State Space Analysis and Control Design of Two-Mass Resonant System

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***Abstract*— In this paper a two-mass resonant system with a speed controller by resonance ratio control (RRC) has been simulated. The state space technique for two-mass system is used. A linear transfer function model is also developed. The gains of the controller can be designed by coefficient diagram method (CDM). The simulation results are illustrated that the proposed controller improves dynamic performance.**

Keywords- Two mass system; speed control; resonance ratio control; optimal criterion.

I. INTRODUCTION

In general motion control application the flexible system composed of a motor and load with flexible shaft. In mechanical resonance system, the measurement of system variables is hard to obtain. When not all system variables are measurable, the controller design system is very difficult [1-2]. A simplest model of a multi-mass resonance system is two-mass resonance system. Various controllers for two-mass such as classical control [3, 4], approximations and system identification [5], resonance ratio control [6, 7] and optimal control [8] are proposed. A systematic comparative study of compensation schemes such as resonance ratio control and PID control for the coordinated motion control of two-inertia mechanical systems presented in [9]. Neural estimators of the load speed and torsional torque in the open- and closed-loop control structure proposed in [10], which additional feedbacks from the shaft torque and the difference between the motor and the load speeds is used. The tasks of speed controller for two-mass resonant system are suppression of shaft torsional vibration, faster tracking the load speed of the speed reference without overshoot, rejection the effect of the load disturbance torque and robust controller. In this paper, a resonance ratio control using by optimal criterion for two-mass system is design. The remainder of this paper is organized as follows. In section 2, the state-space model of two-mass mechanical system in open-loop is demonstrated. In section 3, the control structure with a feedback signal from the shaft torque is described. The simulation results in the open- and close-loop system are show in section 4. Then, brief summary follow in section 5.

II. TWO-MASS MECHANICAL SYSTEM

The two-mass resonant system which has a motor and a load connected with a flexible shaft is shown in Fig. 1.

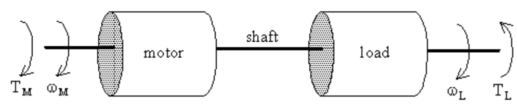

Figure 1. Two-mass resonant system

Such systems are often found in the steel mill drives. The state equation of the two-mass resonant system is as follows:

$$\frac{d}{dt}\omega_M = -\frac{B_M}{J_M}\omega_M - \frac{1}{J_M}T_S + \frac{1}{J_M}T_M \quad (1)$$

$$\frac{d}{dt}T_S = (K_S - \frac{B_M B_S}{J_M})\,\omega_M - (K_S - \frac{B_L B_S}{J_L})\,\omega_L$$
$$- B_S(\frac{1}{J_M} + \frac{1}{J_L})\,T_S + \frac{B_S}{J_M}T_M + \frac{B_S}{J_L}T_L \quad (2)$$

$$\frac{d}{dt}\omega_L = -\frac{B_L}{J_L}\omega_L + \frac{1}{J_L}T_S - \frac{1}{J_L}T_L \quad (3)$$

where:
J_M, J_L: the motor inertia, the load inertia
B_M, B_L: the motor viscous damping coefficient, the load viscous damping coefficient
K_S: the shaft stiffness
ω_M, ω_L: the motor speed, the load speed
T_M, T_S, T_L: the motor torque, the shaft torsional torque, the load disturbance torque

Increasing the inertia ratio K_J=J_L/J_M and shaft stiffness will decrease the mechanical vibration of the system.

With the state equations the circuit can be easily modeled by using the functional blocks. The simplified block diagram of the two-mass system is shown in Fig. 2. Generally, the speed ω_M and position θ_M of the motor shaft differ from the respective variables ω_L and θ_L, on the load side. During transients, speeds of motor and load differ, and torsional torque is given by:

$$T_S(s) = B_S\,[\omega_M(s) - \omega_L(s)] + K_S\,[\theta_M(s) - \theta_L(s)] \quad (4)$$

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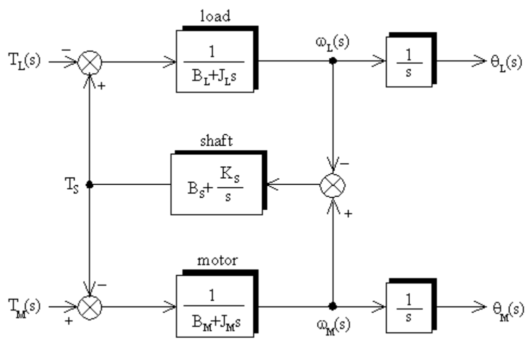

Figure 2. Block Diagram of the two-mass mechanical system

The transfer function from T_M to ω_M, which plays an important role in the closed loop design, is given by:

$$H_M(s) = \frac{\omega_M(s)}{T_M(s)} = \frac{1}{J_L J_M} \frac{J_L s^2 + (B_L + B_S)s + K_S}{\Delta(s)} \quad (5)$$

where the characteristic equation of the open loop system is given as:

$$\Delta(s) = s^3 + \left(\frac{B_M + B_S}{J_M} + \frac{B_L + B_S}{J_L}\right)s^2 + \left(\frac{K_S(J_M + J_L) + B_S(B_M + B_L) + B_M B_L}{J_M J_L}\right)s + \frac{K_S(B_M + B_L)}{J_M J_L} \quad (6)$$

Since damping losses usually considered being relatively low, they are neglected with out significantly affecting the accuracy of the forgoing analysis. Neglecting friction terms, the transfer function $H_M(s)$ is given by:

$$H_M(s) = \frac{1}{J_M} \frac{s^2 + \omega_A^2}{s(s^2 + \omega_R^2)} \quad (7)$$

We see that ω_R represents the mechanical resonance frequency of the open-loop system. The anti-resonance frequency ω_A is lower thanω_R.

$$\omega_A = \sqrt{\frac{K_S}{J_L}} \quad (8)$$

$$\omega_R = \sqrt{K_S\left(\frac{1}{J_M} + \frac{1}{J_L}\right)} \quad (9)$$

The control bandwidth in closed-loop motion control system is limited by the ω_A. An increase in the motor inertia constant decreases the ω_R without affecting the ω_A. Conversely, increasing the mechanical stiffness of the shaft (K_S) increases both ω_R and ω_A, and also the mechanical bandwidth in closed-loop system. Therefore, in the large systems, which have large inertias, generally produce low natural frequencies. The system parameters used in this paper for two cases are listed on Table I.

TABLE I. TWO-MASS SYSTEM PARAMETERS

Symbol	Value	Unit
K_S	138	Nm/rad
B_S	0.1000	Nm/rad/s
J_M	0.0480	kg.m^2
B_M	0.0013	Nm/rad/s
J_L	0.0086	kg.m^2
B_L	0.0690	Nm/rad/s
K_J	0.1792	-
ω_A	126.7	rad/s
ω_R	137.6	rad/s

Mechanical systems are notoriously under damped, hence ascertaining the damping factors amounts to an educated guess. The dominant eigenvalues for open-loop system are P_1=-1.24 and $P_{2,3}$= -10.26±j137.14. The damping factor of the original plant without the controller is η=0.0498. The step response of shaft torque due change in motor torque and load torque without controller is shown in Fig. 3. In this figures, the dashed line and the thick solid line show corresponds relative to motor torque and relative to load torque. Figs. 4-5 show the open-loop frequency response from the motor speed, shaft torque and load speed to the motor torque and to load torque, respectively.

III. RESONANCE RATIO CONTROL

In this section various close-loop transfer functions describing the dynamic of the system following change in reference speed and disturbance torque are derived. Fig. 6 show the complete block diagram of the two-mass system with resonance ratio controller. The controller structure consists of an integral controller and proportional compensation with a additional feedback signal from the shaft torque with transfer function $G_R(s)$.

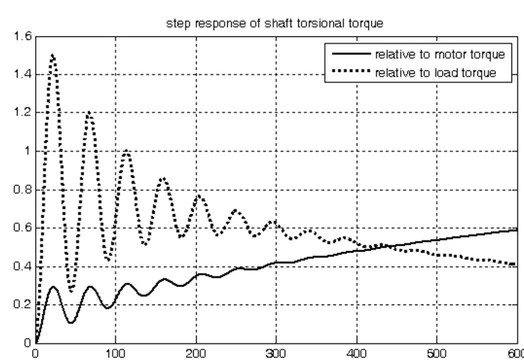

Figure 3. Step response of shaft torque in open-loop system

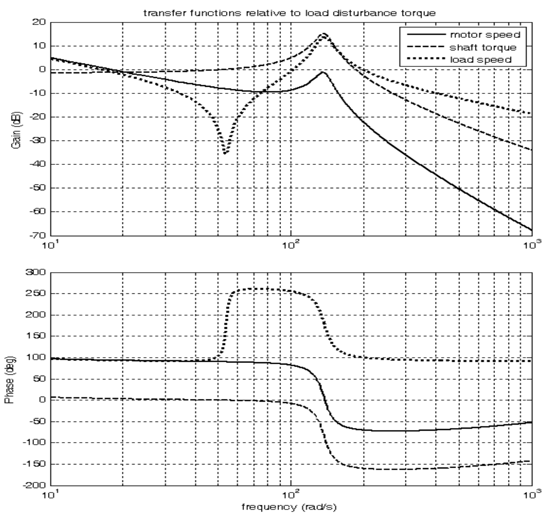

Figure 4. Frequency response characteristics of transfer function relative to motor torque

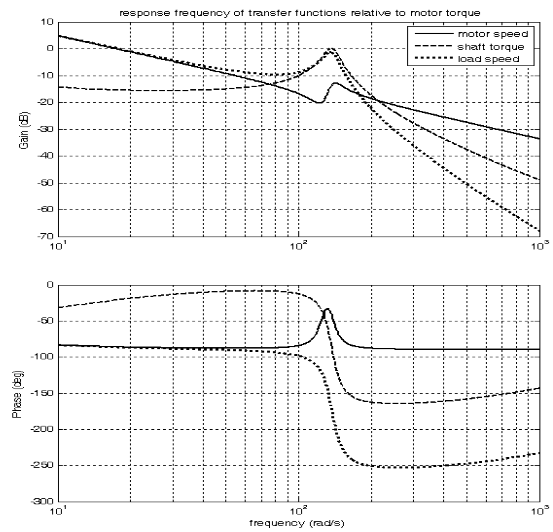

Figure 5. Frequency response characteristics of transfer function relative to motor torque

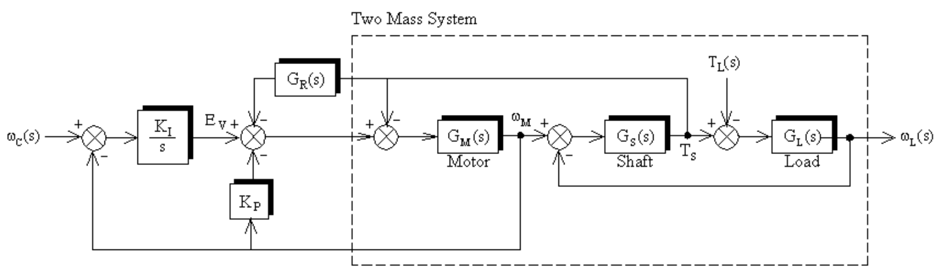

Figure 6. Block diagram of control system

The motor speed and load speed are depends on the disturbance torque T_L and reference speed ω_C:

$$\omega_M(s) = H_{RS}(s)\,\omega_C(s) - H_{RT}(s)\,T_L(s) \quad (10)$$

$$\omega_L(s) = H_{RC}(s)\,\omega_C(s) - H_{RL}(s)\,T_L(s) \quad (11)$$

If $G_S(s)$, $G_M(s)$ and $G_L(s)$ are the transfer functions of shaft, motor and load, respectively, the close-loop transfer functions of the reference speed to motor speed and the reference speed to load speed (tracking dynamics) can be expressed in (12) and (13), respectively:

$$H_{RS}(s) = \frac{\omega_M(s)}{\omega_C(s)} = \frac{K_I\,G_M(s)[1+G_S(s)G_L(s)]}{\Delta(s)} \quad (12)$$

$$H_{RC}(s) = \frac{\omega_L(s)}{\omega_C(s)} = \frac{K_I\,G_M(s)G_S(s)G_L(s)}{\Delta(s)} \quad (13)$$

The close-loop transfer functions of the disturbance torque to motor speed and disturbance torque to load speed (regulation dynamics) can be expressed in (14) and (15), respectively:

$$H_{RT}(s) = \frac{\omega_M(s)}{T_L(s)} = \frac{s\,G_S(s)G_M(s)G_L(s)[1+G_R(s)]}{\Delta(s)} \quad (14)$$

$$H_{RL}(s) = \frac{\omega_L(s)}{T_L(s)} = \frac{G_L(s)}{\Delta(s)} [s+G_M(s)(sK_P+K_I+s\,G_S(s)+s\,G_S(s)G_R(s))] \quad (15)$$

where $\Delta(s)$ is given by:

$$\Delta(s) = [1+G_S(s)G_L(s)][s+(K_I+K_P\,s)G_M(s)] + s\,G_S(s)G_M(s)[1+G_R(s)] \quad (16)$$

If the resonance ratio control is PD, $G_R(s)=K_C+K_Ds$, the state-space equations of two-mass system with controller are given by:

$$\frac{d}{dt}\omega_M = [-\frac{K_DK_S}{J_M} + \frac{K_D}{J_M}\frac{B_S(B_M+K_P)}{J_M} - \frac{K_P+B_M}{J_M}]\omega_M + [\frac{K_DB_S}{J_M}(\frac{K_C}{J_M}+\frac{1}{J_L}) - \frac{K_C+1}{J_M}]T_S$$

$$+\frac{K_D}{J_M}[K_S-\frac{B_S\,B_L}{J_L}]\,\omega_L+\frac{1}{J_M}E_V+\frac{B_S}{J_L}T_L \qquad (17)$$

$$\frac{d}{dt}\omega_L=\frac{-B_L}{J_L}\omega_L+\frac{1}{J_L}T_S-\frac{1}{J_L}T_L \qquad (18)$$

$$\frac{d}{dt}T_S=[K_S-\frac{B_S\,(B_M+K_P)}{J_M}]\,\omega_M$$
$$+[-K_S+\frac{B_S\,B_L}{J_L}]\,\omega_L-B_S(\frac{K_C}{J_M}+\frac{1}{J_L})\,T_S+\frac{B_S}{J_L}T_L \qquad (19)$$

$$\frac{d}{dt}E_V=K_I(\omega_C-\omega_M) \qquad (20)$$

where K_C and K_D are gains proportional and derivative of controller.

The characteristic equation polynomial in close-loop with RRC is given by:

$$\Delta_{RRC}(s)=J_M\,s^4+(K_S\,K_D+K_P)s^3$$
$$+(J_M\,\omega_A^2+K_I+K_SK_C+K_S)s^2+K_P\,\omega_A^2\,s+K_I\,\omega_A^2 \qquad (21)$$

The Coefficient Diagram Method (CDM) is an indirect pole placement method to design an appropriate characteristic polynomial. The CDM can give a controller design which is both stable and robust, and it has the desired system response speed. CDM needs some design parameters with respect to the characteristic polynomial coefficients which are the equivalent time constant (τ) and the stability index (γ_k) [11]. The controller gains obtained from CDM criterion as followers:

$$K_I=\frac{J_M\,\gamma_1^3\,\gamma_2^3\,\gamma_3}{\omega_A^2\,\tau^4} \qquad (22)$$

$$K_P=\frac{J_M\,\gamma_1^3\,\gamma_2^3\,\gamma_3}{\omega_A^2\,\tau^3} \qquad (23)$$

$$K_C=[-J_M\omega_A^2+(\frac{\omega_A^2\,\tau^2}{\gamma_1}-1)\frac{J_M\,\gamma_1^3\,\gamma_2^3\,\gamma_3}{\omega_A^2\,\tau^3}-K_S]\frac{1}{K_S} \qquad (24)$$

$$K_D=\frac{J_M\,\gamma_1^3\,\gamma_2^3\,\gamma_3}{K_S\,\omega_A^2\,\tau^3}(\frac{\omega_A^2\,\tau^2}{\gamma_1^2\,\gamma_2}-1) \qquad (25)$$

In this criterion, for $K_C>0$ and $K_D>0$, the stability index must be (26) and (27), respectively.

$$\gamma_2<\frac{\tau^2\,\omega_A^2}{\gamma_1^2} \qquad (26)$$

$$\gamma_3>\frac{(K_J+1)\,\tau^4\omega_A^4}{\gamma_1^2\,\gamma_2^2\,(\tau^2\omega_A^2-\gamma_1)} \qquad (27)$$

IV. SIMULATION RESULTS

The simulation results of the speed control in the two-mass using the proposed controller will be shown in this section in order to demonstrate the efficiency of the controller. In order to confirm the performance of the controller, we accomplish a simulation study using Matlab Simulink as shown in Fig. 7.

The controller parameters from CDM criterion without and with derivative of the proposed control scheme of the system are listed in Tables II and III, respectively. In to cases the equivalent time constant is 0.0304 sec.

TABLE II. CONTROLLER PARAMETERS WITHOUT DERIVATIVE

index stability	K_P	K_I	K_D	K_C
γ_1=2.5 γ_2=2.37 γ_3=1	9.37	308.09	0	4.4292
γ_1=3 γ_2=1.65 γ_3=1.6	12.49	410.79	0	5.1564
γ_1=2.5 γ_2=2.37 γ_3=1.5	14.05	462.14	0	9.9345

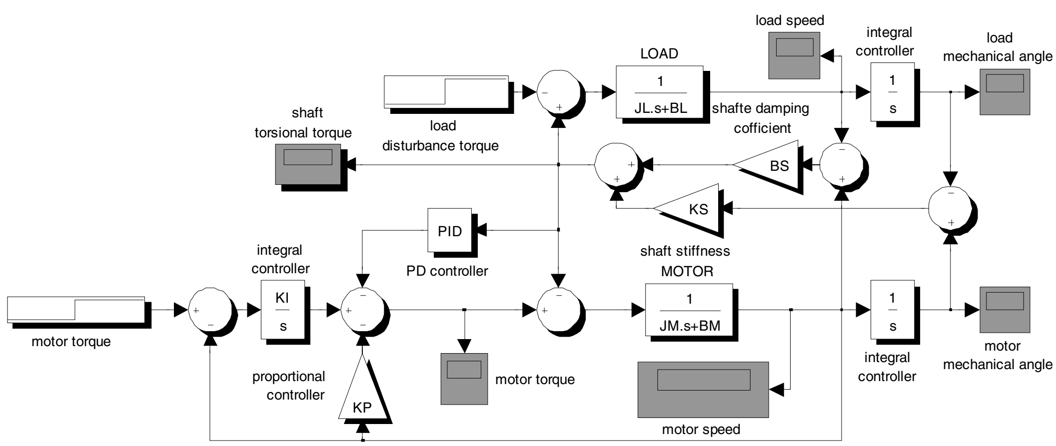

Figure 7. Block diagram of control system in Matlab/Simulink

TABLE III. CONTROLLER PARAMETERS WITH DERIVATIVE

index stability	K_P	K_I	K_D	K_C
γ_1=2.5 γ_2=2 γ_3=1	6.62	217.49	0.0091	1.2215
γ_1=2.5 γ_2=1.8 γ_3=1.5	8.05	264.25	0.0188	2.90
γ_1=3 γ_2=1.6 γ_3=1.6	11.72	384.84	0.0028	4.46

The dominant eigenvalues for close-loop system are $P_{1,2}$=-101.44±j144.85 and $p_{2,3}$=-66.13±j23.81 for without derivative and $p_{1,2}$=-51.78±j130.57, p_3=-128.78 and p_4=-50.93 for with derivative. Fig. 8 show the step response of motor speed in closed-loop system with RRC controller with and without derivative. Fig. 9 show the frequency response characteristics of transfer function of the reference speed to motor speed.

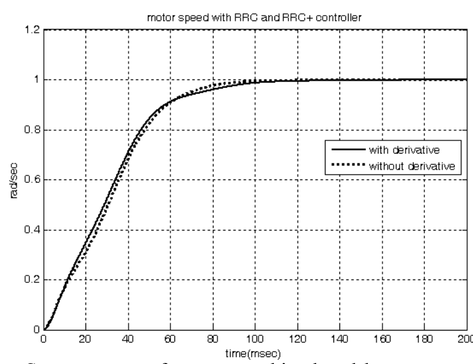

Figure 8. Step response of motor speed in closed-loop system with RRC controller

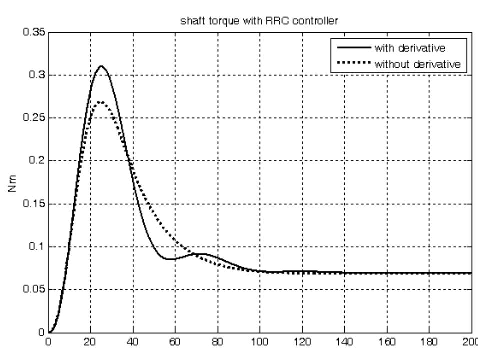

Figure 9. Shaft torque in closed-loop system with RRC controller

V. CONCULSION

A space-state mathematical analysis and design methods for a two-inertia system is develop. The simulation results in controlling the speed of the-mass system by the proposed controller are investigated to verify the effectiveness of the proposed method.

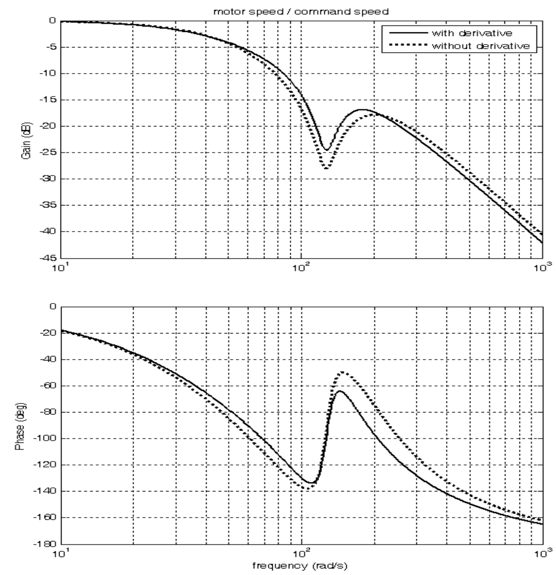

Figure 10. Frequency response characteristics of transfer function of the reference speed to motor speed

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