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Simple Analytical and Robust Controller Design for Two-Mass Resonant System

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***Abstract*— The main objective of this research work is analysis and design controller for torque control of two-mass resonant system. The integral of time multiplied by absolute error (ITAE) is applied to assign the closed-loop poles of the system characteristic equation. The simulation results show that the transfer of torque has a better performance.**

Keywords- Two mass system; torque controller; ITAE criterion.

I. INTRODUCTION

In some industrial applications like the rolling mill drives, elevator systems, flexible arms and so on, the mechanical part of the system has very low resonant frequency, because of a long flexible shaft between the motor and the load machine. The torsional vibration is generated by the torsion of the spindle which couples the motor to the load [1, 2]. A typical configuration of steel rolling mil system is show in Fig. 1.

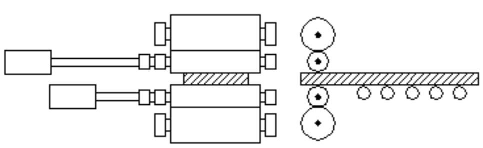

Figure 1. The steel rolling mill system

A mechanical system composed of some masses connected with flexible shafts can be modeled by a multi-mass system. The simplest model for the torsional system is the two-inertia model. Mechanical resonance is one of the most common problems designers face when trying to maximize either command response or dynamic stiffness. To vibration suppression of the two-mass resonant system, various controllers such as classical pole-placement method [3], linear H_∞ controller [4] and PID controller [5, 6] are proposed. A vibration control of multi-mass resonant system based on the phase-lead compensator based on the disturbance observer technique proposed in [7]. In [8], the discrete-time sliding mode control has been applied for two-mass system coupled by flexible shaft where a full order observer is used to estimate the necessary state variables. In [9] torque controller designs with PID-P controller using the coefficient diagram method (CDM). In [10] pole-placement controller such as coefficient diagram method and integral of time multiplied by absolute error is used to assign closed-loop poles of the system characteristic equation.

Always, a simple PID controller used in two-mass system for suppresses the mechanical resonance and improve the transfer performance of torque. In this paper, a PID-P controller by using ITAE criterion is suggested and compared with PID controller. Section II describes the modeling of the two-mass resonant system with controller. Section III presents some simulation results that show the transient performance of the proposed algorithm and compares it with the conventional PID controller. A brief summary is given in section IV.

II. MATHEMATICAL MODEL OF THE STUDY SYSTEM

Fig. 2 show the system configuration of a two-mass resonant system. The system parameters used in this paper listed on Table I.

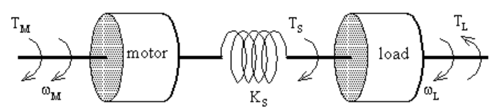

Figure 2. Two-mass resonant system

TABLE I. NOMINAL PARAMETERS OF THE TWO-MASS PLANT

Symbol	Quantity	Value
K_S	shaft stiffness	410 Nm/rad
B_S	shaft damping coefficient	0.025 Nm/rad/s
J_M	motor inertia	0.02 kg.m^2
B_M	motor viscosity coefficient	0.002 Nm/rad/s
J_L	load inertia	0.333 kg.m^2
B_L	load viscosity coefficient	0.065 Nm/rad/s
K_J	inertia ratio	2.0060
ω_R	resonant frequency	60.8 rad/s
ω_A	anti-resonant frequency	35.1rad/s

In this section, the dynamic analysis of two-mass resonant system has been made using of the small signal model for the open loop and closed loop system. This analysis based on state space equation and transfer functions.

A. Open loop system

A simple state-space model of the two-mass system with three state variables: ω_L (load speed), ω_M (motor speed) and

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T_S (shaft torsional torque) and two inputs: T_M (motor torque) and T_L (load disturbance torque) is given as follows:

$$\frac{d}{dt}X = \begin{bmatrix} -\frac{B_M}{J_M} & -\frac{1}{J_M} & 0 \\ K_S - \frac{B_M B_S}{J_M} & -B_S(\frac{1}{J_M}+\frac{1}{J_L}) & -(K_S - \frac{B_L B_S}{J_L}) \\ 0 & \frac{1}{J_L} & -\frac{B_L}{J_L} \end{bmatrix} X + \begin{bmatrix} \frac{1}{J_M} & 0 \\ \frac{B_S}{J_M} & \frac{B_S}{J_L} \\ 0 & -\frac{1}{J_L} \end{bmatrix} U \quad (1)$$

where:

$$X = [\omega_M \quad T_S \quad \omega_L]^T \quad (2)$$

$$U = [T_M \quad T_L]^T \quad (3)$$

The block diagram of the compliantly coupled mechanism is shown in Fig. 3.

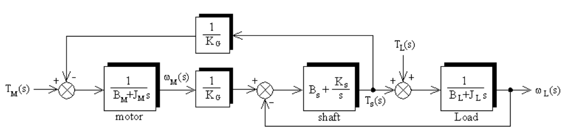

Figure 3. Block diagram of two-mass resonant system

Usually in mechanical systems, damping is small and it can be neglected without affecting the accuracy of the analysis of the control system. The characteristic equation in open-loop system with ignored of viscosity coefficient of load and motor is:

$$\Delta(s) = s(s^2 + 2\eta\omega_R s + \omega_R^2) \quad (4)$$

The damping ratio η and resonant frequency ω_R are given by:

$$\omega_R = \sqrt{\frac{K_S}{J_L}(1+K_J)} \quad (5)$$

$$\eta = \frac{B_S}{2}\sqrt{\frac{1}{K_S J_L}(1+K_J)} \quad (6)$$

where $K_J=J_L/J_M$ is inertia ratio. The anti-resonant frequency is:

$$\omega_A = \sqrt{\frac{K_S}{J_L}} \quad (7)$$

If K_J and B_S are small, mechanical vibration occurs easily.

B. Close loop system

The configuration for control close-loop system is shown in Fig.4, consists of two loops namely PID controller and feedback compensation, in which $T_C(s)$ is torque command input, $G_T(s)$ is transfer function of PID controller and $G_F(s)$ is the transfer function of feedback compensation.

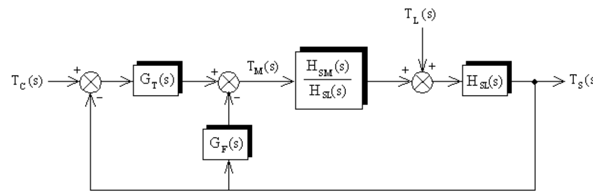

Figure 4. Torque control system using PID controller with feedback compensation

The symbol $H_{SL}(s)$ and $H_{SM}(s)$ denotes the open loop transfer function from the shaft torque to load torque and the shaft torque to motor torque, respectively. The s domain form of controller transfer function is as follows:

$$G_T(s) = K_{PT} + \frac{K_{IT}}{s} + K_{DT}s \quad (8)$$

$$G_F(s) = K_{PF} + \frac{K_{IF}}{s} + K_{DF}s \quad (9)$$

where K_{PT} and K_{PF} are the proportional gain, K_{IT} and K_{IF} are integration gain, and K_{DT} and K_{DF} are derivative gain. The transfer functions of the open-loop system that the relationship describing the dynamics of the system under consideration can be represented as:

$$H_{SL}(s) = \left.\frac{T_S(s)}{T_L(s)}\right|_{T_M=0} = \frac{1}{J_L}\frac{s(B_S s + K_S)}{\Delta(s)} \quad (10)$$

$$H_{SM}(s) = \left.\frac{T_S(s)}{T_M(s)}\right|_{T_L=0} = \frac{1}{J_M}\frac{s(B_S s + K_S)}{\Delta(s)} \quad (11)$$

The closed loop transfer function from the torque command input to the shaft torsional torque is given by:

$$H_{CF}(s) = \left.\frac{T_S(s)}{T_C(s)}\right|_{T_L=0} = \frac{G_T(s)H_{SM}(s)}{1+[G_T(s)+G_F(s)]H_{SM}(s)} \quad (12)$$

The basic characteristic of the transient response of a closed loop system is closely related to the location of the closed loop poles. The closed loop poles are roots of the characteristic equation:

$$\Delta_{FT}(s) = J_M s^3 + K_S (K_{DT} + K_{DF}) s^2 + (J_M \omega_R^2 + K_{PT}K_S + K_{PF}K_S)s + K_S (K_{IT} + K_{IF}) \quad (13)$$

III. CONTROLLER DESIGN

The gains controller is evaluated using the integral of time multiplied by the absolute error (ITAE) criterion for a step reference input. The ITAE performance index provides the best selectivity by minimizing overshoot and settling time for a given undershoot. As for the ITAE criterion, the standard form coefficients for a step input can be expressed as (7), where ω_n represents the 3dB bandwidth [11].

$$\Delta(s) = s^3 + 1.75\,\omega_n s^2 + 2.15\,\omega_n^2 s + \omega_n^3 \quad (14)$$

The coefficients that will minimize the ITAE performance criterion for a step input have been determined for the general closed-loop transfer function. The gains of the controller obtained from ITAE criterion are given by:

$$\begin{cases} K_D = K_{DT} + K_{DF} = \dfrac{1.75\,\omega_n J_M}{K_S} \\ K_P = K_{PT} + K_{PF} = \dfrac{J_M (2.15\,\omega_n^2 - \omega_R^2)}{K_S} \\ K_I = K_{IT} + K_{IF} = \dfrac{J_M\, \omega_n^3}{K_S} \end{cases} \quad (15)$$

By analyzing the root Hurwitz, a sufficient condition of system stability with PID control as:

$$K_D > \frac{J_M K_I}{K_P K_S + J_M \omega_R^2} \quad (16)$$

$$K_I < \frac{K_D (K_P K_S + J_M \omega_R^2)}{J_M} \quad (17)$$

IV. SIMULATION RESULTS AND DISCUSSIONS

To prove the effectiveness of the proposed technique, various simulation results using Matlab/Simulink are shown under system different cases.

The dominant eigenvalues for open-loop system are P_1=-0.1365 and $p_{2,3}$=-0.1515±j60.8365. The resonant frequency is ω_R=60.8 rad/s and anti-resonant frequency is ω_A=35.1 rad/s. The damping factor of the original plant without the controller is η=0.0019. Since all eigenvalues of the system are on the left hand of the plane, the system is stable but highly damped. The frequency responses of variables state (shaft torsional torque, motor speed and load speed) to motor torque in open loop system are show in Fig. 5. The step response of shaft torque relative to motor torque (line) and load torque (dash-line) for open-loop system show in Fig. 6. The step response of motor speed relative to motor torque in open-loop system is show in Fig. 7.

The controller parameters for three different 3dB bandwidths are given in Table II. The step response of motor torque and shaft torque in close-loop system for different control structures consists only PID controller, PID-P controller, PI-PD controller and I-PD controller are show in Figs. 8 and 9, respectively. It is seen that the PID controller with compensation reducing the overshoot and improving the transient response.

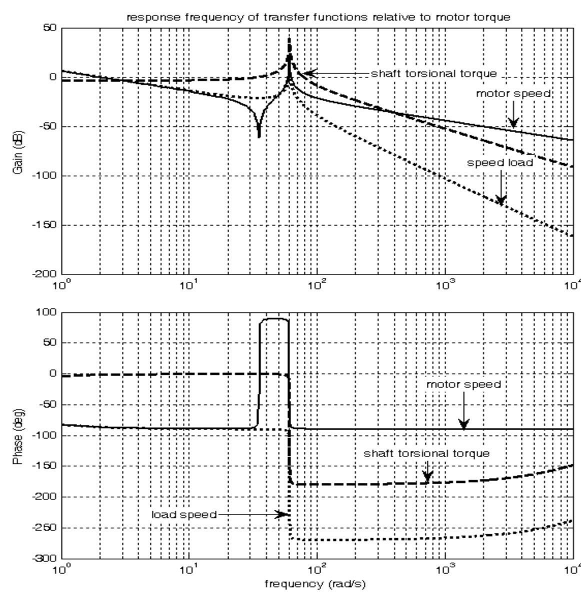

Figure 5. The frequency response of variables state to motor torque in open loop system

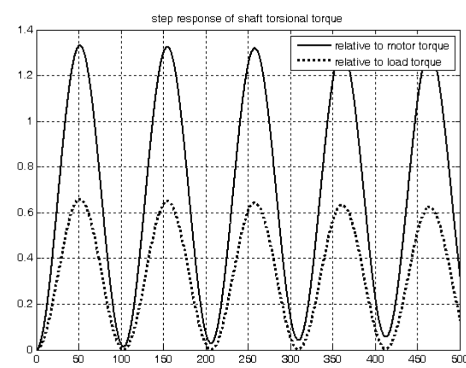

Figure 6. Step response of the shaft torque in open-loop system

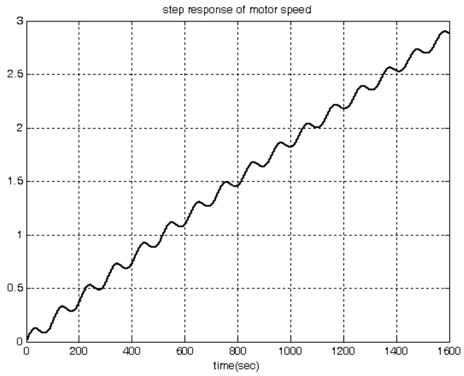

Figure 7. Step response of the motor speed

TABLE II. CONTROLLERS PARAMETERS

ω_n	K_P	K_I	K_D
ω_R	1.7233	91.1638	0.0431
0.6820 ω_R	0	28.9178	0.0294
$(\omega_R + \omega_A)/2$	0.5040	44.6723	0.0340

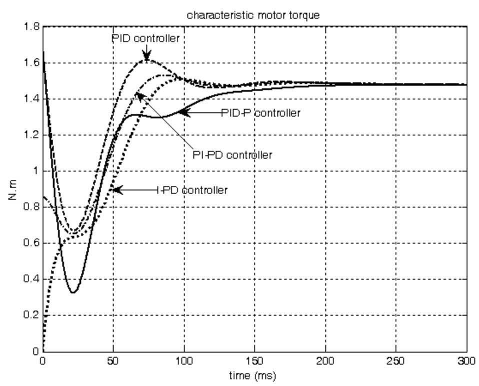

Figure 8. Step response of the motor torque in close-loop system

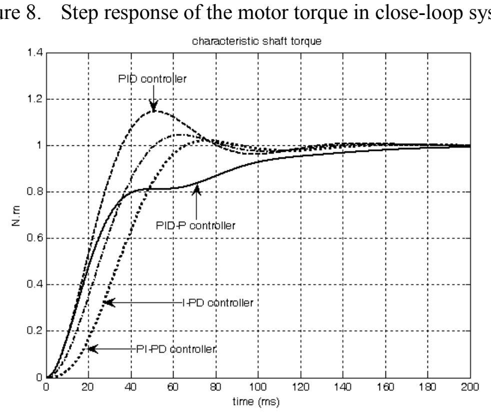

Figure 9. Step response of the shaft torque in close-loop system

The different control structures have the same characteristic equations and it can be seen that the zero are different. Therefore the overshoot in the shaft torque, for a step change in the input reference, is expected to smaller for the I-PD control.

V. CONCLUSION

Different control structures are presented and compared. The design method of the control structure relies on the ITAE criterion. The simulation results confirm the improvement of torque transfer performance in closed loop operation using PID controller with feedback compensation.

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