Control of Bipedal Walking Robot through Direct Regulation of the Zero Moment Point

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***Abstract*— By controlling the zero moment point (ZMP) The unexpected rotation of the supporting foot can be avoided. this paper analyze a control strategy for simultaneously regulating the position of the ZMP and the joints of the robot , the proposed controller is based on a path-following control strategy. The objective of the control law is not to track a (time based), but only the associated path in joint space.**

***Keywords*- zero moment; point. biped robot; periodic motion; path-following control; reference joint path.**

I. Introduction

Research on biped humanoid robots is one of the most exciting topics in the robotic Projects. Synthesis and analysis of bipedal locomotion is a complex task which requires knowledge of the dynamics of multi-link mechanisms, collision theory, control theory, and nonlinear dynamical systems theory of robot. this paper control policies are built around the notion of controlling the zero moment point (ZMP) [1-3] .ZMP is very important for walking robot and generally used as dynamic criterion for gait planning and control. As long as the ZMP remains inside the convex hull of the foot support region, CoP=ZMP and the supporting foot does not rotate. the modification of the reference motion has an important effect on the stability of the gait in the sense of the convergence toward a periodic motion and its resulting. Our control strategy is based on simultaneous control of the joint and on the evolution of the ZMP position. The unexpected rotation of the supporting foot is avoided via the control of the position of the zero moment point. Since the joints and the position of the ZMP are controlled simultaneously, the system becomes under-actuated in the sense that the number of inputs is less than the number of outputs . the control strategy based on a path-following control strategy previously developed for dealing with the underactuation present in planar robots without actuated ankles [4-5]. this paper controller is related to the work in [6] but in that work a planar biped considered with massless feet so no tourqe used at the swinging ankle . In this paper, the position of the ZMP will be prescribed, which is important for robustly avoiding unexpected rotations of the foot in the presence of perturbations or for taking into account the desired rotation of the supporting foot toward the end of the single-support phase. The modification of the reference motion can be related to the work presented in [7], but in this paper The modification of the joint motion to obtain the desired ZMP evolution is provided by a temporal modification only, the joint path is preserved. Since the references for the joint and the ZMP position are not functions of time, the control does not attempt to resynchronize its motion with time.The Poincare return map can be used to study the existence and stability of periodic motions under this control law. In this paper ,first the model of the biped is presented then the proposed control low are defined and finally some simulation result are presents.

II. Biped Model

The biped studied walks in the vertical sagittal plane identified with a vertical x–z Plane. The robot is composed of a torso and two legs, and each leg is composed of two links and a foot. The ankles, the knees, and the hips are one degree-of-freedom rotational frictionless joints. The walking gait consists of single-support phases where the stance foot is flat on the ground The vector $q=[q_1,q_2,q_3,q_4,q_5,q_6]$ of configuration variables (see Fig. 1) describes the shape of the biped during single support . The torques are grouped into a torque vector:

$$\tau = [\tau_1, \tau_2, \tau_3, \tau_4, \tau_5, \tau_6] \qquad (1)$$

In the simulation, we use the biped parameters given in Table 1.The dimensions of the feet are, h_p=0.08 m, l_g=0.06 m, l_d=0.2 m.

TABLE I. Biped Parameters for Simulation

	feet	femur	tibia	torso
length m	0.26	0.4	0.4	0.625
mass kg	1	3.2	6.8	17
inertia kg/m	0.012	0.048	0.069	1.869

978-0-7695-3925-6/09 $26.00 © 2009 IEEE
DOI 10.1109/ICCEE.2009.149

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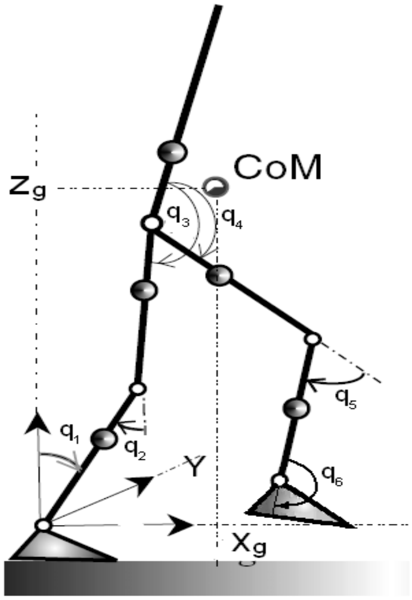

Figure 1. Studied biped and a choice of coordinates

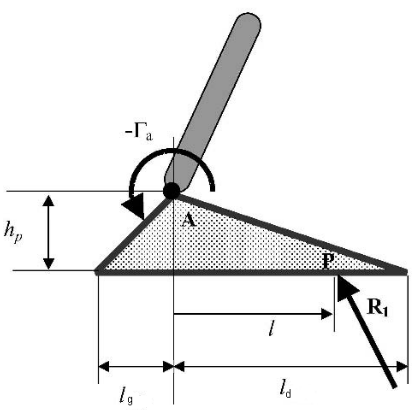

Figure 2. The equilibrium of the supporting foot

III. DYNAMIC MODELLING

The legs swap their roles from one step to the next one. Thus the study of a step allows us to deduce the complete behavior of the robot. Only a single support phase and an impact model are derived.

A. The Single Support Phase Model

Using Lagrange's formalism, the i_{th} line of the dynamic model for i = 1. . . 6 (q_i is the i_{th} element of vector q) is:

$$\frac{d}{dt}\left(\frac{\partial k}{\partial \dot{q}_i}\right) - \frac{\partial k}{\partial q_i} + \frac{\partial p}{\partial q_i} = Q_i \qquad (2)$$

The kinetic energy K is independent of the coordinate frame chosen.

Since coordinate q_1 defines the orientation of the biped as a rigid body, the inertia matrix is independent of this variable, it depends only of "internal" variables represented by vector $q_c = [q_2,q_3,q_4,q_{5,}q_{6,}]$ The dynamic model of robot can be written:

$$M(q_c)\ddot{q} + h\left(q,\dot{q}\right) = \tau \qquad (3)$$

Where M (qc) is a (6 × 6) inertia matrix and vector h(q,q˙) contains the centrifugal, Coriolis and gravity forces.
During single support, the position of the center of mass of the biped denoted by xg (q), zg (q) and expressed as a function of the angular coordinates. if the abscissa of the stance ankle is 0, xg denote by:

$$\frac{\partial p}{\partial q_1} = -mgx_g \qquad (4)$$

m is the mass of the biped and g is the gravity acceleration.

B. The Reaction Force

When leg 1 is on the ground, a ground reaction force R_1 exists. The global equilibrium in translation of the robot makes it possible to calculate this force. Thus, we have:

$$m\frac{\partial x_g(q)}{\partial q}\ddot{q} + m\dot{q}^T\frac{\partial^2 x_g(q)}{\partial q^2}\dot{q} = R_{x1} \qquad (3)$$

$$m\frac{\partial z_g(q)}{\partial q}\ddot{q} + m\dot{q}^T\frac{\partial^2 z_g(q)}{\partial q^2}\dot{q} + mg = R_{z1} \qquad (4)$$

Where $\frac{\partial^2 z_g(q)}{\partial q^2}$ and $\frac{\partial^2 x_g(q)}{\partial q^2}$ are (6 6) matrix.

the reaction force must be inside the friction cone. These conditions can be written at each time by:

$$R_{1z} > 0 \qquad (7)$$

Figure (3) show the evolution of reaction force during one step.

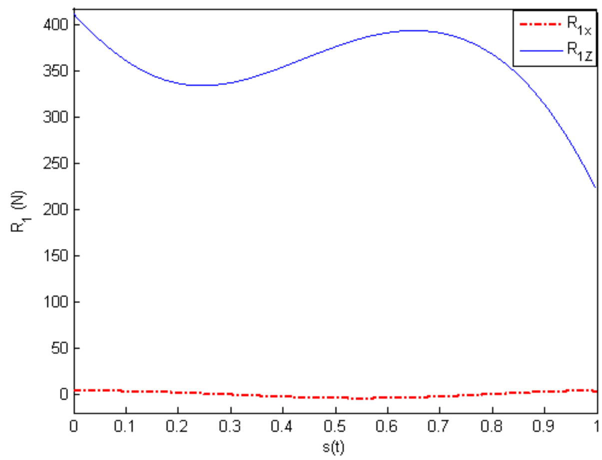

Figure 3. Evolution of reaction force

C. EQUILIBRIUM *IN R*OTATION

The robot is submitted to the reaction force exerted by the ground at the ZMP, and the force of gravity. Since the stance ankle A is stationary during the single-support phase, the equilibrium of the foot around the axis of the ankle can be written (see Fig. 2) as:

$$\dot{\sigma} = mgx_g - lR_{zl} - h_p R_{xl} \tag{8}$$

By definition, the angular momentum is linear with respect to the joint velocities and can be written as:

$$\sigma A = \frac{\partial_k}{\partial \dot{q}_1} = N(q)\dot{q} \tag{9}$$

Where σA is the angular momentum of the biped about A and N(q) is the first line of the inertia matrix $M(q_c)$.
The location of the ZMP is then defined directly by the robot dynamics. With, using (8)–(19), we have:

$$(N_0(q) + lN(q)\ddot{q} + h_0(q,\dot{q}) + l h_1(q,\dot{q})) = 0 \tag{10}$$

where:

$$N_0 = N(q) + mh_p \frac{\partial x_g(q)}{\partial q} \tag{11}$$

$$N_L = m \frac{\partial x_g(q)}{\partial q} \tag{12}$$

$$h_0 = m\dot{q}^T \frac{\partial^2 z_g(q)}{\partial q^2}\dot{q} + mh_p \dot{q}^T \frac{\partial^2 x_g(q)}{\partial q^2}\dot{q} - mgx_g \tag{13}$$

$$h_1 = m\dot{q}^T \frac{\partial^2 z_g(q)}{\partial q^2}\dot{q} + mg \tag{14}$$

IV. DEFINITION OF THE CONTROL LAW

The control law must ensure that the joint coordinates follow the joint reference path $q_d(s)$ and the position of the ZMP tend to $l_d(s)$. The torques act on the second derivative of q and directly on l. It follows from the definition of the joint reference path that the desired velocity and acceleration of the joint variables are [8]:

$$\dot{q}_d(t) = \frac{dq_d(s(t))}{ds}\dot{s} \tag{15}$$

$$\ddot{q}_d(t) = \frac{dq_d(s(t))}{ds}\ddot{s} + \frac{d^2 q_d(s(t))}{ds^2}\dot{s}^2 \tag{16}$$

Because the control objective is only to track a reference path, the evolution of s(t) is free and the second derivative $\ddot{s}$ can be treated as a "supplementary control input." This allows the control law to be designed for a system with equal number of inputs and outputs. The control inputs are the six torques τ_j, j = 1. . . 6, plus $\ddot{s}$ and the chosen outputs are the six components of $q_d(s(t))$-q(t) and $l_d(s(t))$-l(t) The control law is based on the computed torque. The joint tracking errors are defined as:

$$e_q(t) = q_d(s(t) - q(t)) \tag{17}$$

$$\dot{e}_q(t) = \frac{dq_d(s(t))}{ds}\dot{s} - \dot{q}(t) \tag{18}$$

The desired behavior of the configuration variables in closed loop is:

$$\ddot{q} = \ddot{q}_d + \psi(q,\dot{q},s,\dot{s}) \tag{19}$$

where $\psi(q, \dot{q}, s, \dot{s})$ is the term that imposes $q_t \rightarrow q_d(s(t))$ in finite time; in fact, the settling time can be chosen to be less than the time duration of a step. In this section, we present a class of continuous time-invariant feedback controllers that globally finite-time stabilizes the double integrator, by a finite-time-stabilizing feedback law, we mean a feedback controller that renders the origin of the closed-loop system a finite-time-stable equilibrium thus we seek a continuous feedback law, that finite-time stabilizes the double integrator.
Proposition 1: The origin of is a globally finite-time-stable equilibrium under the feedback control law with:

$$\psi_k = -\mathrm{sign}(\varepsilon \dot{e}_{qi}) \left| \varepsilon \dot{e}_{qi} \right|^{v} - \mathrm{sign}(\phi_l) \left| \phi_l \right|^{v} \qquad (20)$$

The continuously differentiable Lyapunov function candidate is[9]:

$$V(x, y) = \frac{2 - v}{3 - v} \left| \phi_v \right|^{\frac{3-v}{2-v}} + sy\phi_v + \frac{r}{3 - v} \left| \varepsilon \dot{e}_{qi} \right| \qquad (21)$$

where r and s are positive numbers Along the closed-loop trajectories, thus we have:

$$\dot{V}(x, y) = -ry^2 - s\left| \phi_v \right|^{\frac{2}{2-v}} + \left| \varepsilon \dot{e}_{qi} \right|^{1-v} \left| \phi_v \right|^{\frac{1+v}{2-v}} \qquad (22)$$

$-s\phi_v \mathrm{sign}(\varepsilon \dot{e}) \left| \varepsilon \dot{e}_{ql} \right|^{v} - (r+s)\mathrm{sign}(\varepsilon \dot{e}_{qi}\, \phi_v) \left| \phi_v \right|^{\frac{v}{2-v}}$ which is continuous everywhere since $v \in (0,1)$ for r > 1 and s < 1, both v and - $\dot{v}$ take positive values. Taking into account the expression for the reference motion, can be written as

$$\ddot{q} = \frac{dq_d s(t)}{ds} \ddot{s} + v(s, \dot{s}, q, \dot{q}) \qquad (23)$$

with:

$$u(s, \dot{s}, q, \dot{q}) = \left| d^2 q^d(s) / ds^2 \right| \dot{s}^2 + \psi \qquad (24)$$

For the position of the ZMP, the desired closed-loop behavior is:

$$l(t) = l_d(s(t)) \qquad (25)$$

Combining expression (23) with the dynamic model (1) of the robot and the relation (10) for the ZMP determines the feedback controller. Thus, the control law must be such :

$$M(q) \left(\frac{dq_d(s)}{ds} \ddot{s} + h(q, \dot{q}) \right) = \tau \qquad (26)$$

$$(N_0(q) + l_d(s) N_L(q)) \left(\frac{dq_d(s)}{ds} \ddot{s} + u \right) + h_0(q, \dot{q})$$

$$+ l_d(s) h_l(q, \dot{q}) = 0 \qquad (27)$$

It follows that, in order to obtain the desired closed-loop behavior, it is necessary and sufficient to choose:

$$\ddot{s} = \frac{-(N_0(q) + l_d(s) N_L(q) v - h_0(q, \dot{q}) - l_d(s))}{(N_0(q) + l_d(s) N_l(q)) \left[dq_d(s)/ds \right]} \qquad (28)$$

$$\tau = M(q) \left(\frac{dq_d(s)}{ds} \ddot{s} + v \right) + h(q, \dot{q}) \qquad (29)$$

by (14), q(t), l(t) converges to q_d (s(t)), l_d (s(t)) respectively, in finite time, and that Without initial errors, a tracking of q_d (s(t)) and l_d (s) is obtained.

Analyzing the angular momentum σ is sufficient to study the evolution of parameter s. The motion of the robot can in turn be deduced from the evolution of parameter s. the velocity of the robot is proportional to ds/dt. Thus, the angular momentum can be expressed by:

$$\sigma = f(s)\dot{s} \qquad (30)$$

Scalar function f(s) depends on vector q (s) and on the biped parameters. Let us assume that function q (s) and the biped parameters are such that f(s)≠0 for 0≤s≤1. If f(s)≠0 in the interval 0≤s≤1, then f(s)<0 or f(s)>0 in this interval. The sign of f(s) changes with the sense of the axis Z. In the following we assume that q (s) is such that f(s)>0. If f(s)≠0, we obtain from (26):

$$\dot{s} = \frac{\sigma}{f(s)} \qquad (31)$$

if s(0) and $\dot{s}$ are known. We choose s(0)=0 and we define $\dot{s}(0)$ to minimize the error on the joint velocity we have:

$$\varepsilon = \left| \dot{q}(0) - \dot{q}_r(0) \right|^2 = \left| \dot{q}(0) - \frac{dq_r(s(0))}{ds} \dot{s}(0) \right|^2 \qquad (32)$$

V. SIMULATION RESULTS

The block diagram of the control system show in Fig.4. The evolution of the ZMP position is show in Fig. 6 it is linear function of s. The reference q_d (s) can be the result of an optimization process.

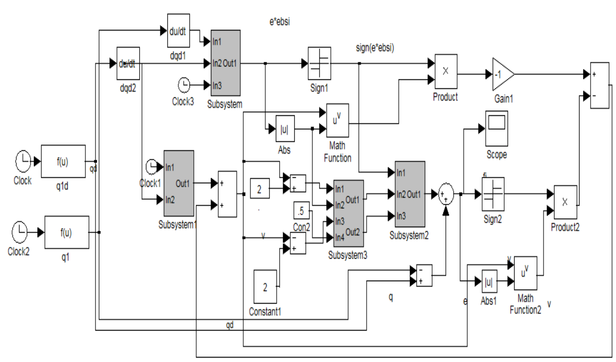

Figure 4. Block diagrams of the control system

In the simulation presented here, the desired evolution for q_d (s) are:

$$q_d = a_1 s^4 + a_2 s^3 + a_3 s^2 + a_4 s + a_5 \tag{33}$$

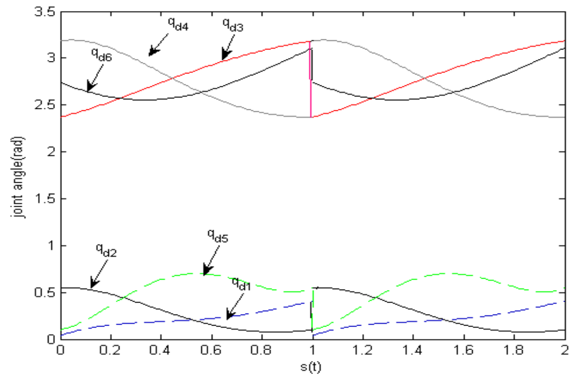

Figure 5. Optimal qd

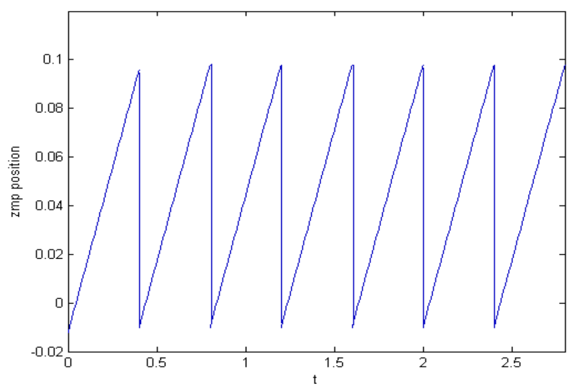

Figure 6. Horizontal position of the ZMP

As show in fig.6 the position of zmp convergence toward the periodic motion. For a sufficiently high initial velocity $\dot{s}(0)$, successful stepping pattern can be achieved. $\dot{s}$ against s show in Fig. 7

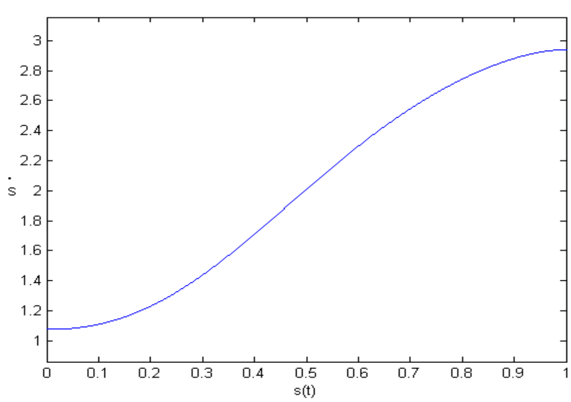

Figure 7. The phase plane for the zero dynamics

VI. Conclusion

For a planar biped, a control strategy was proposed based on tracking a reference path in the joint space instead of a reference function of time. This allows the simultaneous control of the path positions of the joints and the ZMP. The biped adapts its time evolution according to the effect of gravity. Walking with more human-like characteristics can be handled by this control law. Easily testable analytical conditions have been presented for the existence and uniqueness of a periodic motion.

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