\boldsymbol{\psi}\left(V^{*T}\bar{\mathbf{z}}\right) = \boldsymbol{\psi}\left(\hat{V}^T\bar{\mathbf{z}}\right) - \hat{\boldsymbol{\psi}}'\tilde{V}^T\bar{\mathbf{z}} + O\left(\tilde{V}^T\bar{\mathbf{z}}\right)^2 \tag{27}$$

where $\hat{\boldsymbol{\psi}}' = \mathrm{diag}(\hat{\psi}'_1, \hat{\psi}'_2, \ldots, \hat{\psi}'_l)$, with $\hat{\psi}'_i = \psi'(\hat{V}_i^T\bar{z}) = \{d[\psi_i(z_a)]/dz_a\}|_{z_a=\hat{V}_i^T\bar{z}}$, where $i = 1, 2, \ldots, l$; and $O(\tilde{V}^T z)^2$ denotes the sum of the high-order terms in the Taylor series expansion. In the sequel, the following fact is used from Karimi and Menhaj (2010):

$$\begin{aligned} &\hat{W}^T\boldsymbol{\psi}\left(\hat{V}^T\bar{z}\right) - W^{*T}\boldsymbol{\psi}\left(V^{*T}\bar{z}\right) \\ &\quad = \tilde{W}^T\left(\hat{\boldsymbol{\psi}} - \hat{\boldsymbol{\psi}}'\hat{V}^T\bar{\mathbf{z}}\right) + \hat{W}^T\hat{\boldsymbol{\psi}}'\tilde{V}^T\bar{\mathbf{z}} + d_u \end{aligned} \tag{28}$$

where $\hat{\boldsymbol{\psi}} = \boldsymbol{\psi}(\hat{V}^T\bar{\mathbf{z}})$; and the residual term d_u is defined as

$$d_u = \tilde{W}^T\hat{\boldsymbol{\psi}}'V^{*T}\bar{\mathbf{z}} - W^{*T}O\left(\tilde{V}^T\bar{\mathbf{z}}\right)^2 \tag{29}$$

Lemma

The positive constants α_0, α_1, α_2, and α_3 exist such that the residual term d_u is bounded by

$$|d_u| \le \alpha_0 + \alpha_1|e| + \alpha_2\|\tilde{W}\| + \alpha_3\|\tilde{W}\||e| \tag{30}$$

Karimi et al. (2010) provides the proof.

An adaptive algorithm is then proposed as

$$u = \hat{W}^T\boldsymbol{\psi}\left(\hat{V}^T\bar{\mathbf{z}}\right) + u_k(\mathbf{z}) + u_b \tag{31}$$

$$u_b = -k_s|e|e \tag{32}$$

where $k_s > 0$ is a design parameter. The first term, $\hat{W}^T \boldsymbol{\psi}(\hat{V}^T \bar{\mathbf{z}})$, represents the MFNN employed to approximate the ideal controller. The second term, $u_k(z)$, is a prior continuous controller, possibly proportional plus integral (PI), proportional integral derivative (PID), or some other type of controller, designed in advance via heuristics or conventional control methods. If such prior knowledge is not available, u_k can simply be set to zero. The third term, u_b, is called the bounding control term that is applied for guaranteeing the boundedness of the system states. The following adaptive rules are proposed to update the parameters $\hat{W}$ and $\hat{V}$:

$$\dot{\hat{W}} = -\Gamma_w \left[\left(\hat{\boldsymbol{\psi}} - \hat{\boldsymbol{\psi}}' \hat{V}^T \bar{\mathbf{z}} \right) e + \delta_w (1 + e^2) \hat{W} \right] \tag{33}$$

$$\dot{\hat{V}} = -\Gamma_v \left(\bar{\mathbf{z}} \hat{W}^T \hat{\boldsymbol{\psi}}' e + \delta_v \hat{V} \right) \tag{34}$$

where $\hat{\boldsymbol{\psi}} = \boldsymbol{\psi}(\hat{V}^T \bar{z})$; and $\Gamma_w = \Gamma_w^T > 0$, $\Gamma_v = \Gamma_v^T > 0$, $\delta_w > 0$, and $\delta_v > 0$ are constant design positive parameters. The following theorem proves the stability of the proposed controller.

Theorem

Given Assumptions 1–3 and the system described by Eq. (8), the control law, Eq. (31), with the adaptation laws, Eqs. (33) and (34), make the tracking error of the system converge to a neighborhood of zero, and all signals in the closed-loop system are bounded as well.

Proof

Consider the following Lyapunov function:

$$V_1 = \frac{1}{2} \left[\frac{e^2}{f_u} + \tilde{W}^T \Gamma_w^{-1} \tilde{W} + \mathrm{tr}\left(\tilde{V}^T \Gamma_v^{-1} \tilde{V} \right) \right] \tag{35}$$

where $\tilde{W} = \hat{W} - W^*$; and $\tilde{V} = \hat{V} - V^*$. The time derivative of the Lyapunov function $\dot{V}_1$ along the error dynamics, Eqs. (21), (33), and (34), becomes

$$\dot{V}_1 = e \left[\frac{-k_1 e}{f_u} + \tilde{W}^T \left(\hat{\boldsymbol{\psi}} - \hat{\boldsymbol{\psi}}' \hat{V}^T \bar{\mathbf{z}} \right) + \hat{W}^T \hat{\boldsymbol{\psi}}' \tilde{V}^T \bar{\mathbf{z}} + d_u - k_s|e|e - \varepsilon(\mathbf{z}) \right] - \frac{\dot{f}_u}{2f_u^2} e^2 + \tilde{W}^T \Gamma_w^{-1} \dot{\tilde{W}} + \mathrm{tr}\left(\tilde{V}^T \Gamma_v^{-1} \dot{\tilde{V}} \right) \tag{36}$$

Substituting Eq. (30) into Eq. (36) produces

$$\dot{V}_1 \leq \frac{-k_1 e^2}{f_u} + e \left[\tilde{W}^T \left(\hat{\boldsymbol{\psi}} - \hat{\boldsymbol{\psi}}' \hat{V}^T \bar{\mathbf{z}} \right) + \hat{W}^T \hat{\boldsymbol{\psi}}' \tilde{V}^T \bar{\mathbf{z}} \right] - k_s|e|^3 + |e| \left(\alpha_0 + \alpha_1 |e| + \alpha_2 \|\tilde{W}\| + \alpha_3 \|\tilde{W}\| |e| + \varepsilon_l \right) - \frac{\dot{f}_u}{2f_u^2} e^2 + \tilde{W}^T \Gamma_w^{-1} \dot{\tilde{W}} + \mathrm{tr}\left(\tilde{V}^T \Gamma_v^{-1} \dot{\tilde{V}} \right) \tag{37}$$

Using the network parameter adaptive laws, Eqs. (33) and (34), the preceding inequality becomes

$$\dot{V}_1 \leq -k_s|e|^3 - \frac{k_1 e^2}{f_u} - \frac{\dot{f}_u}{2f_u^2} e^2 + |e| \left(\alpha_0 + \alpha_1 |e| + \alpha_2 \|\tilde{W}\| + \alpha_3 \|\tilde{W}\| |e| + \varepsilon_l \right) - \delta_w (1 + e^2) \tilde{W}^T \hat{W} - \delta_v \mathrm{tr}\left(\tilde{V}^T \hat{V} \right) \tag{38}$$

Using the inequalities $2\tilde{W}^T \hat{W} \leq \|\tilde{W}\|^2 - \|W^*\|^2$ and $2\mathrm{tr}(\tilde{V}^T \hat{V}) \leq \|\tilde{V}\|_F^2 - \|V^*\|_F^2$, the preceding inequality becomes

$$\dot{V}_1 \leq -k_s|e|^3 - \frac{k_1 e^2}{f_u} - \frac{\dot{f}_u}{2f_u^2} e^2 + |e| \left(\alpha_0 + \alpha_1 |e| + \alpha_2 \|\tilde{W}\| + \alpha_3 \|\tilde{W}\| |e| + \varepsilon_l \right) - \frac{\delta_w}{2} (1 + e^2) \left(\|\tilde{W}\|^2 - \|W^*\|^2 \right) - \frac{\delta_v}{2} \left(\|\tilde{V}\|_F^2 - \|V^*\|_F^2 \right) \tag{39}$$

The inequality, Eq. (39), can be written as

$$\dot{V}_1 \leq -k_s|e|^3 - \frac{k_1 e^2}{f_u} + \left(\frac{|\dot{f}_u|}{2f_u^2} + \alpha_1 + \frac{\delta_w}{2} \|W^*\|^2 \right) e^2 - \frac{\delta_w}{2} e^2 \|\tilde{W}\|^2 - \frac{\delta_w}{2} \|\tilde{W}\|^2 - \frac{\delta_v}{2} \|\tilde{V}\|_F^2 + (\alpha_0 + \varepsilon_l)|e| + \alpha_2 \|\tilde{W}\| |e| + \alpha_3 \|\tilde{W}\| e^2 + \frac{\delta_w}{2} \|W^*\|^2 + \frac{\delta_v}{2} \|V^*\|_F^2 \tag{40}$$

From Assumption 2, $0 < f^L \leq f_u$, where $|\dot{f}_u| \leq H$, which, in turn, yields $|\dot{f}_u|/2f_u^2 \leq H/2(f^L)^2$. Then, the following upper bound for the time derivative of V_1 is obtained

$$\dot{V}_1 \leq -k_s|e|^3 - \frac{k_1 e^2}{f_u} + \left[\frac{H}{2(f^L)^2} + \alpha_1 + \frac{\delta_w}{2} \|W^*\|^2 \right] e^2 - \frac{\delta_w}{2} e^2 \|\tilde{W}\|^2 - \frac{\delta_w}{2} \|\tilde{W}\|^2 - \frac{\delta_v}{2} \|\tilde{V}\|_F^2 + (\alpha_0 + \varepsilon_l)|e| + \alpha_2 \|\tilde{W}\| |e| + \alpha_3 \|\tilde{W}\| e^2 + \frac{\delta_w}{2} \|W^*\|^2 + \frac{\delta_v}{2} \|V^*\|_F^2 \tag{41}$$

Using the inequality $\alpha ab \leq (\delta/4) a^2 b^2 + (\alpha^2/\delta)$ (for any real positive constants α and δ) and Assumption 3, Eq. (41) can be written as

$$\dot{V}_1 \leq -k_s|e|^3 - \frac{k_1 e^2}{f_u} - \frac{\delta_w}{2} \|\tilde{W}\|^2 - \frac{\delta_v}{2} \|\tilde{V}\|_F^2 + \beta_0 e^2 + \beta_1 |e| + \beta_2 \tag{42}$$

where β_0 to β_2 are positive constants defined by

Table 1. Simulation Parameters

Parameter	Value
Cabin volume	$100\ \mathrm{m}^3$
Cabin temperature	20°C
Supply flow	1,600 kg/h
Diameter of exhaust valve	170 mm
Exhaust valve flow coefficient (subsonic mode)	0.95
Exhaust valve flow coefficient (supersonic mode)	0.99

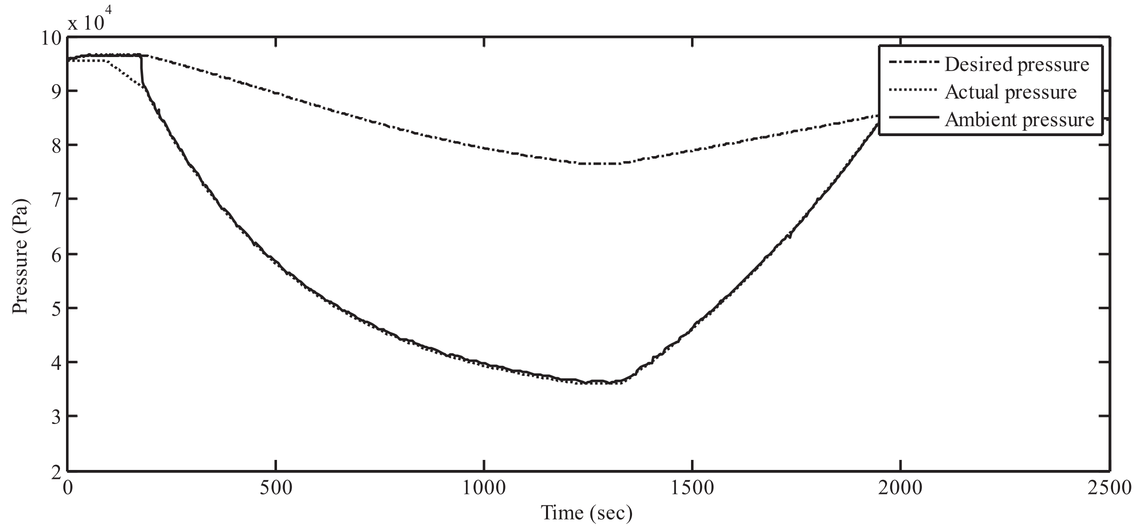

Fig. 2. Performance of PI controller

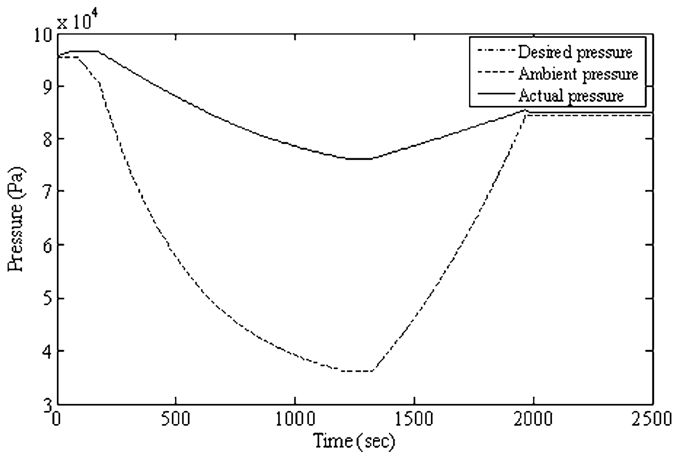

Fig. 3. Output tracking of desired cabin pressure using NN controller

$$\beta_0 = \frac{H}{2(f^L)^2} + \alpha_1 + \frac{\delta_w}{2}\|w_m\|^2 + \frac{\alpha_3^2}{\delta_w}$$
$$\beta_1 = \varepsilon_l + \alpha_0 \tag{43}$$
$$\beta_2 = \frac{\delta_w}{2}\|w_m\|^2 + \frac{\delta_v}{2}\|v_m\|^2 + \frac{\alpha_2^2}{\delta_w}$$

Using the inequality $\alpha ab \le (\delta/4)a^2b^2 + \alpha^2/\delta$, $2\beta_0 e^2 \le (k_s/2)|e|^3 + (2\beta_0^2/k_s)|e|$ and $[(2\beta_0^2/k_s) + \beta_1]|e| \le \beta_0 e^2 + (2\beta_0^2 + \beta_1 k_s)^2/4k_s^2\beta_0$. Adding these inequalities gives

$$\beta_0 e^2 + \beta_1|e| \le \frac{k_s}{2}|e|^3 + \frac{(2\beta_0^2 + \beta_1 k_s)^2}{4k_s^2\beta_0} \tag{44}$$

Now the inequality Eq. (42) can be written as

$$\dot{V}_1 \le -\frac{k_s}{2}|e|^3 - \frac{k_1 e^2}{f_u} - \frac{\delta_w}{2}\|\tilde{W}\|^2 - \frac{\delta_v}{2}\|\tilde{V}\|_F^2 + D \tag{45}$$

where $D = \beta_2 + (2\beta_0^2 + \beta_1 k_s)^2/4k_s^2\beta_0$. Now the following are defined:

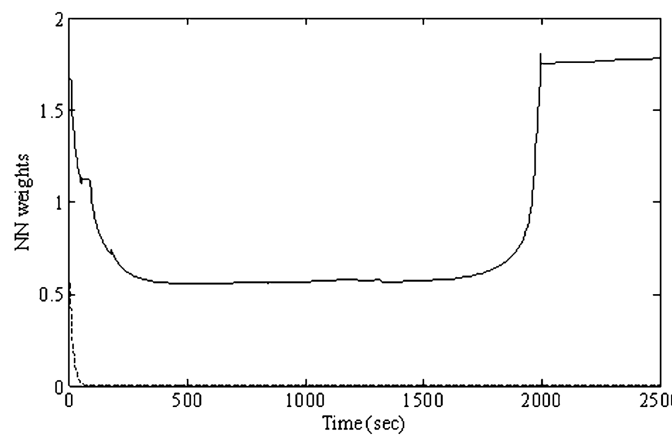

Fig. 4. NN weights $\|\hat{w}\|$ (solid data line) and $|\hat{v}|_F$ (dashed data line)

$$\Xi_e \triangleq \left\{ e \,|\, |e| \le \min\left[\left(\frac{2D}{k_s}\right)^{1/3}, \sqrt{Df_u/k_1}\right]\right\}$$
$$\Xi_w \triangleq \left[\left(\tilde{W}, \tilde{V}\right) \Big| \|\tilde{W}\| \le \sqrt{2D/\delta_w},\ \|\tilde{V}\|_F \le \sqrt{2D/\delta_v}\right]$$
$$\Xi \triangleq \left[\left(e, \tilde{W}, \tilde{V}\right) \Big| \frac{k_s}{2}|e|^3 + \frac{k_1 e^2}{f_u} + \frac{\delta_w}{2}\|\tilde{W}\|^2 + \frac{\delta_v}{2}\|\tilde{V}\|_F^2 \le D\right] \tag{46}$$

Because D, f_u, k_1 and δ_v are positive constants, it can be concluded that Ξ_e, Ξ_w, and Ξ are compact sets, and then $\dot{V}_1 < 0$ when V_1 is outside the Ξ. According to the light of the Lyapunov-like theorem presented on p. 172 of the paper by Khalil (2002), the closed-loop system is globally uniformly ultimately bounded, and a finite time, T_0, exists such that $t > T_0$, and e converges to a neighborhood of the zero. The upper bound of e in this neighborhood is $\min[(2D/k_s)^{1/3}, \sqrt{Df_u/k_1}]$ and can be arbitrarily reduced as D decreases by choosing δ_w and δ_v properly. It is easy to see that an increase in the number of the multilayer perceptron (MLP) hidden layer neurons causes D to decrease sufficiently. This completes the proof (quod erat demonstrandum).

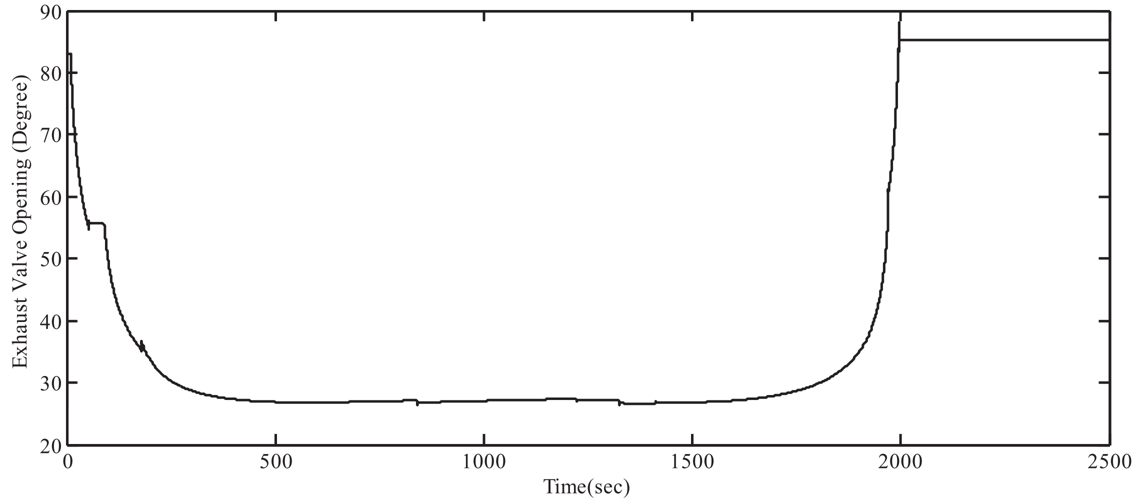

Fig. 5. History of control input u

Simulation

In this section, the proposed adaptive controller is applied to control the aircraft cabin pressure system. As mentioned in "Mathematical Model and Error Dynamic Derivation" [Eqs. (4)–(7)], Eqs. (47) and (48) define the nonlinear equations that describe operation of the aircraft cabin pressure system, and in an actual flight, both of these modes happen.

In subsonic mode, $P_c/P_{\rm atm} < 1.893$ and

$$\frac{dx}{dt} = \frac{RT_c}{V_c}\left[W_i - 0.95\mu_{\rm sub}A_0(1-\cos u)\sqrt{\frac{3}{\gamma RT}P_{\rm atm}(x-P_{\rm atm})}\right] \tag{47}$$

In supersonic mode, $P_c/P_{\rm atm} \geq 1.893$ and

$$\frac{dx}{dt} = \frac{RT_c}{V_c}\left[W_i - \mu_{\rm super}A_0(1-\cos u)x\sqrt{\frac{\gamma}{RT}\left(\frac{2}{\gamma+1}\right)^{\gamma+1/\gamma-1}}\right] \tag{48}$$

where state variable $x = P_c$ is the aircraft cabin pressure; the control input $u = \theta$ is the angle of outflow valve; and $y = P_c$ is the output of the system. Table 1 gives the parameters of the aircraft cabin pressure.

Obviously, the model is in a nonaffine form. To show the effectiveness of the proposed method, two different controllers are studied for the purpose of comparison. A fixed-gain PI control law is first used as follows (Yurkevich 2008):

$$u = \left(0.01e + \frac{1}{1{,}000}\int_0^t e d\tau\right) \tag{49}$$

The parameters of the PI controller are selected to give an adequate response for the aircraft cabin pressure system. Fig. 2 shows the desired, ambient, and actual pressures. The PI controller cannot provide a good tracking response due to the effects of the nonlinearities.

The adaptive controller based on the MLP, proposed in "Proposed Adaptive Controller: Design and Stability Analysis," is then applied to this system. The controller is taken as

$$u(t) = \hat{W}^T \boldsymbol{\psi}\left(\hat{V}^T \bar{z}\right) + u_{pi} - k_s|e_s|e_s \tag{50}$$

where $\hat{W}$ and $\hat{V}$ are updated by the adaptive laws Eqs. (29) and (30). The parameters in the weight-updating laws are chosen as $\Gamma_w = 2$, $\Gamma_v = 100$, $\delta_w = 0.25$, and $\delta_v = 7$. It is found that the magnitude of this term is much smaller than that of the neural networks. Therefore, the neural networks play a major role in improving the control performance. The input vector of MNNs is chosen as $\bar{\mathbf{z}} = [x, x_d, v, 1]^T$. The gain $k_s = 0.1$ and the NN node number $l = 3$ are used in the simulation. The initial conditions of cabin pressure $[x(0)]$ depend on airport elevation. The initial weight of $\hat{W}$ and $\hat{V}$ are simply set to zero.

Fig. 3 shows the simulation result for the designed controller. Comparing the results in Figs. 2 and 3, it can be observed that the proposed adaptive neural network controller presents a more desirable performance and the actual pressure follows the desired pressure completely. Fig. 4 indicates the boundedness of the NN weight estimates. Fig. 5 shows the history of the control input.

Conclusion

This paper developed an adaptive controller using multilayer neural networks for the nonaffine-nonlinear dynamic of an aircraft cabin pressure control system. The proposed adaptive controller guarantees the closed-loop ultimately boundedness and convergence of the tracking error to a small neighborhood of zero. Simulation results have been presented to confirm the validity of the proposed controller.

Acknowledgments

The authors gratefully acknowledge the helpful comments and suggestions of the editor and reviewers, which have improved the presentation.

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