پاسخ تکلیف سری دوم درس پردازش سیگنال‌های دیجیتالی

(Digital Signal Processing)

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حل ۱:

(a) We first perform a partial-fraction expansion of $X(e^{j\omega})$:

$$\begin{aligned} X(e^{j\omega}) &= \frac{1-a^2}{(1-ae^{-j\omega})(1-ae^{j\omega})} \\ &= \frac{1}{1-ae^{-j\omega}} + \frac{ae^{j\omega}}{1-ae^{j\omega}} \\ x[n] &= a^n u[n] + a^{-n}u[-n-1] \\ &= a^{|n|} \end{aligned}$$

(b)

$$\begin{aligned} \frac{1}{2\pi}\int_{-\pi}^{\pi} X(e^{j\omega})\cos(\omega)d\omega &= \frac{1}{2\pi}\int_{-\pi}^{\pi} X(e^{j\omega})\frac{e^{j\omega}+e^{-j\omega}}{2}d\omega \\ &= \frac{1}{2}\left(\frac{1}{2\pi}\int_{-\pi}^{\pi} X(e^{j\omega})e^{j\omega}d\omega + \frac{1}{2\pi}\int_{-\pi}^{\pi} X(e^{j\omega})e^{-j\omega}d\omega\right) \\ &= \frac{1}{2}(x[n-1]+x[n+1]) \\ &= \frac{1}{2}(a^{|n-1|}+a^{|n+1|}) \end{aligned}$$

حل ۲:

We have

$$r[n] = \begin{cases} 1, & \text{for } 0 \le n \le M \\ 0, & \text{otherwise} \end{cases}$$

Taking the Fourier transform

$$\begin{aligned} R(e^{j\omega}) &= \sum_{n=0}^{M} e^{-j\omega n} \\ &= \frac{1-e^{-j\omega(M+1)}}{1-e^{-j\omega}} \\ &= e^{-j\frac{M}{2}\omega}\left(\frac{e^{j\frac{M+1}{2}\omega}-e^{-j\frac{M+1}{2}\omega}}{e^{j\omega}-e^{-j\omega}}\right) \\ &= e^{-j\frac{M}{2}\omega}\left(\frac{\sin(\frac{M+1}{2}\omega)}{\sin(\omega/2)}\right) \end{aligned}$$

We have

$$w[n] = \begin{cases} \frac{1}{2}(1+\cos(\frac{2\pi n}{M}), & \text{for } 0 \le n \le M \\ 0, & \text{otherwise} \end{cases}$$

We note that,

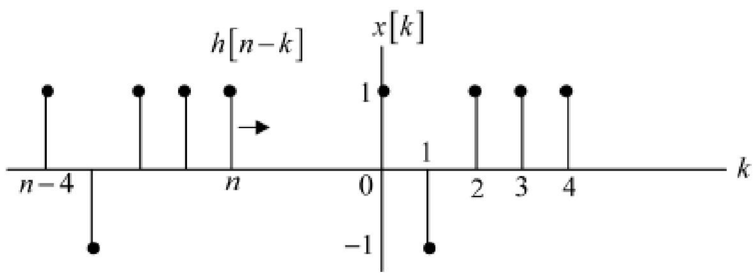
$$w[n] = r[n] \cdot \frac{1}{2}[1 + cos(\frac{2\pi n}{M})].$$

Thus,

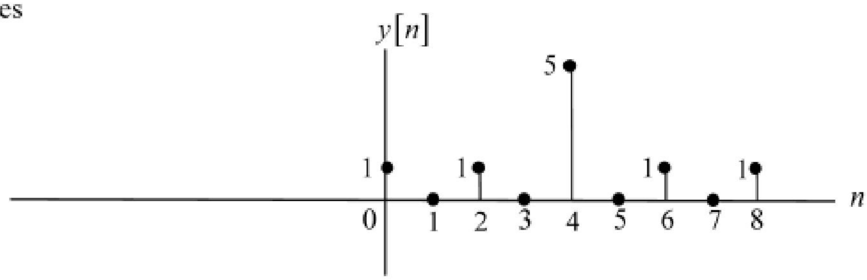
$$\begin{aligned} W(e^{j\omega}) &= R(e^{j\omega}) * \sum_{n=-\infty}^{\infty} \frac{1}{2}(1 + cos(\frac{2\pi n}{M}))e^{-j\omega n} \\ &= R(e^{j\omega}) * \sum_{n=-\infty}^{\infty} \frac{1}{2}(1 + \frac{1}{2}e^{j\frac{2\pi n}{M}} + \frac{1}{2}e^{-j\frac{2\pi n}{M}})e^{-j\omega} \\ &= R(e^{j\omega}) * (\frac{1}{2}\delta(\omega) + \frac{1}{4}\delta(\omega + \frac{2\pi}{M}) + \frac{1}{4}\delta(\omega - \frac{2\pi}{M})) \end{aligned}$$

حل ۳:

Using "flipping and shifting,"



gives



حل ۴:

$$x[n] = -b^n u[-n-1] = \begin{cases} -b^n, & n \le -1 \\ 0, & \text{otherwise} \end{cases}$$

Then

$$X\left(e^{j\omega}\right) = \sum_{n=-\infty}^{\infty} x[n] e^{-j\omega n}$$

$$= \sum_{n=-\infty}^{-1} -b^n e^{-j\omega n}.$$

Let $k = -n$. Then

$$X\left(e^{j\omega}\right) = \sum_{k=1}^{\infty} -b^{-k} e^{j\omega k}$$

$$= -\left\{\left(\sum_{k=0}^{\infty} b^{-k} e^{j\omega k}\right) - 1\right\}$$

$$= 1 - \sum_{k=0}^{\infty} \left(b^{-1} e^{j\omega}\right)^k$$

$$= 1 - \frac{1}{1 - \frac{e^{j\omega}}{b}},$$

where the last step is true only for $\left|b^{-1} e^{j\omega}\right| < 1$, or $\left|b^{-1}\right| < 1$, or $|b| > 1$. Now we have

$$X\left(e^{j\omega}\right) = \frac{1 \quad \frac{e^{-j\omega}}{b} \quad 1}{1 - \frac{e^{-j\omega}}{b}}$$

$$= \frac{-be^{-j\omega}\left(-\frac{1}{b} e^{j\omega}\right)}{-be^{-j\omega}\left(1 - \frac{1}{b} e^{j\omega}\right)}$$

$$X\left(e^{j\omega}\right) = \frac{1}{1 - be^{-j\omega}}$$

only when $|b| > 1$.

Now suppose

$$Y\left(e^{j\omega}\right) = \frac{2e^{-j\omega}}{1 + 2e^{-j\omega}} = 2 \frac{1}{1 - (-2) e^{-j\omega}} e^{-j\omega}$$

Using the above transform pair and then shifting to the right by one,

$$y[n] = 2\left[-(-2)^{n-1} u[-(n-1)-1]\right] = -2(-2)^{n-1} u[-n]$$

$$= (-2)^n u[-n].$$