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Analysis and Design of a Linear Quadratic Regulator Control for Static Synchronous Compensator

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Abstract— **STATCOM control can improve the transmission capacity considerably and can thus enhance the transient stability margin of the power system. In this paper, an optimal control method based on linear quadratic regulator (LQR) control design for transient dynamic performance of STATCOM. The dynamic analysis is verified by transfer function simulation using Matlab and time domain simulation of the open-loop and close-loop system.**

Keywords- STATCOM; eigenvalue analysis; frequency response; LQR control.

I. INTRODUCTION

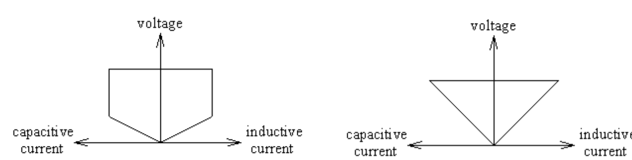
STATCOM is a voltage sourced converter based shunt flexible ac transmission system (FACTS) device, which is connected as a shunt to the network, for capable enhancing the power system damping by injection controllable reactive power into the system [1, 2]. With properly designed control, STATCOM can enhance system transfer limit and improve its dynamic behavior significantly especially in the interconnected power systems [3]. The FACTS controllers consist of the three main control schemes: steady-state power flow control, transient control for improving the first swing stability and power oscillation damping control to damp the power system oscillations [4, 5].

Many control strategies for STATCOM are reported in the literature. In [6] a vector control scheme for control of reactive current using STATCOM based on a nonlinear state feedback controller is proposed. In [7] is present a new control strategy based on input-output feedback linearization approach for a three-phase PWM converter, which consists of applying an adaptive back stepping control. An alternate algorithm to solve the optimal power flow problem incorporating FACTS devices in multi machine power system using genetic algorithm proposed in [8]. Linear quadratic regulator (LQR) approach is an optimal control method and is also a pole placement method. In [9] a hybrid PI/LQR control method and its parameter selection for STACOM control is proposed, which the simulation results show that this method is an effective method for STATCOM control applications.

In this paper a mathematical model of a STATCOM is presented from the control point of view. This paper is organized as follows. Sections II review the STATCOM system configuration and find the mathematical model to represent its dynamic characteristics. Section III presents a theoretical analysis of the LQR controller. Simulation results are provided and discussed in section IV. A brief summary is given in section V.

II. STATCOM MODELING

With FACTS technology bus voltages, line impedances and phase angles in the power system can be regulated rapidly and flexibly. Shunt FACTS devices may be variable impedance, variable source or a combination of theses. They inject current into the system at the point of connection. Shunt FACTS devices are classified into two categories, namely variable impedance type (SVC) and switching converter type (STATCOM). The voltage-current characteristic of SVC and STATCOM are shown in Fig.1. As can be seen in the linear operating range the voltage-current characteristic and functional compensation capability of the SVC and STATCOM are similar. The main advantage of a STATCOM over an SVC is its reduced size, which results from the elimination of ac capacitor banks and reactors. Also STATCOM can serve as a controllable current source without changing the network structure parameters and beyond the limitation of bus voltage.



(a) STATCOM (b) SVC

Figure 1. Voltage-current characteristics of the shunt FACTS device

The basic principle of operation of a STATCOM is the voltage source converter (VSC), which in general converters an input dc voltage into a three-phase output voltage at fundamental frequency, with rapidly controllable amplitude and phase angle [10]. In general, a STATCOM system can be divided into three key parts: the converter power stage, the passive components and the control system. Figs. 2 and 3 show a functional model and the equivalent circuit of a

STATCOM respectively. In this representation, the series inductance L_S accounts for the leakage of the transformer and R_S represents the active losses of the inverter and transformer. The dc capacitor (C_{DC}) has the function of establishing an energy balance between the input and output during the dynamic change of the var output. The size of the capacitor is primarily determined by the ripple input current encountered with the particular converter design. The charged capacitor C_{DC} provides a dc voltage to the converter, which produces a set of controllable three-phase output voltages with the frequency of the ac power system. The R_{DC} represent the sum of the switching losses of the inverter and power losses in the capacitor.

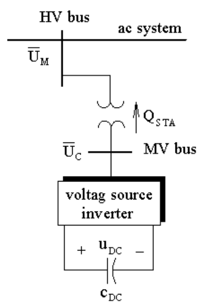


Figure 2. Shunt connected FACTS devices STATCOM

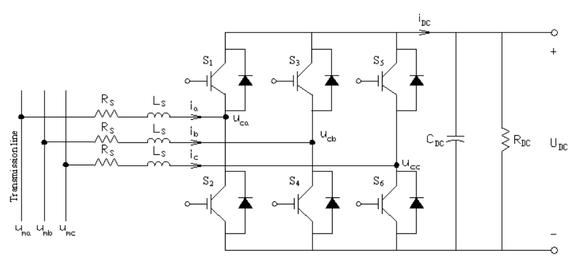


Figure 3. Three-phase PWM converter topology

By varying the amplitude of the output voltage U_C, the reactive power exchange between the converter and the ac system can be controlled. In the dc side, a resistance R_{DC} which represents the sum of the switching losses of the inverter and the power loss in the capacitor is in shunt with the capacitor. The three-phase VSC is modeled as a linear network with a topology that changes depending on the state of the six switching devices. The VSC is assumed to be lossless [11]. There are eight possible gating combinations as shown in Table I.

It is assumed that the source is a balanced, sinusoidal three-phase voltage supply with angular frequency ω:

$$\begin{cases} u_{ma}(t) = U_M \cos(\omega t + \theta) \\ u_{mb}(t) = U_M \cos(\omega t - \frac{2\pi}{3} + \theta) \\ u_{mc}(t) = U_M \cos(\omega t - \frac{4\pi}{3} + \theta) \end{cases} \quad (1)$$

TABLE I. THE SWITCHING STATES IN A THREE-PHASE VOLTAGE SOURCE INVERTER AND OUTPUT VOLTAGE

Mode	Switches			Line to line voltage		
	S_1	S_3	S_5	U_{ab}	U_{bc}	U_{ca}
A	ON	ON	ON	0	0	0
B	ON	ON	OFF	0	U_{DC}	$-U_{DC}$
C	ON	OFF	ON	U_{DC}	$-U_{DC}$	0
D	ON	OFF	OFF	U_{DC}	0	$-U_{DC}$
E	OFF	ON	ON	$-U_{DC}$	0	$-U_{DC}$
F	OFF	ON	OFF	$-U_{DC}$	U_{DC}	0
G	OFF	OFF	ON	0	$-U_{DC}$	U_{DC}
H	OFF	OFF	OFF	0	0	0

The loop equations for the circuit may be written in vector form as [12]:

$$\frac{d}{dt} i_{abc} = -\frac{R_S}{L_S} i_{abc} + \frac{1}{L_S}(U_{M,ABC} - U_{C,ABC}) \quad (2)$$

where the phase currents, converter phase voltages (inverter ac side) and transmission line voltages are:

$$i_{abc} = [i_a(t) \quad i_b(t) \quad i_c(t)]^T \quad (3)$$

$$U_{c,abc} = [u_{ca}(t) \quad u_{cb}(t) \quad u_{cc}(t)]^T \quad (4)$$

$$U_{m,abc} = [u_{ma}(t) \quad u_{mb}(t) \quad u_{mc}(t)]^T \quad (5)$$

The ac and dc side converter voltage are related by:

$$\frac{d}{dt} u_{DC} = \frac{1}{C_{DC}} i_{DC} - \frac{U_{DC}}{R_{DC} C_{DC}} \quad (6)$$

where U_{DC} is dc link capacitor voltage and i_{DC} is dc bus current.. The dc bus current bases on switching states are shown in Table II. Therefore ac and dc side currents are related by:

$$i_{DC} = S_A i_a + S_B i_b + S_C i_c \quad (7)$$

TABLE II. DC BUS CURRENT BASE ON THE SWITCHING STATES

Mode	dc current
A	0
B	$i_a+i_b=-i_c$
C	$i_a+i_c=-i_b$
D	$-i_b-i_c=i_a$
E	$i_b+i_c=-i_a$
F	$-i_a-i_c=i_b$
G	$-i_a-i_b=i_c$
H	0

where S_A, S_B and S_C are inverter switching function in three-phase and represent the gating signals applied to the upper three switches. The output voltage of the inverter is a three phase positive sequence system and its magnitude is in proportion with the capacitor voltage. The ac and dc side converter voltage are related by:

$$\begin{cases} u_{ca} = U_{DC}\,(S_A - \dfrac{S_A + S_B + S_C}{3}) \\ u_{cb} = U_{DC}\,(S_B - \dfrac{S_A + S_B + S_C}{3}) \\ u_{cc} = U_{DC}\,(S_C - \dfrac{S_A + S_B + S_C}{3}) \end{cases} \qquad (8)$$

The state space model of STATCOM in rotating dq coordinate system has three state variables i_{Sd}, i_{Sq} and U_{DC} and two control variables S_D and S_Q:

$$\frac{d}{dt} i_{Sd} = -\frac{R_S}{L_S} i_{Sd} + \omega i_{Sq} + \frac{1}{L_S}(U_{cd} - U_{md}) \qquad (9)$$

$$\frac{d}{dt} i_{Sq} = -\frac{R_S}{L_S} i_{Sq} - \omega i_{Sd} + \frac{1}{L_S}(U_{cq} - U_{mq}) \qquad (10)$$

$$\frac{d}{dt} U_{DC} = \frac{3}{2C_{DC}} S_D i_{Sd} + \frac{3}{2C_{DC}} S_Q i_{Sq} - \frac{1}{R_{DC} C_{DC}} U_{DC} \qquad (11)$$

where ω is rotation speed, S_D and S_Q are d-axis and q-axis synchronous reference frame inverter switching function, U_{cd} and U_{cq} d-axis and q-axis are synchronous reference frame source voltage, i_q and i_d are synchronous reference frame STATCOM current. S_D and S_Q represent dq transfo-rmed switching functions. The circuit equivalent of STACOM in dq synchronous frame is given in Fig. 4.

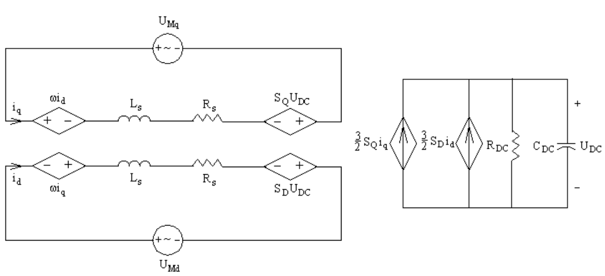


Figure 4. The circuit equivalent of STATCOM in qd reference frame

The set equations can be written in matrix form as:

$$\frac{d}{dt}\begin{bmatrix} i_{sd} \\ i_{sq} \\ U_{DC} \end{bmatrix} = \begin{bmatrix} -\frac{R_S}{L_S} & \omega & 0 \\ -\omega & -\frac{R_S}{L_S} & 0 \\ 0 & 0 & -\frac{1}{R_{DC}C_{DC}} \end{bmatrix} \begin{bmatrix} i_{sd} \\ i_{sq} \\ U_{DC} \end{bmatrix}$$

$$+ \frac{1}{L_S}\begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} U_{Md} \\ U_{Mq} \end{bmatrix}$$

$$+ \begin{bmatrix} -\frac{1}{L_S} U_{DC} & 0 \\ 0 & -\frac{1}{L_S} U_{DC} \\ \frac{3}{2C_{DC}} i_{sd} & \frac{3}{2C_{DC}} i_{qs} \end{bmatrix} \begin{bmatrix} S_D \\ S_Q \end{bmatrix} \qquad (12)$$

If U_M considered as a system constant parameter, it would not exist in small signal model. The full-state linearized model of the system around operating point can be obtained as follows:

$$\frac{d}{dt}\begin{bmatrix} \Delta i_{sd} \\ \Delta i_{sq} \\ \Delta U_{DC} \end{bmatrix} = \begin{bmatrix} -\frac{R_S}{L_S} & \omega & -\frac{1}{L_S} S_{DO} \\ -\omega & -\frac{R_S}{L_S} & -\frac{1}{L_S} S_{QO} \\ \frac{3}{2C_{DC}} S_{DO} & \frac{3}{2C_{DC}} S_{QO} & -\frac{1}{R_{DC}C_{DC}} \end{bmatrix} \begin{bmatrix} \Delta i_{sd} \\ \Delta i_{sq} \\ \Delta U_{DC} \end{bmatrix}$$

$$+ \begin{bmatrix} \frac{1}{L_S} S_{QO} U_{DCO} & -\frac{1}{L_S} U_{DCO} \cos\phi_o \\ -\frac{1}{L_S} S_{DO} U_{DCO} & -\frac{1}{L_S} U_{DCO} \sin\phi_o \\ \frac{3(S_{DO} I_{sqo} - S_{QO} I_{sdo})}{2C_{DC}} & \frac{3(I_{sdo}\cos\phi_o + I_{sqo}\sin\phi_o)}{2C_{DC}} \end{bmatrix} \begin{bmatrix} \Delta\phi \\ \Delta K_M \end{bmatrix} \qquad (13)$$

where the operating points are I_{do}, I_{qo}, U_{dco}, S_{DO} and S_{QO}. The block diagram of the system show in Fig. 5.

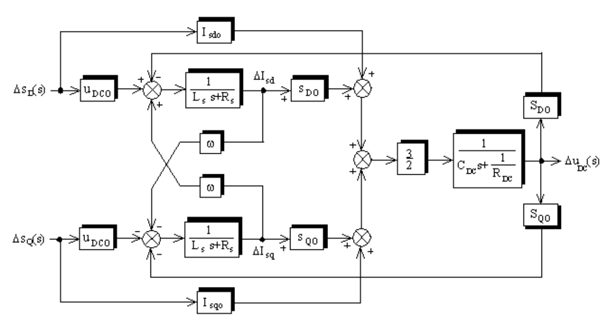


Figure 5. Block diagram of linear system

The dynamic response of a linear system is governed by the magnitude and location of its eigenvalues, or poles. The characteristic equation in open loop system is given by:

$$\Delta(s) = s^3 + a_2 s^2 + a_1 s + a_0 \qquad (14)$$

where:

$$a_2 = \frac{1}{R_{DC} C_{DC}} + \frac{2 R_S}{L_S}$$

$$a_1 = \frac{R_S}{L_S}\left(\frac{2}{R_{DC} C_{DC}} + \frac{3S_{QO}^2}{2 R_S C_{DC}} + \frac{R_S}{L_S}\right) + \omega^2 + \frac{3S_{DO}^2}{2 L_S C_{DC}}$$

$$a_0 = \frac{R_S}{L_S}\left(\frac{R_S}{R_{DC} C_{DC} L_S} + \frac{3S_{QO}^2}{2 L_S C_{DC}} + \frac{3S_{DO}^2}{2 L_S C_{DC}}\right) + \frac{\omega^2}{R_{DC} C_{DC}} \qquad (15)$$

III. STATCOM Control Analysis

The effect of the linear state feedback on STATCOM dynamic transient response is show in this section. An important property of state feedback is that it can be used to control the eigenvalues of a dynamical equation. One possibility for stabilization is to use a state feedback controller in which all the state variables can be measured or one version would be an output feedback controller in which only an output vector with a dimension typically less than that of the state can be measured. It is possible to place the closed-loop poles anywhere we wish in the complex s plane. Fig. 6 shows the pole placement controller for STATCOM applications, where K is a suitable gain matrix.

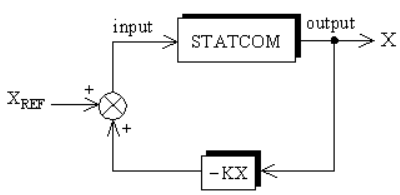


Figure 6. Pole placement controller block

LQR problem uses a state feedback control law to minimize the cost function defined a priori. The LQR is a useful tool in modern optimal control design, consisting of explicit matrix design equation easily solved in a digital computer. This method determines the place of the system poles by minimizing a given performance index. It is well known that LQR problem uses a state feedback control law to minimize the cost function defined a priori.

$$I = \int_0^\infty (X^T Q X + U^T R U + 2 X^T N U)\, dt \qquad (16)$$

where Q and R are real symmetric positive definite matrices. The matrix Q and R determine the relative importance of the error and the expenditure of this energy. The matrix [Q N; N^T R] should be positive semi-definite. The LQR control signal is given by $U_{opt}=KX_{opt}$, where $K=-R^{-1}B^TP$. The gains in the feedback matrix K can be obtained by solving the algebraic Riccati equation.

IV. Simulation Results

The dynamic characteristic of the system are typically defined in terms of the response to a unit-step input. This depends highly on the performance criteria of the design such as the rise time, overshoot, settling time, largest magnitude of the actuating signals, and so forth. The data used for the calculation of all operating curves is summarized in Table III. The operating point is U_{DC0}=3210V, S_{Q0}=0, S_{D0}=1.2732, I_{sd0}=21.4A, and I_{sq0}=-807A. The dominant eigenvalues for open-loop system are p_1=-6.6 and $p_{2,3}$=-9.73±j387.6. Note that these eigenvalue indicate that the STATCOM is a highly damped and stable system at this operating point. Fig. 7 show thr frequency response of capacitor voltage in open loop system. The reactive and active current deviations in the open-loop system show in Figs. 8 and 9.

TABLE III. Parameters of the STATCOM

quantity	symbol	value
line reactor inductance	L_S	3 mH
line reactor resistance	R_S	0.03 Ω
dc side capacitor	C_{DC}	0.1 F
capacitor shunt resistance	R_{DC}	78.5 Ω
angular frequency	ω	377 rad/s

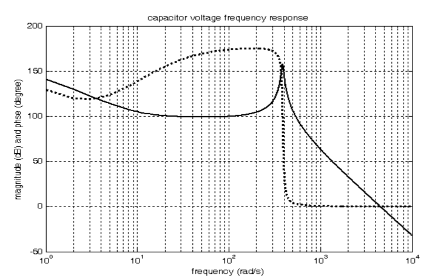


Figure 7. Capacitor voltage frequency response without controller

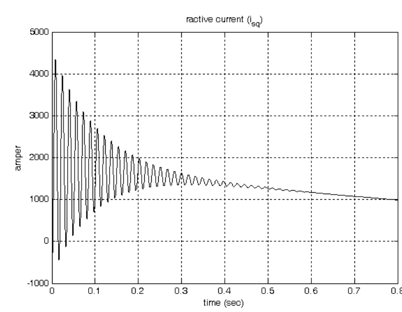


Figure 8. Reactive current without controller

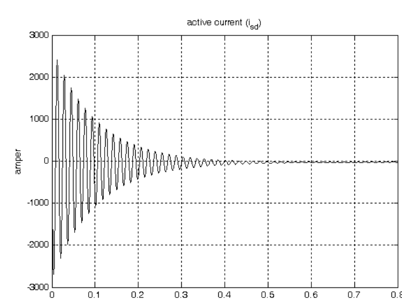


Figure 9. Active current without controller

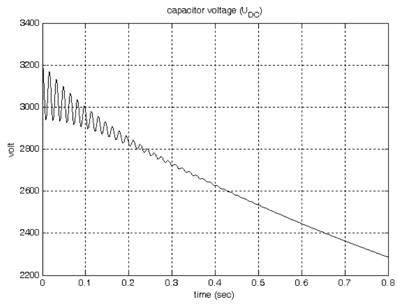


Figure 10. Capacitor voltage without controller

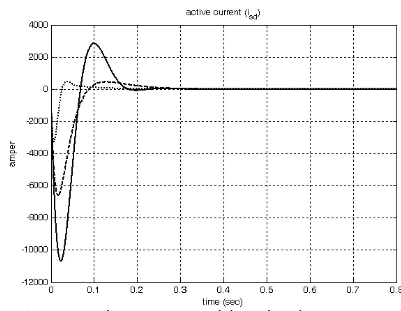


Figure 11. Reactive current with pole placement controller

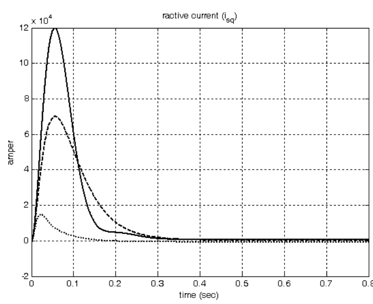


Figure 12. Active current with pole placement controller

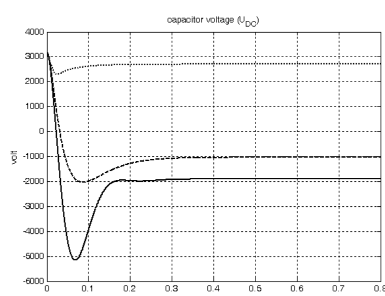


Figure 13. Capacitor voltage with pole placement controller

There are large oscillations in dq current component during 0-0.4 sec in the open-loop system response. The capacitor voltage deviation as shown in Fig. 10 is not constant. The close-loop dynamic performance on the operating point, show in Figs. 11-123, in which the poles in (-20,-25±j37), (-20, -30, -80) and (-20, -100±j100) and show with solid line, dash line and dot line, respectively.

V. Conclusion

Shunt compensators are primarily used to regulate the voltage in a bus by providing or absorbing reactive power. LQR controller method is give high response to the system and steady operating point. Modeling and designed analytically of full state feedback controller for STATCOM have been presented by deriving the small signal plant. The analysis is validated by time domain simulation and the controlled system is tested to be successful by Matlab program.

Acknowledgment

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References

[1] C.K.Lee, S.K.Lenung, S.Y.R.Hui, H.S.H.Chung, “Circuit level comparison of STATCOM technologies”, IEEE Tran. On Pow. Elec., Vol.18, No.4, pp.1084-1092, July 2003.

[2] G.Shahgholian, S.Eshtehardiha, H.Mahdavinasab, M.R.Yousefi, “A novel approach in automatic control based on the genetic algorithm in STATCOM for improvement power system transient stability”, IEEE/ICIS, pp.14-19, 2008.

[3] L.O.Mak, Y.X.Ni, C.M.Shen, “STATCOM with fuzzy controllers for interconnected power systems”, pp.88-95, Elec. Pow. Sys. Res., 2000.

[4] S.M.Alamelu, R.P.K.Devi, “Novel optimal placement of UPFC based on sensitivity analysis and evolutionary programming”, Jour. Of Engi. And Appl., pp.59-63, 2008.

[5] S.Panda, N.P.Padhy, “Power system with PSS and FACTS controller: Modeling, simulation and simultaneous tuning employing genetic algorithm”, Inte. Jou. Of Elec. Com. Sys. Eng., Vol.1, No.1, pp.9-18, winter 2007.

[6] R.W.Menzies, Y.Zhuang, “Advance static compensation using multilevel GTO inverter”, IEEE Tran. On Pow. Deli., pp.732-737, April 1995.

[7] A.Allag, M.Y.Hammoudi, S.M.Mimoune, M.Y.Ayad, “Adaptive backstepping voltage controller design for an PWM ac-dc converter”, Jour. Of Appl. Scien., No.1, pp.62-69, 2007.

[8] K.Vijayakumar, R.P.Kumudinidevi, “A hybrid genetic algorithm for optimal power flow incorporating FACTS devices”, Asia. Jour. Of Scie. Rese., No.4, pp.403-411, 2008.

[9] J.Huilam, Z.Qiang, L.Guixin, X.Jianqiang, "An optimal fuzzy controller for synchronous generator excitation control system", IEEE/PES, pp.1-6, August 2005.

[10] N.Mithulananthan, C.A.Canizares, J.Reeve, G.J.Rogers, “Comparison of PSS, SVC, and STATCOM controllers for damping power system oscillations”, IEEE Tran. On Pow. Sys., Vol.18, No.2, pp.786-792, May 2003.

[11] P.W.Lehn, “Exact modeling of the voltage source converter”, IEEE Tran. On Pow. Del., Vol.17, No.1, pp.217-222, January 2002.

[12] P.Rao, M.L.Crow, Z.Yang, “STATCOM control for power system voltage control applications”, IEEE Trans. On Pow. Del., Vol.15, No.4, pp.1311-1317, October 2000.