State Space Modeling and Eigenvalue Analysis of the Permanent Magnet DC Motor Drive System

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***Abstract*— In this paper the state space model is available for the permanent magnet (PM) direct current (DC) motor drive. The simulation results using the motor model shows the transient response for different applied voltage and load torques. Also, motor behavior for different parameters of proportional-integral-derivative (PID) controller using eigenvalue is studied.**

Keywords-permanent magnet dc motor; state space model; control; coefficient diagram method.

I. INTRODUCTION

In recent years, permanent magnet (PM) motors are used in wide variety of applications in following market segments where simplicity of structure, high efficiency and a low initial cost are primary importance. The PM motor drives are attractive for high speed operations when variable-speed is required. They can be designed in different forms and exhibit high efficiency in a wide range of operation [1-2]. The dc motor offers a wide range of control of speed and torque as well as excellent acceleration and deceleration. The use of PM in place of field windings offers the advantages of lower manufacturing costs, simple construction, lower starting torque, less air noise and higher efficiency. The material used to constant a PM dc motor may be one of three types: ferrite magnets, alnico magnets or rare earth magnets. High performance DC motor drives employ PM motors and require precise complex position-speed reference trajectory tracking, with fast response, small steady state error, small overshoot (or undershoot), fast rise time and minimum settling time [3]. Control of electric drives is one of the main topics in modern electronics. In [4] the problem is studied speed control of a PMDC motor with unknown parameters and unknown external load torque using adaptive control. A second order sliding mode control technique for PMDC motor speed control is present in [5]. The effects of armature reaction and saturation in the constructed prototype and design concepts for the mitigation of these effects present in [6]. In [7], a steady state reference current determination technique based on a neural network for the PM brushless DC motor drive system for improve system response speed, and reduce overshoot and oscillation is proposed. A fuzzy logic control for permanent magnet DC motor to achieve a robust controller under disturbances and un-modeled dynamics acting, such as load torque, dead zone, measurement noise and nonlinearities is presented in [8].

This paper is organized as follows. In section II, the model of the PMDC motor is presented in continuous time. The dynamic behavior of the motor using transfer function is described in section III. Section IV explains the structure and the design process of controller by coefficient diagram method (CDM). The simulation results are shown in section V. Finally, results and conclusion are in section VI.

II. SYAYTEM MODEL AND CONTROLLER

In this section describes the dynamics of the PMDC motors by a set state-variable equation. Mathematical models are of fundamental importance in understanding any physical system. A dynamic model for a PMDC motor is derived from both the electrical circuit and mechanical equations of motion. The equations describing the characteristics of a PMDC motor is follow as:

$$\frac{d}{dt} i_A(t) = -\frac{R_A}{L_A} i_A(t) - \frac{K_T}{L_A}\omega_M(t) + \frac{1}{L_A} U_T(t) \qquad (1)$$

$$\frac{d}{dt}\omega_M(t) = \frac{K_T}{J_M} i_A(t) - \frac{B_M}{J_M}\omega_M(t) - \frac{1}{J_M} T_L(t) \qquad (2)$$

$$\frac{d}{dt}\theta_M(t) = \omega_M(t) \qquad (3)$$

where θ_M is angular position, ω_M is rotor speed, i_A is motor current, B_M is viscous friction constant, J_M is inertia of rotor, T_L is load torque, R_A is armature resistance, L_A is armature inductance, K_T is back electromotive force (emf) constant or torque constant and U_T is applied voltage to motor. In PMDC motor, the electromagnetic torque (T_E) and the back-emf (u_b) are proportional to motor current and speed motor, respectively. The back electromotive force (K_T) determined by the strength of magnet, reluctance of iron and number of turns of armature winding. The stator magnetic flux remains essentially constant at all levels of armature current, therefore the torque-speed curve of the PMDC motor is linear.

Transfer function of the motor is important to design the proper controller that will results improve the high perform-

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ance of the system. An armature controlled PMDC motor as shown in Fig.1 is modeled to use in simulations of this study.

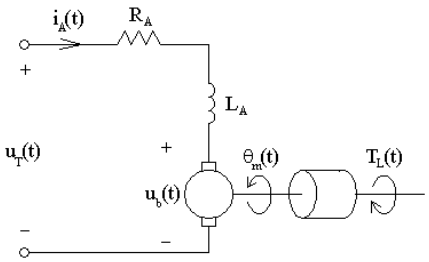

Figure 1. Model of armature controlled dc motor

Fig.2 show the open loop block diagram of the motor-load combination base on (1)-(3) by transfer block, which has two inputs. The rotor speed is:

$$\omega_M(s) = H_U(s)U_T(s) - H_T(s)T_L(s) \tag{4}$$

where two closed-loop transfer functions are:

$$H_U(s) = \left.\frac{\omega_M(s)}{U_T(s)}\right|_{T_L(s)=0} = \frac{1}{\Delta_O(s)}\frac{K_T}{B_M R_A} \tag{5}$$

$$H_T(s) = \left.\frac{\omega_M(s)}{-T_L(s)}\right|_{U_T(s)=0} = \frac{1}{\Delta_O(s)}\frac{T_M s + 1}{B_M} \tag{6}$$

The characteristic equation of the system open-loop, mechanical time constant (T_M) and electrical time constant (T_A) of the motor are:

$$\Delta_O(s) = T_M T_A s^2 + (T_M + T_A)s + 1 \tag{7}$$

$$T_M = \frac{J_M}{B_M} \tag{8}$$

$$T_A = \frac{L_A}{R_A} \tag{9}$$

PMDC motor can be found in systems where motor speed control below base speed only is required and is achieved by armature voltage control through electronic methods. One of the most useful control algorithms in linear and nonlinear control systems is PID control [9]. The PID controller is widely used in the industry owing to its simplicity and robustness, a functional block diagram of proposed speed control is shown in Fig.3. The motor speed is compared the reference speed (ω_R) to form the error speed. The transfer function of the PID controller is:

$$G_C(s) = K_P + \frac{K_I}{s} + K_P s \tag{10}$$

where K_P, K_I and K_D are proportional gain, integral gain and derivative gain, respectively. The motor speed in the closed-loop system as shown Fig. 2 is:

$$\omega_M(s) = \frac{1}{1 + H_T(s)G_C(s)}[H_U(s)G_C(s)\omega_R(s) - H_T(s)T_L(s)] \tag{11}$$

III. DYNAMIC ANALYSIS

Linearized models are useful for control system using linear analysis techniques such as frequency response and root locus. Analytical model is important tools for the prediction of dynamic performance and stability limits with different control laws and system parameters. One of the most important characteristics of the transient response of a system is the stability of the system.

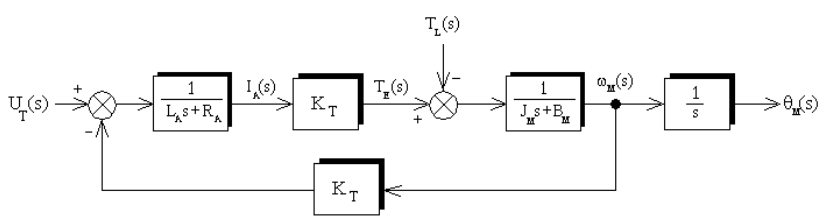

Figure 2. Open loop block diagram of PMDC motor drive

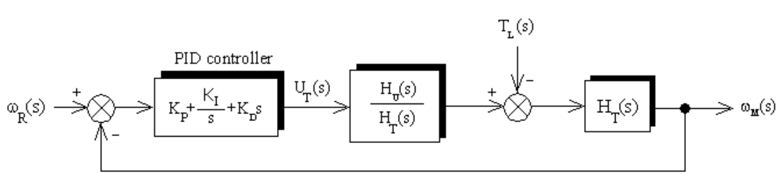

Figure 3. Functional scheme for the speed control system

The location of the poles in the s-plane is good indicators of the relative stability and transient response of the closed-loop control system. The eigenvalue of the Linearized model correspond to the poles of the system. The motor speed in the closed-loop system is:

$$\omega_M(s) = G_R(s)\,\omega_R(s) - G_L(s)\,T_L(s) \qquad (12)$$

The PMDC motor dynamic performance is described by the two closed-loop transfer functions: one is relating ω_M with ω_R and another relating ω_M with $T_L(s)$:

$$G_R(s) = \left.\frac{\omega_M(s)}{\omega_R(s)}\right|_{T_L(s)=0} = \frac{K_T(K_D s^2 + K_P s + K_I)}{\Delta_C(s)} \qquad (13)$$

$$G_L(s) = \left.\frac{\omega_M(s)}{-T_L(s)}\right|_{\omega_R(s)=0} = \frac{s(T_M s + 1)R_A}{\Delta_C(s)} \qquad (14)$$

The characteristic equation of the system close-loop is:

$$\Delta_C(s) = B_M R_A T_M T_A s^3 + (B_M T_M R_A + B_M T_A R_A + K_T K_D)s^2 + (B_M R_A + K_T K_P)s + K_T K_I \qquad (15)$$

The motor parameters are used for simulation studies shown in Table 1. The step response of the Electromagnetic torque and motor speed in open loop system are shown in Figs. 4 and 5. The frequency response of the motor current in open loop system is show in Fig. 6. The effects of changes in the control gain can be evaluated from the root locus shown in Figs. 7 and 8.

TABLE I
PARAMETERS VARIOUS MOTORS

parameter	motor A	motor B	motor C	unit
B_M	4.32×10^{-4}	1×10^{-5}	3×10^{-3}	N.m.s/r
L_A	8.05	5.35	162.73	mH
R_A	1.4	3.2	7.72	Ω
K_T	0.095	0.3	1.25	Nm
J_M	7.49×10^{-4}	5×10^{-4}	2.36×10^{-2}	kg.m^2
T_A	5.7500	1.6719	21.0790	mH/Ω
T_M	1.7330	50	7.8667	Kg.m.r/N/s

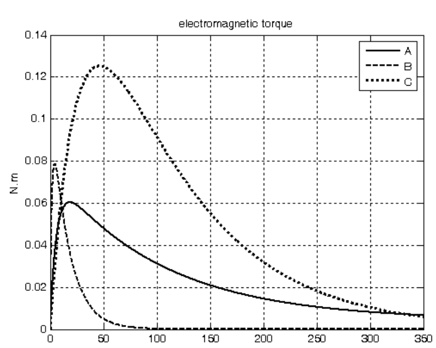

Figure 4. Electromagnetic torque step response

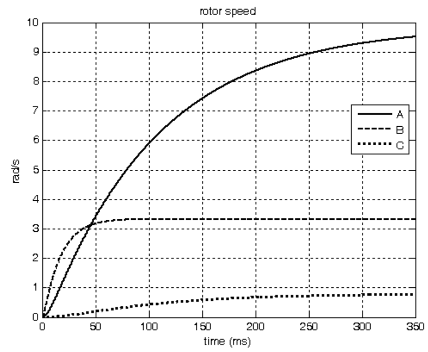

Figure 5. Motor speed step response

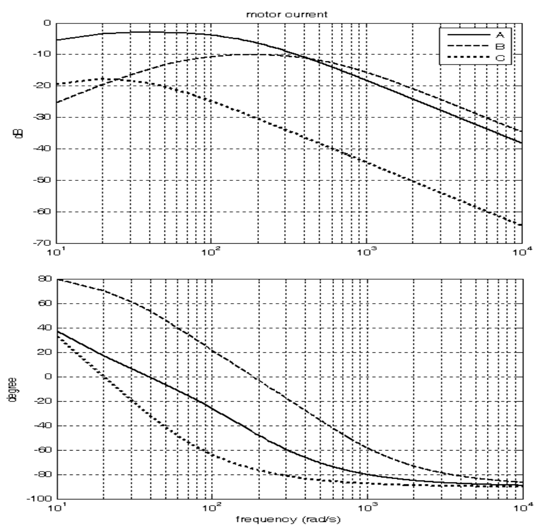

Figure 6. Motor speed step response

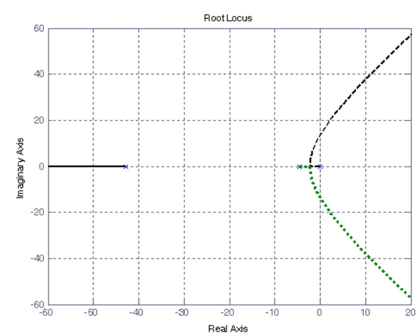

Figure 7. The root locus for the parameter K_I

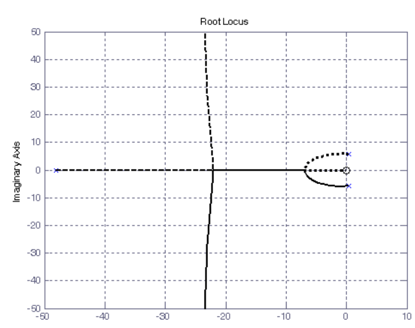

Figure 8. The root locus for the parameter K_P

If controller is only proportional, the steady state change in speed due to a step change in ω_R is $\Delta\omega_{MR}$ and a step change in T_L is $\Delta\omega_{ML}$:

$$\Delta\omega_{MR} = \frac{K_T K_P}{K_T K_P + B_M R_A} \quad (16)$$

$$\Delta\omega_{ML} = \frac{R_A}{K_T K_P + B_M R_A} \quad (17)$$

Also, the damping ratio (η) and undamped natural frequency (ω_n) of the closed-loop system are:

$$\eta = \frac{B_M R_A (T_A + T_M)}{2\sqrt{K_T K_P + B_M R_A}} \quad (18)$$

$$\omega_n = \sqrt{K_T K_P + B_M R_A} \quad (19)$$

IV. DESIGN CONTROLLER

The Coefficient Diagram Method (CDM) is an indirect pole placement method to design an appropriate characteristic polynomial. The CDM can give a controller design which is both stable and robust, and it has the desired system response speed. The PI controller gains from CDM criterion given by:

$$K_I = \frac{B_M R_A (T_A + T_M)^3}{\gamma_1 \gamma_2^2 K_T T_A^2 T_M^2} \quad (20)$$

$$K_P = \frac{B_M R_A}{K_T}\left[\frac{(T_A + T_M)^2}{T_A T_M \gamma_2} - 1\right] \quad (21)$$

where γ_1 and γ_2 are the stability indexes and τ is the equivalent time constant is:

$$\tau = \frac{\gamma_1 \gamma_2 T_A T_M}{T_A + T_M} \quad (22)$$

In PI controller, for $K_P>0$, the stability index must be:

$$\gamma_2 < \frac{(T_A + T_M)^2}{T_A T_M} \quad (23)$$

The PID controller gains from CDM criterion given by:

$$K_I = \frac{\gamma_2 \gamma_1^2 B_M R_A T_A T_M}{K_T \tau^3} \quad (24)$$

$$K_P = \frac{B_M R_A}{K_T}\left[\frac{\gamma_2 \gamma_1^2 T_A T_M}{\tau^2} - 1\right] \quad (25)$$

$$K_D = \frac{B_M R_A T_A T_M}{K_T}\left(\frac{\gamma_1 \gamma_2}{\tau} - \frac{T_A + T_M}{T_A T_M}\right) \quad (26)$$

In PID controller, for $K_P>0$ and $K_D>0$, the stability index must be:

$$\gamma_1 \gamma_2 > \frac{T_A + T_M}{T_A T_M}\tau \quad (27)$$

$$\gamma_1^2 \gamma_2 > \frac{\tau^2}{T_A T_M} \quad (28)$$

Therefore we can write as:

$$\gamma_1 < \frac{\tau}{T_A + T_M} \quad (29)$$

$$\gamma_2 < \frac{T_A T_M}{(T_A + T_M)^2} \quad (30)$$

The simulation result for P controller and PI controller are shown in Figs. 9 and 10. It is seen that the increase of ω_n and decrease of η relationship to open-loop system are caused by the proportional controller.

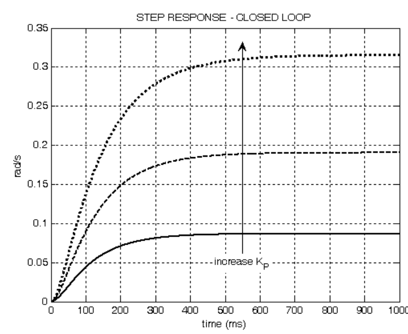

Figure 9. Transient responses for different proportional gain with P controller

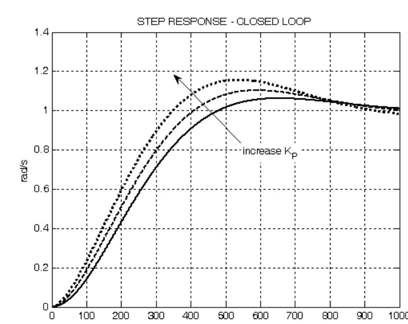

Figure 10. Transient responses for different proportional gain with PI controller

V. CONCLUSION

The mathematical state space model for the PMDC motor has been developed which include PID controller. The simulations for different parameters such as controller gains are shown. CDM approach applied to the design of speed controller.

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