# PM locating for cogging torque reduction in a single-phase surface-mounted PM motor

**Jawad Faiz, A.H. Tavakol-zadeh, Gh. Shahgholian**

*Surface-mounted permanent magnet (PM) is a well-known structure in PM machines. Low armature reaction and capability of standing high power load can be achieved by choosing the slotted surface PMs in machines. This structure has uniform air gap. Existing slots leads to a cogging toque which can be alleviated using different methods. Most design-based methods are: skewing PMs slots, changing PM arc length, changing radial depth of pole shoe, and using small slots in each pole. It is shown that PMs skewing by an half pole pitch is a proper technique in redcing the cogging torque. However, this technique needs structural changes which can decrease or vary the amplitude of the main torque. Most cogging torque alleviation techniques reduce the back emf, and therefore the starting torque. So, key point in the motor design is minimizing the cogging torque and maximizing the output torque of the motor.*

## Introduction

Permanent magnet (PM) motors are widely used electrical motors because of their high torque density and efficiency. Although a surface-mounted PM (SMPM) motor has a simple structure compared to an inset-PM type motor, it has some drawbacks such as difficulty in its demagnetization. Also, the SMPM motor is under heavy centrifugal force. PM merely covers full pole pitch, because the flux passing N and S poles penetrates between the poles with no link with the stator windings. Generally, cogging torque presented in this motor must be minimized using analytical [1] or numerical methods. Impact of slot and stator design on the cogging torque has been studied in [2], [3]. The effect of both slot opening shifting and pole arc length on the cogging torque has been proposed in [4]. A four sliced method on an SMPM motor has been applied in [5]. Slot shapes and various windings topologies impacts on the cogging torque have been considered in [6]. A frozen permeability FEM has been applied to show the load effect upon the cogging torque and back-emf waveform [7]. Furthermore, the effectiveness of the rotor skew on the minimization of the cogging torque has been shown in [8].

This paper describes the cogging torque and its causes and gives the methods of torque and cogging torque evaluation. Different reshaped PMs are employed and compared in a smooth pole single-phase PM motor with distributed winding in order to reduce the cogging torque. The merits of SMPM motor in this respect are described and the results of applications of cogging torque reduction methods are compared.

## Surface-mounted PM Machines

Slot-less electrical motors can develop a smooth torque leading to a low-noise machines. In the case of slot-less and tooth-less structure, there is a large air gap for current dependent magnetic field of the windings. This can diminish armature reaction leading to a better performance in over-loads region. Back-electromotive force (emf) in the stator windings does not vary by the change of motor load. Motor with low inductance needs a large switching frequency converter which increases the converter losses. If motor structure has slots, Ferrite PMs approaches a reasonable torque density and this reduces the materials volume and core losses. The residual flux within the core can reduce the required iron volume in the motor.

Different methods may be applied to reduce the cogging torque and its relevant noise. SMPM motor has negligible armature reaction and it can stand the over-load. An uniform air gap, in slot-less SMPM leads to a low stator inductance which is traditionally desirable in the design of a fast torque controller for motor. It is clear that there are saliencies in the SMPM motors but PM motors with inset PMs have characteristics similar with salient pole synchronous machines. An SMPM motor with conventional winding has no reluctance torque; therefore, its converter design is easier than that of the PMs-inset motors.

Advantages of SMPM include:

1. Use of less magnetic materials compared to the PMs-inset machine.
2. Manufacturing technique is well-known, easy and economical.
3. PMs are stuck on the metallic non-magnetic cylinder by paste and ribbons.

Disadvantages of SMPM include:

1. Lower capability against centrifugal force over high speeds.
2. Considerable cogging torque.

## Cogging Torque Calculation

Although PM machines are high performance devices, presence of current and voltage harmonics in the machine generates torque ripples. On the other hand, interaction between rotor PMs and stator teeth leads to the cogging torque. This torque creates distortion, vibration and noise in of the motor and may be 25% up to the rated torque. This cogging torque must be reduced to 1-2% of the rated torque. The reasons for this cogging torque include non-sinusoidal current waveform, non-similarity of back-emf and current waveforms, and stator slots. An alternative form of this torque is as follows [9], [10]:

(1) $$T_{cog} = \frac{\pi L_{ef} N_L}{4\mu_0}(R_2^2 - R_1^2)\sum_{i=1}^{\infty} i G_{a_i N_L} B_{a_i N_L} \sin(i N_L \theta)$$

where $N_L$ is the least common multiple of slot number and pole number, $L_{ef}$ is the effective length, $R_2$ is the air gap external radius and $R_1$ is the air gap inner radius and:

(2) $$B_{a_i N_L} = \frac{2N_p}{i\pi N_L} B_g^2 \sin(i N_L \frac{\alpha_p}{N_p}\pi)$$

(3) $$\alpha_p = \frac{\sum_{i=1}^{\infty} \frac{2}{i\pi} f(i) \sin(\frac{i\pi}{2})}{\sum_{i=1}^{\infty} f(i)}$$

(4) $$f(i) = \sin(\frac{i\pi\alpha_p}{2}) \frac{1}{(\frac{iN_p}{2})-1} (\frac{R_m}{R_s})^{\frac{iN_p}{2}+1} \times \left\{ \frac{(\frac{iN_p}{2}-1) + 2(\frac{R_r}{R_m})^{\frac{iN_p}{2}+1} - (\frac{iN_p}{2}+1)(\frac{R_r}{R_m})^{iN_p}}{\frac{\mu_r+1}{\mu_r}\left[1-(\frac{R_r}{R_s})^{iN_p}\right] - \frac{\mu_r-1}{\mu_r}\left[(\frac{R_m}{R_s})^{iNp} - (\frac{R_r}{R_m})^{iN_p}\right]} \right\}$$

(5) $$G_{a_i N_L} = \frac{2N_s}{\pi} \times \left[\int_{\frac{b0}{2}}^{\frac{\pi}{N_s}} \cos(i N_L \theta) d\theta + \int_0^{\frac{b0}{2}} \frac{(\frac{hm}{\mu_r} + g C_\phi)^2 \cos(i N_L \theta)}{[\frac{h_m}{\mu_r} + (g - R_2 + R_2 \cos(\frac{b_0}{2} - \theta)) + \frac{\pi R_2}{2}(\frac{b_0}{2} - \theta))C_\varphi]^2}\right]$$

(6) $$C_\phi = \frac{R_2 - g - h_m/2}{R_2 - g/2}$$

where $\alpha_p$ is the ratio of PM arc and pole pitch, $\mu_r$ is the iron relative permeability, $R_s$ is the stator core radius, $R_r$ is the rotor core radius, $R_m$ is the rotor core radius including PM and $B_\delta$ is the air gap peak flux density. The above-equations are based on Fig. 1.

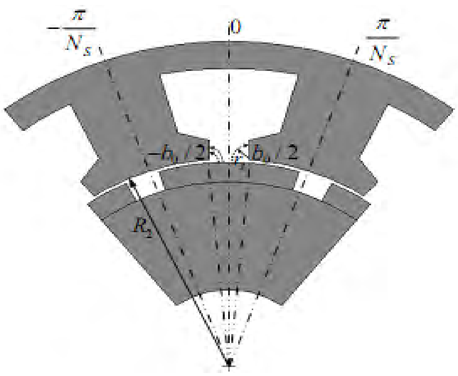


*Fig. 1. Slot and PM in PM motor.*

In an SMPM, the cogging torque depends on the motor type and its design details. Therefore, this torque depends on the design of individual parts and design technique. The contribution of PM in the cogging torque development is as follows:

(7) $$T_{cog} = \sum_{k=4}^{\infty} T_{pN_sK} \sin(N_s k\theta)$$

where $T_{PNsK}$ is the PM factor depending on the air gap flux density. The effective air gap flux density, $B_i$, is:

(8) $$B_i = K_{fi} K_{osi} K_{si} B_g$$

where $K_{fi}$ comes from $i^{th}$ spatial harmonic and $K_{osi}$ is the slot opening factor of $i^{th}$ harmonic, $K_{si}$ is the skewing factor of $i^{th}$ harmonic and $B_g$ is the air gap flux density. So, the cogging torque can be reduced by skewing the PM slots, varying the PM arc length, varying radial depth and using small slots in the pole [11].

## Simulation Results

The single-phase SMPM motor specifications have been given in Table 1.

**Table 1**

*Specifications on SMPM motor*

| | |
|---|---|
| Number of poles | 6 |
| Frequency | 50 Hz |
| Speed | 1000 rpm |
| Core length | 5 cm |
| Rotor radius | 6.68 cm |
| Stator radius | 9.79 cm |
| Type of PM | NdFeB |
| PM thickness | 1 cm |

Fig. 2 shows the PM motor with PM used in the rotor and no air gap between PMs [9, 10]. It is impossible to have continuous skewing in the rotor PMs and they are axially divided into several rows. The PM is skewed non-continuously in three pieces (a half of pole pitch) as shown in Fig. 3 [9].

Space between the poles is normally covered by the pieces of non-magnetic materials. Therefore, no flux passes over the air gap between the PMs. The complete arc of the pole is 180 electrical degrees generating a full voltage waveform which contains current harmonics. By reducing the pole arc length and filling the remaining rooms by soft metals, near sinusoidal output flux waveform containing less harmonics is achieved and rotor losses become lower; therefore, the optimal pole arc length is as follows:

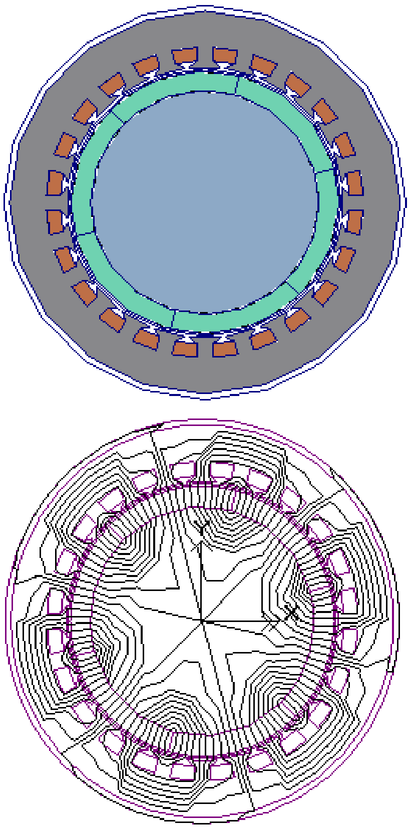

*Fig. 2. Continuous PM inserted on rotor.*

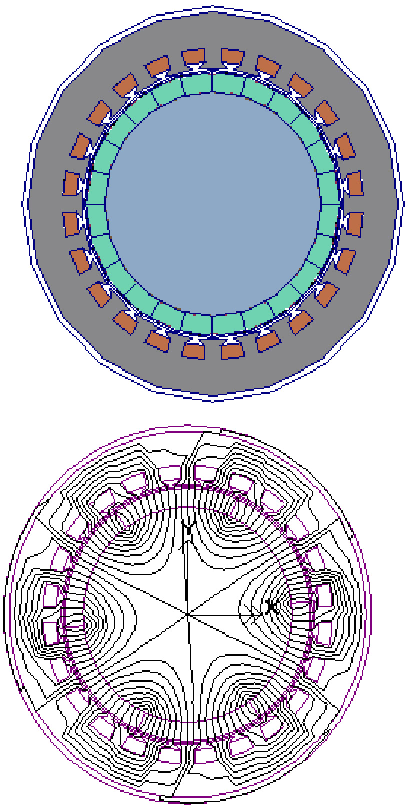

*Fig. 3. Locating PMs with skewing.*

(9) $$\alpha_m = \frac{n+\nu}{N_{sm}} < 10, < \nu < 1$$

where $\alpha_m$ is the pole arc and pole pitch ratio, $n$ is an integer, $N_{sm}$ is the number of slots under each pole, and υ is the parameter developing minimum cogging torque. There is a single value for $\upsilon$ which minimizes the $i^{th}$ harmonic of cogging torque. If υ tends to zero, the pole arc will be 40.8 degrees and the motor will be as shown in Fig. 4 [9], [10].

PM shifting in the conventional PM motors is the reason for cogging torque due to each PM of the phase and in total this generates a large cogging torque. To prevent this incremental effect, the PMs can be shifted against each other. This slightly depends on integer number of slots under each pole. So, each structure must be individually addressed. In motors with integer number of slots under each pole, each pole is seen as a complete multiple of the stator teeth. Therefore, impact of the teeth of each PM appears in each phase and total cogging torque is determined as follows:

(10) $$T_{cog} = N_p \sum_{k=1}^{\infty} T_{N_S K} \sin(N_s k\theta)$$

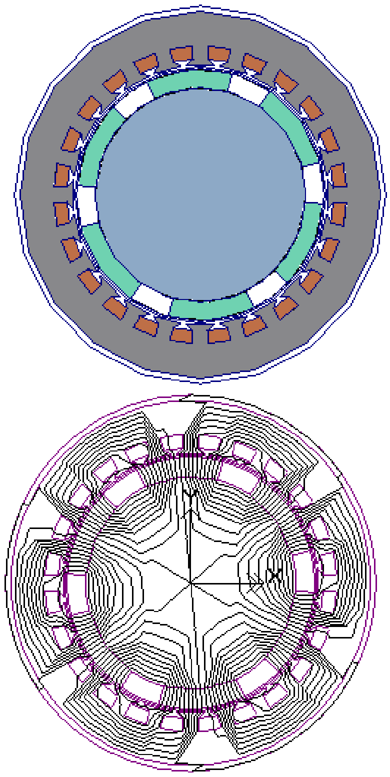

*Fig. 4. Locating ruptured PMs.*

The base frequency in (10) is $N_s$ times mechanical rotation; it means that for each machine with integer number of slots under the pole, the least common multiple of the slots number and pole numbers m is simply equal to the number of slots $N_s$. The reason is that each PM of phase has a rest period and is shifted with each other properly. The total cogging torque in the motor is:

(11) $$T_{cog} = \sum_{h=0}^{N_t-1} \sum_{k=1}^{\infty} T_{P N_s K} \sin(N_s k(\theta - h\theta_0))$$

where $\theta_0$ is the shift angle of each PM against the rest. To conceal the impact on the cogging torque harmonics and the high harmonics, $\theta_0$ must be as follows:

(12) $$\theta_0 = \frac{2\pi}{N_s N_t}$$

Consequently the total cogging torque is as follows:

(13) $$T_{cog} = \sum_{k=1}^{\infty} T_{N_s N_p K} \sin(N_s N_p k\theta)$$

If in a SMPM motor with 40.8 degrees PM pitch shifts 10 degrees, the structure will be as shown in Fig. 5.

The total cogging torque in PM motor with the slot under each pole per phase and PM piece is determined as follows [11]:

(14) $$T_c = \sum_{l=1}^{N} \sum_{i=1}^{\infty} T_i^l \sin(iN_s(\theta + \alpha_l))$$

(15) $$\alpha_l = ks_w + \sum_{j=1}^{k} \beta_j$$

where $N$ is the number of pieces in each PM, $K$ is the number of pieces that must be calculated, $s_w$ is the width of each piece in electrical degree, $\beta_j$ is the air gap width between the PM pieces. Here, each PM is divided into 5 pieces and number of calculated pieces is 4. Width of each piece is 24.48 electrical degrees and air gap between them is 1 degree as shown in Fig.6.

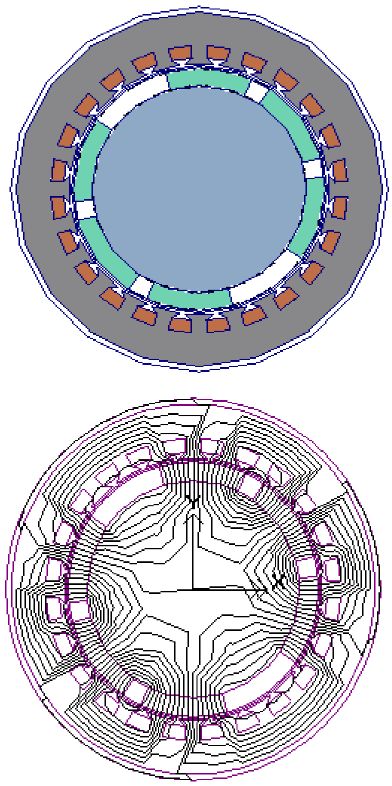

*Fig. 5. SMPMs with PM pitch of 40.8 degrees shifted 10 degrees.*

## Comparison of Different Configurations

For cogging torque reduction in a continuous PM machine (Fig. 2), PMs are skewed and ruptured in rotor and compared in Fig. 7. On the other hand, PMs in rotor are ruptured (Fig. 4), skewed, shifted and sliced in rotor and compared in Fig. 8. The impact of the skewing PM is clear while the PM is fixed non-continuously. It can be realized that skewing PM has the most significant impact on the cogging torque.

## Discussion and Conclusion

It was shown that PM skewing by half pole pitch in PM motor is the best technique for decreasing the cogging torque. However, building such PM motor is

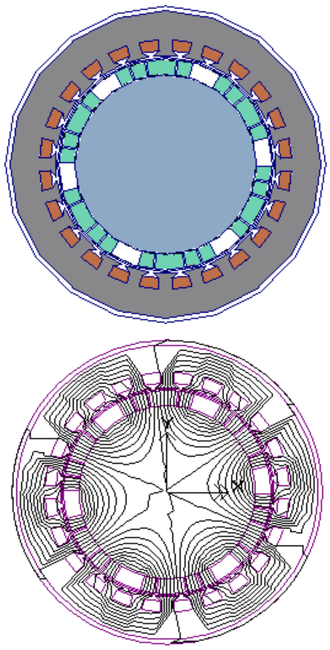

*Fig. 6. Sliced PM locating.*

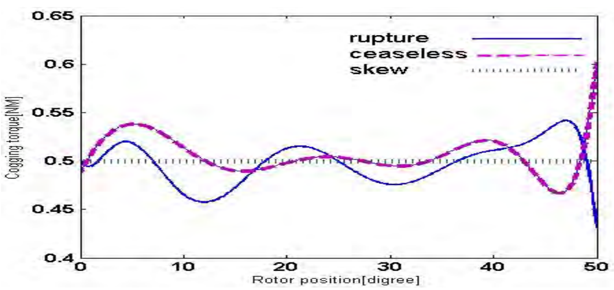


*Fig. 7. Comparison of cogging torque reduction methods in continuous PM-inset rotor.*

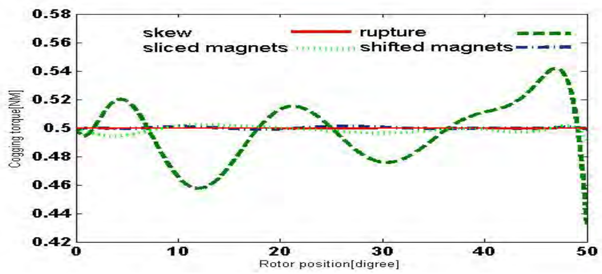


*Fig. 8. Comparison cogging torque reduction methods in ruptured PM rotor.*

difficult and costly. In SMPM motor, excitation field is quasi-rectangular. The cogging torque amplitude can be reduced by some percentage with no skewing. For triple non- continuity, frequency of the cogging torque increases and this is very useful in direct drive PM generators. Skewing reduces the load torque ripples but does not decrease the amplitude of the cogging torque in the same level and harmonic frequency of minimum load torque is not doubled.

Also some methods vary the spatial harmonics of the torque by the air gap flux density ($B_g$) variations. In the SMPM, the continuous current distribution is changed by PM reshaping. The PM can be reshaped as tangential of the axial in order to generate a continuous sinusoidal current. By reshaping the PMs, several harmonics can be properly eliminated from the torque. The reason is that the PM volume and stacking factor of rotor is reduced and no-load voltage decreases. To increase the rotor stacking factor the 3$^{rd}$ harmonic can be added to the PMs. Reshaping the PM does not change the assembly process. The reshaping PMs and locating them in skewing mode result in the best quality torque development similar with buried rectangular PMs. Reshaping and skewing PM double the frequency of the cogging torque. It is recommended that if skewing is difficult asymmetrical locating can be replaced. For instance, PM in one part can be located as such that PMs spaces become less than a full pole pitch (creating more rooms between the parts). This locating can reduces the cogging torque noticeably; there is a similar feature when segmented PM structure is applied. However, in asymmetrical structure there is no capability of multiplying the cogging torque ripple frequency as skewing mode. The major drawback of this method is combination of the generated sub-harmonics due to PM asymmetry. The rotor sub-harmonics are too harmful. Asymmetrical locating and skewing the stator slots can restrict the sub-frequencies without touching the rotor. However, stator asymmetrical slots prevent the sub-harmonics in the induced torque, but the load torque ripple in one harmonic is lower than that of the slot-less case. In the segmented structure, there is a specific number of the PM segments and the air gap between them which widen the pole width, reduce the torque and increase the sub-harmonics.

The cogging toque can be reduced by the above-mentioned techniques but these methods come along with structures change which can also reduce the main torque which is more harmful. So, choosing these structures depend on the designer skill, design object and efficiency of the proposed machine. In other words, the most techniques for cogging torque reduction lead to the decrease of the back emf and starting torque. Therefore, the key point in the motor design is the minimizing of the cogging torque while maximizing the output torque.

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## References


[1] Zarko, D., D. Ban, T.A. Lipo. Analytical Solution for Cogging Torque in Surface Permanent-Magnet Motors Using Conformal Mapping. IEEE Trans. on Magn., Vol. 44, No. 1, pp. 52-65, Jan. 2008.

[2] Zhu, L., S.Z. Jiang, Z.Q. Zhu, C. C. Chan. Optimal slot opening in permanent magnet machines for minimum cogging torque. Przegland Elektrotechniczny (Electrical Review), ISSN 0033-2097, pp. 315-319, R. 87 NR 3/2011.

[3] Chen, N., S.L. Ho, W.N. Fu. Optimization of Permanent Magnet Surface Shapes of Electric Motors for Minimization of Cogging Torque Using FEM. IEEE Trans. on Magn., Vol. 46, No. 6, pp. 2478 – 2481, June 2010

[4] Dosiek, L., P. Pillay. Cogging Torque Reduction in Permanent Magnet Machines. IEEE Trans, on Indus. Appli., Vol. 43, No. 6, Nov./Dec. 2007.

[5] Polinder, H., M.J. Hoeijmakers. Eddy-current losses segmented surface-mounted magnets of pm machines. IEEE Porc-Elcter. Power Appl., Vol. 146, No. 3, pp. 442–445, May 1999.

[6] Zhu, Z.Q., D. Howe, B. Ekkehard, and B. Ackermann. Instantaneous magnetic field distribution in brushless permanent magnet motors, part I:open-circuit field. IEEE Trans. Magn., Vol. 29, pp. 124–134, Jan. 1993.

[7] Zhu, Z.Q., G. Ombach. Influence of Electric Loading and Magnetic Saturation on Cogging Torque, Back-EMF and Torque Ripple of PM Machines. Trans. on Magn., Vol. 48, No. 10, pp.. 2650=2658, Oct. 2012.

[8] Yuejun, A., A.Hui, S.Dan, Z. Wenqiang, Xue Liping Ombach . Study on cogging torque in PM motor with pole width modulation rotor. IEEE Conf Publications, pp. 371 - 374, 24-27 June 2012.

[9] Bianchi, N., S. Bolognani. Design techniques for reducing the cogging torque in surface-mounted PM motors. IEEE Trans. on Ind. Appl.,Vol. 38, No. 5, pp. 1259–1265, Sep. 2002.

[10] Zhu, Z.Q., L, J. Wu, D.A.. Staton, M. Popseco, D. Hawkins. Comparison of Analytical Models of Cogging Torque in Surface-Mounted PM Machines. IEEE Trans. on Magn., Vo. 59, No. 6, pp. 2414–2425, Oct .2012.

[11] Wang, Y., J. Zhu, S. wang, Y. Guo, W. XU. Nonlinear Magnetic Model of Surface Mounted PM Machines Incorporating Saturation Saliency. IEEE Trans.on Magn., Vol. 45, No. 10, pp. 4684–4687, Oct., 2009.

[12] Gieras, J.F. Analytical approach to cogging torque calculation of PM brushless motors. IEEE Trans. on Ind. Appl., Vol. 40, No. 5, pp. 1310–1316.


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