5.4 A two-mass system consists of a piston of mass m_1, connected by two elastic springs, that moves inside a tube as shown in Fig. 5.23. A pendulum of length l and end mass m_2 is connected to the piston as shown in Fig. 5.23. (a) Derive the equations of motion of the system in terms of $x_1(t)$ and $\theta(t)$. (b) Derive the equations of motion of the system in terms of the $x_1(t)$ and $x_2(t)$. (c) Find the natural frequencies of vibration of the system.

Section 5.3 Free-Vibration Analysis of an Undamped System

5.5 Find the natural frequencies of the system shown in Fig. 5.24, with $m_1 = m$, $m_2 = 2m$, $k_1 = k$, and $k_2 = 2k$. Determine the response of the system when $k = 1000$ N/m, $m = 20$ kg, and the initial values of the displacements of the masses m_1 and m_2 are 1 and -1, respectively.

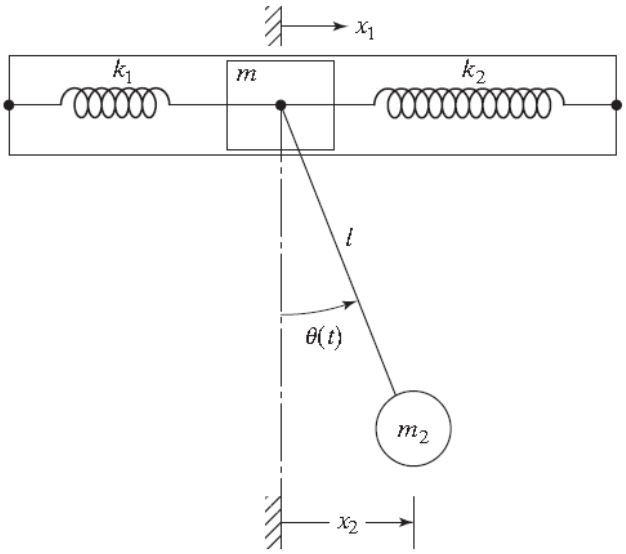

FIGURE 5.23

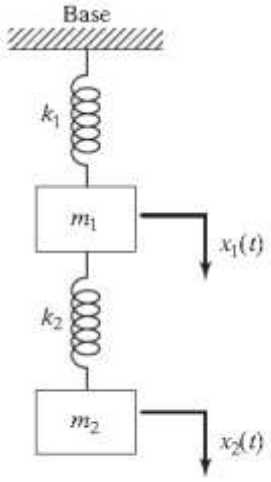

FIGURE 5.24

5.24 A hoisting drum, having a weight W_1, is mounted at the end of a steel cantilever beam of thickness t, width a, and length b, as shown in Fig. 5.34. The wire rope is made of steel and has a diameter of d and a suspended length of l. If the load hanging at the end of the rope is W_2, derive expressions for the natural frequencies of the system.

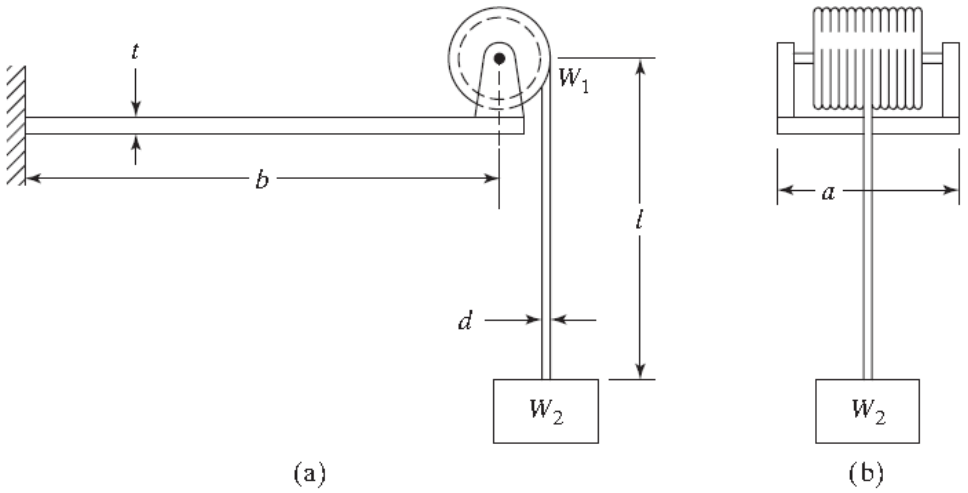

FIGURE 5.34

5.34 The mass and stiffness matrices and the mode shapes of a two-degree-of-freedom system are given by

$$[m] = \begin{bmatrix} 1 & 0 \\ 0 & 4 \end{bmatrix}, \qquad [k] = \begin{bmatrix} 12 & -k_{12} \\ -k_{12} & k_{22} \end{bmatrix}, \qquad \overline{X}^{(1)} = \begin{Bmatrix} 1 \\ 9.1109 \end{Bmatrix}, \qquad \overline{X}^{(2)} = \begin{Bmatrix} -9.1109 \\ 1 \end{Bmatrix}$$

If the first natural frequency is given by $\omega_1 = 1.7000$, determine the stiffness coefficients k_{12} and k_{22} and the second natural frequency of vibration, ω_2.

5.37 Determine the natural frequencies of the system shown in Fig. 5.39 by assuming that the rope passing over the cylinder does not slip.

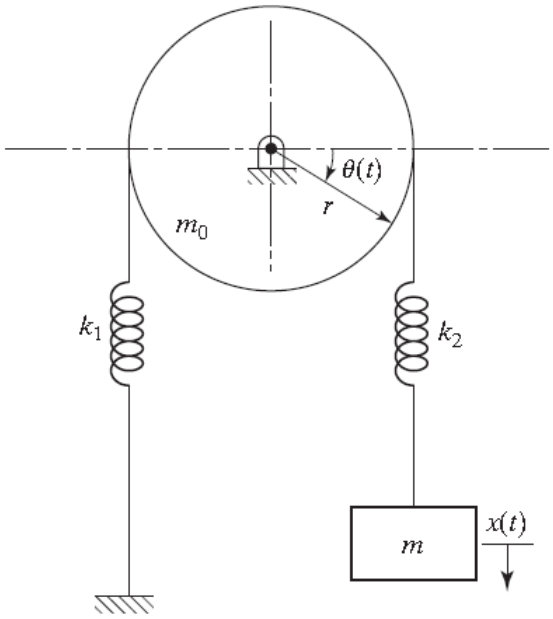

FIGURE 5.39

5.42 A rigid rod of negligible mass and length $2l$ is pivoted at the middle point and is constrained to move in the vertical plane by springs and masses, as shown in Fig. 5.42. Find the natural frequencies and mode shapes of the system.

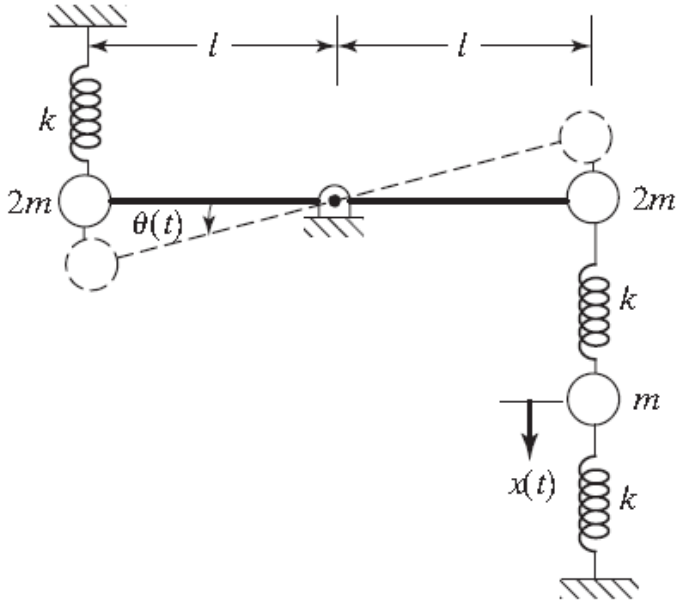

FIGURE 5.42

5.65 Determine the equations of motion and the natural frequencies of the system shown in Fig. 5.56.

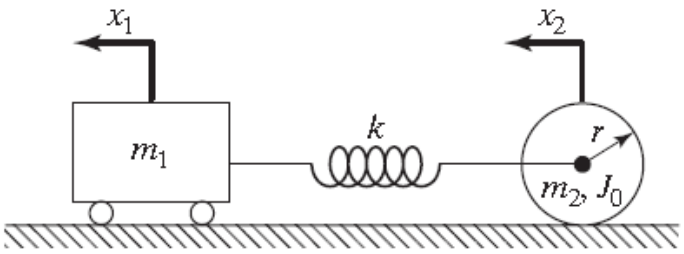

FIGURE 5.56

5.70 A turbine is connected to an electric generator through gears, as shown in Fig. 5.59. The mass moments of inertia of the turbine, generator, gear 1, and gear 2 are given, respectively, by 3000, 2000, 500, and 1000 kg-m^2. Shafts 1 and 2 are made of steel and have diameters 30 cm and 10 cm and lengths 2 cm and 1.0 m, respectively. Find the natural frequencies of the system.

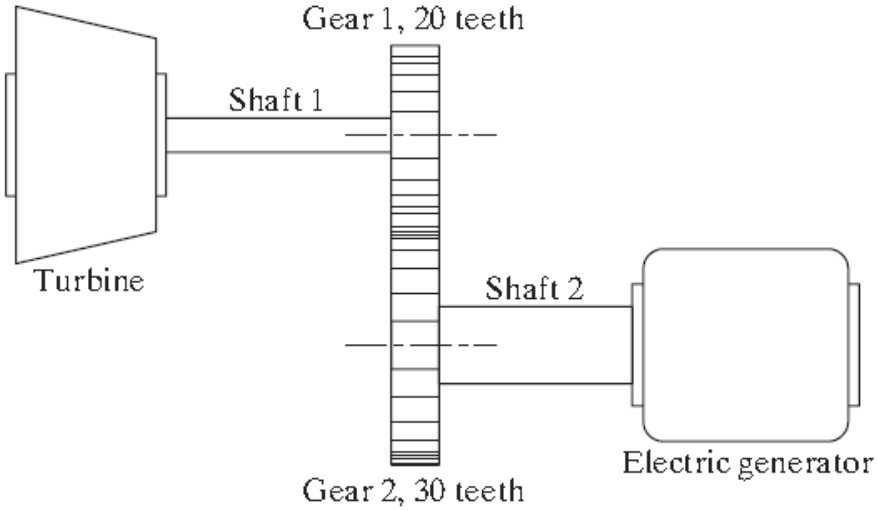

FIGURE 5.59