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Research Paper

Analytical And Numerical Investigation of Energy Absorption in Graded Aluminum Open Cell Foam Under Low Velocity Impact Loading

S. Davari[1,2], S.A. Galehdari[1,2 *], A. Atrian

[1] *Department of Mechanical Engineering, Najafabad Branch, Islamic Azad University, Najafabad, Iran*
[2] *Modern Manufacturing Technologies Research Center, Najafabad Branch, Islamic Azad University, Najafabad, Iran*

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ABSTRACT

Given the significance of energy absorption in various industries, light shock absorbers such as structures made of metal foam have been considered. In this study, analytical equation of plateau stress is presented for an open cell foam based on the Gibson-Ashby model, which follows elastic perfectly plastic behavior. For comparison of acquired analytical equations, the problem for a cell and then for three cells that make up an aluminum open cell foam is simulated in ABAQUS/CAE. Using the stress strain diagram, plateau stress and densification strain equations, the specific energy absorbed of the open cell metal foam is extracted. The capacity of absorb energy for an aluminum open cell foam with three cell is obtained once using analytical equations and again by using numerical simulation in ABAQUS/CAE. Numerical results retain an acceptable accordance with analytical equations with less than 3% occurred error for absorbed energy. To ensure the accuracy of numerical simulation, the results of simulating are compared with the results of the simulation of the same foam in a reference whose accuracy is verified by the experiment. Based on the results, the effective cross-sectional area of the foam with Gibson-Ashby cell does not follow the cross-sectional that is used for the calculation of plateau stress in adsorbent structures. The reason for this is reasonably stated. Then, in order to determine the effective cross-section in this foam, and also with regard to the importance of calculating the force transmitted to the protected structure in impact loading, tow equations are extracted to calculate the effective cross-sectional area and the transfer force. In order to create a suitable impact absorber, grade affection is used to design this foam. Applying sequential quadratic programming method (SQP) and genetic algorithm (GA), to design a graded metal foam with high specific Energy absorption.

Keywords: Open cell foam; Specific energy absorption; Low velocity impact; Graded structure; Optimization.

* Corresponding author. Tel.: +98 9399238836.
E-mail address: ali.galehdari@gmail.com

1 INTRODUCTION

DUE to the importance of energy absorption in various industries, light-weight impact absorbents including honeycomb structures [1] and metal foams have gained increased attention [2]. Metal foam is a porous metal structure with vastly different characteristics compared to a continuous metal part. These metal foams offer physical, mechanical and electrical properties which are different from their bulk metal which has resulted in several applications as impact, vibration and sound absorbents. One of the important applications of metal foams is in automobile and aerospace industries due to their high energy absorption capacity in compression stresses. A metal foam is defined as a metal structure with uniformly distributed gas-filled pores. If these pores are not connected to each other, the structure will be known as a closed cell metal foam while structures with connected pores are known as open cell metal foams [3]. Materials with cellular structure are abundantly used in nature. The examples of these natural porous structures include prismatic or honeycomb cells in wood or cork, the cellular structure in the inner part of plant stems and cellular structures in the inner bones of human skull.

In the previous decade, various structures with high energy absorption capacity have been investigated. Galehdari et.al. [1] carried out an analytical, numerical and empirical investigation of graded honeycomb structures at low speeds. While optimizing the mass and geometry of honeycomb structures, they observed that graded structure and plane loading result in decrease in the force transferred to the protected structure and reduced damage. Sawei et.al. [4] investigated different modeling methods for simulation of metal foams and introduced six models with different advantages and disadvantages. Stone [5] investigated material test methods in different relevant standards and stated that these methods are not sufficient for test of metal foams. He created twenty-one finite element models of open-cell and close-cell metal foams and carried out more than seventy numerical simulations. His finite element models were made from beam elements while using Gibson-Ashby and Kelvin cells. He then repeated a single cell in order to reach the desired foam density. Kremer [6] investigated the effects of metal foams in car seats for reducing damage to the passengers and used head injury critria (HIC) for his tests. Gibson [2] also investigated various natural foam structures including wood, cork, bone, plant stems and other similar structures. Pinnoji et.al. [7] replaced the thermoplastic material in motorcycle helmets with a metal foam and investigated the dynamic behavior of the helmet. They also separately investigated head injury criterion in a helmet with metal foam and one with ABS foam and concluded that using metal foam leads to better results. Moreira et.al. [8] investigated the numerical behavior of open cell aluminum foam under impact scenarios. They investigated the behavior of aluminum foam with different spherical cavity dimensions and presented displacement force diagrams for each cavity dimension. Among the dimensions of 4, 5 and 6 mm holes, the energy absorption property of the 5 mm hole was better than other samples. Li et.al. [9] investigated the effect of radial supports on the behavior of open-cell aluminum foam under quasi-static and dynamic compression. The results showed that this type of support condition increases the stiffness strain in the open cell aluminum foam and increases the absorbed energy per unit volume of the open cell aluminum foam. Ramirez et.al. [10] investigated the elasto-plastic behavior of open cell aluminum foam which was created by aluminum-silicon-magnesium alloy and by infiltration method and by vacuum pressure under compressive loading in the experiment. They created the geometric model by 3D CT scan method and used it in finite element analysis. And they concluded that plane stress and energy absorption increase with decreasing the size of the cavity and increasing the density. Anwar al-Hassan [11] presented a model for closed-cell aluminum foam modeling, whose finite element analysis results were in good agreement with the test results. The model presented in this reference provided a better prediction for the mechanical properties of the foam than the previous models. Goga [12] presented a new model to predict the behavior of foams. The results of this model were consistent with the results of experiments. In this model, the definition of a spring and damper was used to create a foam mathematical model. Bin et.al. [13] studied the repeatability and predictability of void size and relative density of an open-cell aluminum foam with spherical voids using the space-holder method and evaluated the mechanical properties of the foam. Kaoua et.al. [14] used a numerical simulation and a finite element modeling. In this way, the foam structure was created by a regular network of the Kelvin model. For the elements, they used the beam model and defined four types of sections for the beam. The numerical analysis results were in agreement with the experimental results. Branca et.al. [15] used Gresen's model to simulate failure in metal foams. Two aspects were used in Gresen's model and hypoelastic and hyperelastic formulations were compared. Lopatnikov et al. [16] investigated an aluminum metal foam in four different ballistic velocity regimes and presented the time history of the stress wave propagated in the foam along with the impact energy and deformation of the foam and finally obtained a relationship between optimal energy absorption and foam density.

Mukai et al. [17] investigated an open-cell magnesium foam in a dynamic test with a strain rate of 103s-1 and evaluated its energy absorption. Also, this test was performed for foams with different densities and the results were compared with foams of other types.

Zhihua et al. [18] evaluated the static and dynamic behavior of open-cell aluminum foam with different cell sizes but with the same structure. The results showed that the dynamic response of foams is sensitive to the strain rate and depends on the cell size. Davari et.al. [19] designed an aluminum open cell foam as a shock absorber for use in helicopters in emergency landing condition. By design a graded foam and performing optimization operations, they achieved an optimal structure for energy absorption that compliance the criteria of aviation standards. Base on the reviewed articles, the equation of the absorbed energy for Gibson- Ashby open cell foam has not been derived yet. In this paper this equation has been derived. This equation can be used to measure the capacity and optimization of energy absorption of open cell foams.

2 MECHANICS OF METAL FOAM

2.1 Stress – strain graph of energy absorbents

When a foam structure is subjected to compression loading, it first enters an elastic region where the stress increases until reaching its yield stress. Then, the foam enters a plastic region where its cells undergo plastic collapse one by one. At this stage, the stress – strain graph of the foam is an almost horizontal line which shows significant increase in strain without significant increase in stress. This stress is known as the plateau stress. With increased in load, material is then compressed to its ultimate level, reaching a strain known as densification strain and shown with ε_D. After this stage, stress will increase with a sharp slop and material loses its capacity to absorb energy. Figure 1 shows an ideal stress – strain graph for energy absorbent materials.

2.2 Foam behavior under compression loadings

When investigating the behavior of foams under compression loading, it is necessary to investigate various behaviors including linear elastic behavior, non-linear elastic behavior and plastic behavior. During the linear elastic phase, materials don't show suitable energy absorption capabilities. However, non-linear elastic behavior and plastic behavior result in significant strain in foams which is suitable for energy absorption. Among these two, non-linear elastic behavior is often seen in hyper elastic materials such as elastomers which are not the subject of the current study. However, plastic behavior is often seen in metals including aluminum foams under impact loading which is explained in the following sections.

When investigating the mechanical behavior of materials, it is first necessary to have a suitable parametric model which offers a mathematical explanation of mechanical behaviors. To this end, Gibson and Ashby introduced a geometrical model for studying the behavior of foams. They offered two models for open and close-cell foams and proved their accuracy using empirical experiments [3]. This model for open-cell foams is shown in figure 2.

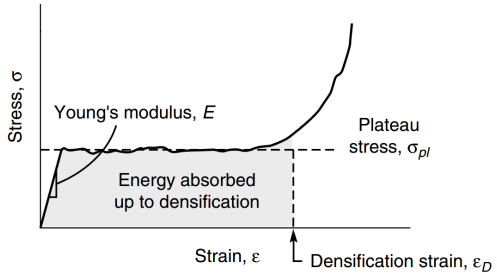

Fig.1
Ideal stress – strain graph of energy absorbent materials [19].

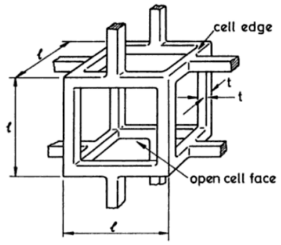

Fig.2
Gibson – Ashby model for open-cell foams [3].

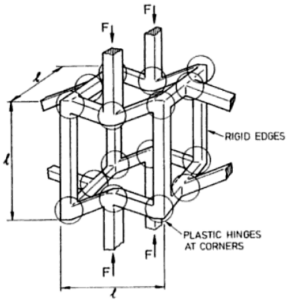

Fig.3
Foam densification due to plastic behavior [3].

According to figure 3, compression loading results in bending of the foundations and creates plastic hinge in junctions. Continuous loading results in densification of the foam. The formation of plastic hinge is shown in figure 3.

If σ_{pl}^{*} is the plateau stress resulting from plastic behavior of the foam, σ_{ys} is the yield stress of base metal, ρ^{*} is the density of the foam and ρ_s is the density of the base metal, then their relation can be defined as shown in equation (1) [3].

$$\frac{\sigma_{pl}^{*}}{\sigma_{ys}} = 0.3\left(\frac{\rho^{*}}{\rho_s}\right)^{\frac{3}{2}} \tag{1}$$

3 THE ENERGY ABSORPTION EQUATION OF OPEN-CELL FOAM

If σ_{pl}^{*} is the plateau stress, it can be calculated with the help of equation (1) as shown in equation (2) [3].

$$\sigma_{pl}^{*} = 0.3\sigma_{ys}\left(\frac{\rho^{*}}{\rho_s}\right)^{\frac{3}{2}} \tag{2}$$

On the other hand, densification strain can be calculated using equation (3) [3].

$$\varepsilon_D = 1 - 1.4\left(\frac{\rho^{*}}{\rho_s}\right) \tag{3}$$

Therefore, a constant value for plateau stress in stress – strain graph is based on the assumption of elastic perfectly plastic behavior and necessary for Gibson – Ashby model. The shaded area in figure 1 is the amount of absorbed energy and is calculated using equation (4).

$$E = \left(0.3\sigma_{ys}\left(\frac{\rho^{*}}{\rho_s}\right)^{\frac{3}{2}}\right)\left(1 - 1.4\left(\frac{\rho^{*}}{\rho_s}\right)\right)V \tag{4}$$

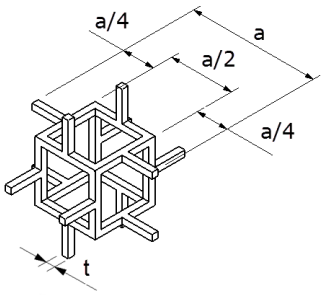

Fig.4
The naming of cell dimensions in a Gibson – Ashby cell.

Where V is the volume of the foam. So we have an energy function that its independent variables are: density of foam (which is dependent on cell dimensions), density of the base metal and yield stress of the base metal. This function will then calculate the amount of energy absorbed by the foam. However, if the goal is to provide an energy absorption equation for open-cell foams based on cell dimensions, these dimensions can be named as shown in figure 4 with bases in other directions shown by b and c.
The volume occupied by these parts is:

$$V_{members} = t^2(3a + 3b + 3c - 16t) \tag{5}$$

While the mass of the basses is calculated using:

$$m = \rho_s \times V = \rho_s t^2(3a + 3b + 3c - 16t) \tag{6}$$

Therefore, foam density can be written as:

$$\rho^* = \frac{\rho_s t^2(3a + 3b + 3c - 16t)}{abc} \tag{7}$$

Finally, energy absorption of a single cell is calculated using equation (8).

$$E = \left(\left(\frac{0.3\sigma_{ys}}{\rho_s^{1.5}}\right)\left(\frac{\rho_s t^2(3a + 3b + 3c - 16t)}{abc}\right)^{1.5} - \left(\frac{0.3\sigma_{ys}}{\rho_s^{1.5}}\right)\left(\frac{1.4}{\rho_s}\right)\left(\frac{\rho_s t^2(3a + 3b + 3c - 16t)}{abc}\right)^{2.5}\right) \tag{8}$$

For example, using equation (8), it is possible to predict that a cell with dimensions of a=b=c=0.04(m) and t=0.002(m) and yield stress of 1.53×10^8(Pa) is capable of absorbing 8.35 joules of energy.

4 NUMERICAL SIMULATION IN ABAQUS ENVIRONMENT

ABAQUS software environment was used to compare the results obtained from analytical equations with the results of numerical simulations. To this end, a single Gibson – Ashby cell with dimensions of a=b=c=0.04(m) and t=0.002(m) was modeled in ABAQUS environment and investigated under impact loading. Geometrical modeling of a Gibson – Ashby cell along with an impacting plane above the cell and a support plane below the cell was carried out in Assembly environment of CATIA software. Then, the geometrical model (including three parts of cell, moving plane and stationary plane) was imported into ABAQUS environment. Model dimensions were presented in the metric system. After introducing material properties to the software, a 5(Kg) weight was modeled on top of striker plane. After assembly of the parts, an explicit dynamic step with time step of 0.02(s) was defined in the Step environment. The contact between cell base and striker and support planes was defined without friction penalty. In the loading section and support conditions, an initial velocity of 1.82 (m/s) was applied to the weight on the striker

plane. The cells used solid element meshing while shell element was used for meshing of the rigid planes. The lower rigid plane was fully fixed and support conditions at the end of cell bases was considered to be symmetrical. This is because this cell is in fact part of a continuous foam structure. Finite element model of the cell and rigid planes is shown in figure 5.

In order to introduce material characteristics to the software, simple tensile strength test results for an Al-6061 sample presented in reference [20] were used. Given the fact that Gibson – Ashby model assumes elastic perfectly plastic stress – strain behavior in materials, the strain for the yield stress is equal to zero which is written in the first row while the ultimate strain is added in the next row while specifying that the stress associated with this strain should be the yield stress of material. Table 1 shows the plastic properties defined in the software.

The simulation results can be compared to the results of analytical equations only when the energy of the impact in the software is fully absorbed by the cell. In other words, after the end of simulation, the striker weight should have a kinetic energy of zero. According to equation (9), the kinetic energy of the striker weight in simulation can be calculated:

$$K = \frac{1}{2}mv^2 = \frac{1}{2} * 5 * 1.82^2 = 8.35\ (J) \tag{9}$$

After the end of analysis, the kinetic energy graph of the striker weight is created using the History output section of ABAQUS software and presented in figure 6. This graph shows that all kinetic energy is absorbed by the cell.

Furthermore, figure 7 shows the position of the striker weight at the final time step which shows full absorption of kinetic energy. On the other hand, deformation of the cell also indicates that the cell has used all its energy absorption capacity. The animated time history output of the software also shows that kinetic energy reaches zero only due to bending capacity of cell parts without compressing two vertical supports after bending of horizontal supports. In other words, observing cell deformation steps under impact loading shows that impact stops before bended bases can reach each other.

Table 1
Defined plastic properties of the material

Stress (Pa)	Strain
1.53×10^8	0
1.53×10^8	0.181478

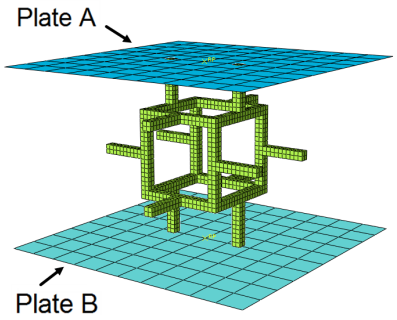

Fig.5
Finite element model of a single cell along with striker and support planes.

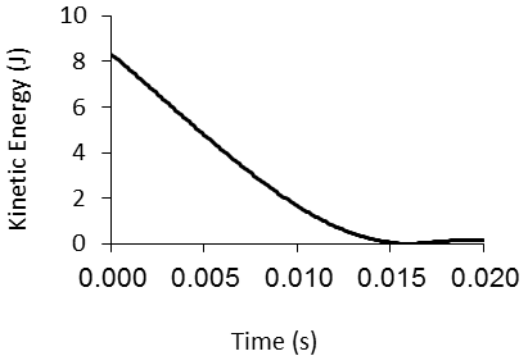

Fig.6
Changes in kinetic energy during simulation.

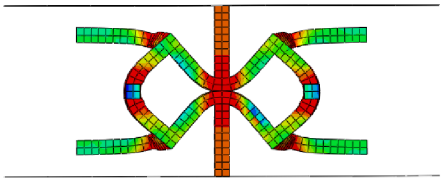

Fig.7
Deformed cell after impact loading.

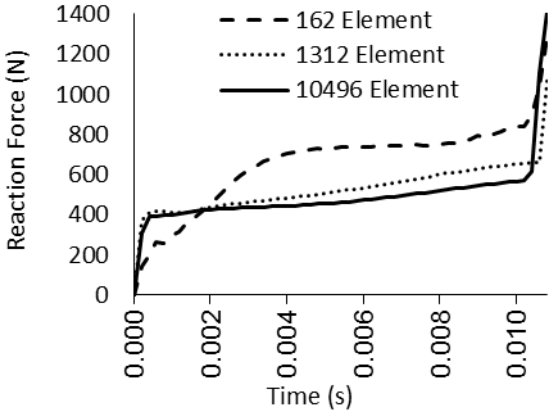

Fig.8
Comparison of force transferred to the support in three different meshing.

Therefore, numerical simulation results are fully compatible with the results obtained from analytical equations.

In order to ensure that the results are independent of the finite element meshing, analysis is repeated three times with different meshing sizes. After each simulation, the force applied to the support is extracted from the software. The support force in the second simulation is different from the first simulation while the results of second and third simulations are similar. This shows that results are convergent after the second simulation which is shown in figure 8.

Since selecting proper element type can affect the analysis results in finite element simulations, the results obtained using solid element and beam element in cell modeling were compared. Figure 9 shows the results of simulation using beam element.

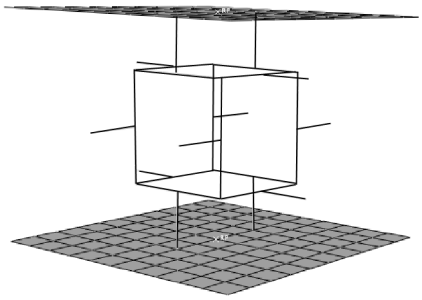

Fig.9
Finite element model of Gibson – Ashby cell using beam element.

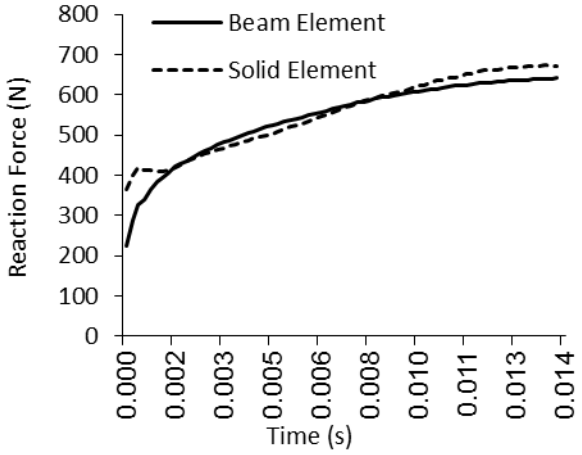

Fig.10
The transferred force for two different elements in impact loading.

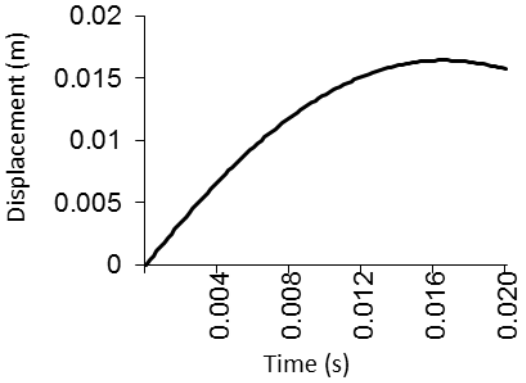

Fig.11
Maximum displacement of striker in impact loading.

After preparing the model, its numerical solution was determined using the same method used for the model with solid element. The results show that the force transferred to the support plain in this model is similar to the model using solid element as shown in figure 10.

However, it's important to note that due to the ability of solid elements in creating self-contact between cell parts and densification, it is necessary to use solid element for modeling Gibson – Ashby cells.

In order to compare the results of impact loading for Gibson – Ashby cell with the results of quasi-static loading, the same model was investigated under quasi-static loading. To this end, maximum displacement of striking plane in impact loading in software simulation results was identified. The maximum displacement can be seen in figure 11.

The maximum striker displacement in impact loading (where initial kinetic energy is fully absorbed) was then defined as mandatory displacement in the amplitude of ABAQUS software. It's worth noting that in this case, it's not necessary to define striking weight or initial velocity for the moving place because plane movement is only controlled by the mandatory displacement. In order to solve this problem, the force transferred to the base is determined and compared to the transferred force under impact loading which is shown in figure 12.

Based on the results, the force transferred to the support plain in the impact loading is higher than the transferred force under quasi-static loading.

5 CONFIDENCE AND ACCURACY OF NUMERICAL SIMULATION RESULTS

The results presented in reference [5] were used to ensure the accuracy of numerical results. This reference conducts an extensive investigation of Gibson – Ashby and Kelvin models in simulation of metal foams in ANSYS software and compares the results of empirical tests with numerical simulations. An important part of this reference is that it includes the results of modeling and simulation for a five-cell Gibson – Ashby foam under different loadings.

The geometrical characteristics of this modeled foam and loadings are also provided. Figure 13 shows the open-cell foam investigated in this work [5].

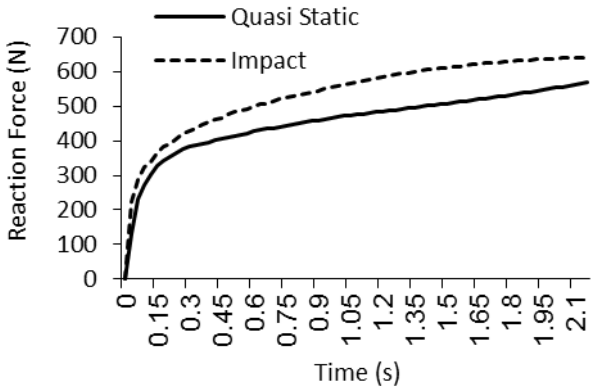

Fig.12
Comparison between transferred force under quasi-static and dynamic loadings.

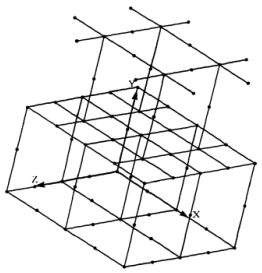

Fig.13
Five-cell foam modelled with beam elements [5].

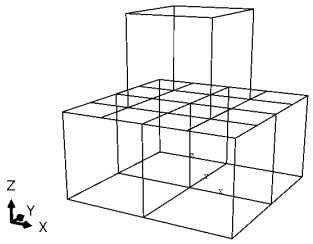

Fig.14
Finite element model of five-cell foam in ABAQUS software.

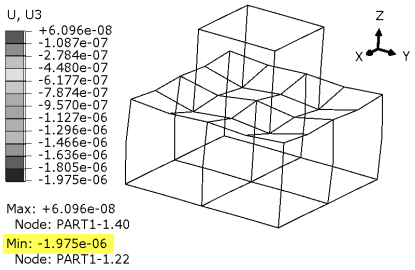

Fig.15
Foam deformation under loading.

This model foam as the dimensions of 1x1x1(in) with base thickness of 0.015(in). The elasticity module of the material is 16500000(psi) and its Poisson ratio is 0.29. Furthermore, loading is carried out by applying a total force of 2(lb) on four nodes of the upper cell. Based on the result, this foam is densified by 0.00005565(in) [5].

In order to compared the results of ABAQUS simulation with these results, this foam was modeled in ABAQUS environment using metric dimensions. Boundary conditions and loadings were also carried out similar to the reference values. Figure 14 shows the finite element model of this foam in ABAQUS environment.

After analysis, as shown in figure 15, highest displacement is 1.975×10^{-6}(m). The displacement of the model in the reference [5] is also equal to 1.4135×10^{-6}(m). This means that the displacement of the ABAQUS model is 0.562×10^{-6}(m) higher than the displacement in reference [5] which is a 39% increase. This difference can't be attributed to error for two reasons. First is that both simulations use finite element analysis and the second reason is that difference in the number of elements between two models or optimization of software finite element algorithms since the publication of reference [5] (1997) can result in different results. This means that the ABAQUS results and the results presented in reference [5] have good compatibility with each other.

6 EQUATION FOR CALCULATING SUPPORT FORCES IN GIBSON-ASHBY OPEN-CELL FOAMS

Other than equation (2), plateau stress can also be determined by dividing the force transferred to the support plain with the effective cross-section area of the foam as shown in equation (10).

$$\sigma_{pl} = \frac{F_T}{A_C} \tag{10}$$

Often, A_c is determined by projecting the absorbent on a horizontal surface (perpendicular to the loading direction). For example, if the absorbent is cubic and the force is applied to one of its faces, its effective cross-section is rectangular.

Densification strain is also different from its usual definition in elasticity equations. In energy absorption, densification strain is determined by dividing the distance between stationary and moving planes (ΔL) by the initial distance between these two planes.

However, in an open-cell Gibson – Ashby foam, if the support force (plateau force) calculated in software is divided by the effective cross-section of the cell, the resulting stress is higher than the plateau stress determined

using analytical equations. In other words, if A_c is the effective foam cross-section, it is not possible to calculate the support force using Gibson – Ashby equations. This means that force applied to the support doesn't follow equation (11) [3].

$$F_T = 0.3\sigma_{ys}\left(\frac{\rho^*}{\rho_s}\right)^{\frac{3}{2}} \times A_C \tag{11}$$

Instead, in practice, the plateau force reported in the software is larger than the value calculated using equation (11). The reason for this contrast is as follows.

If we accept the plateau force presented by the software as the correct value, then this means that the effective cross-section area of a single cell is probably different from the traditional cross-section area used in energy absorbent structures. This probability becomes more likely when we observe that densification strain in a single cell is different from the densification strain calculated using analytical equations. Based on the analytical equations presented in section 3, densification strain can be calculated using equation (3).

If the densification strain of the foam is calculated using equation (3) for different densities, we can see that densification strain in all cases is close to 1. This means that Gibson – Ashby analytical equations assume that foam is fully densified. However, in the simulation of a single Gibson – Ashby cell, densification strain is not close to 1 because a cell reaches densification strain when, under load, its two horizontal bases bend and make contact with each other. In this case, as shows in figure 16, densification strain can be calculated using equation (13).

$$\varepsilon = \frac{\Delta L}{L_0} = \frac{\frac{X}{4}+\frac{X}{4}}{X} = 0.5 \tag{13}$$

This means that densification strain in a single cell is 50% lower than the strain calculated using analytical equations.

Given the fact that absorbed energy is equal to the area under stress – strain graph or the product of plateau stress and densification strain and that this number should be similar in analytical equations and simulations, then it's logical that transferred force in the simulation as the product of plateau stress and effective cross-section area to be higher.

If the aim is to introduce a rule for decreasing simulated strain compared to analytical strain, then it's important to note that this value is dependent on the thickness of basses. This is due to the fact that thicker bases, when bent, are faster to make contact with each other which means densification strain occurs sooner. According to figure 17, densification strain when considering thickness is calculated using equation (14).

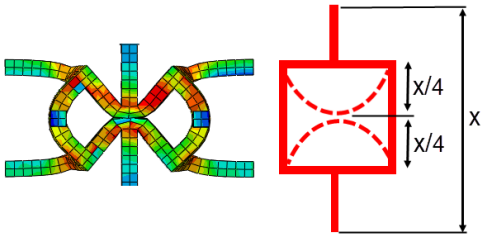

Fig.16
Determination of cell strain.

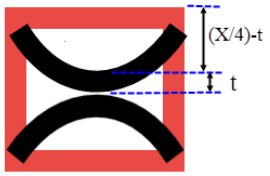

Fig.17
The role of thickness in calculating densification strain.

$$\varepsilon = \frac{\Delta L}{L_0} = \frac{\left(\frac{X}{4} - t\right) + \left(\frac{X}{4} - t\right)}{X} = \frac{\frac{X}{2} - 2t}{X} = \frac{X - 4t}{2X} \tag{14}$$

It's evident that in equation (14), using t=0 results in ε=0.5 which is similar to equation (13).

For ε=0.5, a logical assumption is that plateau stress (and therefore plateau force) in simulation should be doubled in order to reach similar energy absorption capacity. We can further refine this assumption and conclude that the ratio introduced in equation (15) should be added to analytical equation (11) in order to reach similar energy absorption capacity.

$$n = \frac{1 - 1.4\left(\frac{\rho^*}{\rho_s}\right)}{\frac{X - 4t}{2X}} = \frac{2X\left(1 - 1.4\left(\frac{\rho^*}{\rho_s}\right)\right)}{X - 4t} = \frac{\frac{2X\rho_s - 2.8X\rho^*}{\rho_s}}{X - 4t} = \frac{2X\rho_s - 2.8X\rho^*}{\rho_s(X - 4t)} \tag{15}$$

Therefore, we can predict that if the effective cross-section area of the foam traditionally used in energy absorbent structures is shown by A_c, then the force applied to the support of a foam modeled using Gibson – Ashby model is equal to:

$$F_{TG} = n \times F_T = n \times 0.3A_C\sigma_{ys}\left(\frac{\rho^*}{\rho_s}\right)^{\frac{3}{2}} = \left(\frac{2X\rho_s - 2.8X\rho^*}{\rho_s(X - 4t)}\right)\left(0.3A_C\sigma_{ys}\left(\frac{\rho^*}{\rho_s}\right)^{\frac{3}{2}}\right) \tag{16}$$

Furthermore, the effective cross-section area of a Gibson – Ashby cell can be calculate using equation (17):

$$A_{CG} = \frac{A_C}{n} = \frac{A_C}{\left(\frac{2X\rho_s - 2.8X\rho^*}{\rho_s(X - 4t)}\right)} = \frac{A_C\rho_s(X - 4t)}{2X\rho_s - 2.8X\rho^*} \tag{17}$$

These equations are for a single cell. If the foam is made from several cells, then equation (14) can be separately written for each cell before adding all equations together to determine the total densification strain of the foam. For example, if the foam is made from a total of m cells, then densification strain is calculated using equation (18):

$$\varepsilon = \frac{\Delta L}{L_0} = \sum_{1}^{m} \frac{X_m - 4t_m}{2X_m} \tag{18}$$

And the n ratio for a foam made from m number of cells is calculated using equation (19):

$$n = \sum_{1}^{m} \frac{2X_m\rho_s - 2.8X_m\rho^*}{\rho_s(X_m - 4t_m)} \tag{19}$$

7 CAPABILITY OF USING GRADED IMPACT ABSORPTION STRUCTURES

Graded nature of energy absorbent structures is one of the most important and influential characteristics of these structures. This graded structure is abundant in natural impact absorbent structures. For example, figure 18 shows a cross-section of banana peels [10].

Based on this figure and the definition of closed-cell foams, banana peel can be classified as a natural close cell foam. Another example of graded natural structure is bone structure. Figure 19 shows the cross-section of a bone [22].

The internal structure of a bone is in fact a graded and optimized open-cell foam structure. The density of this foam changes in different directions so that it has good compatibility with principal stress directions and the largest shear stress applied to the structure [21].

When considering graded structures, it is necessary to make use of optimization techniques. This necessity is due to the fact that graded structures can be created in numerous different arrangements but only one of these possibilities result in good energy absorption capabilities.

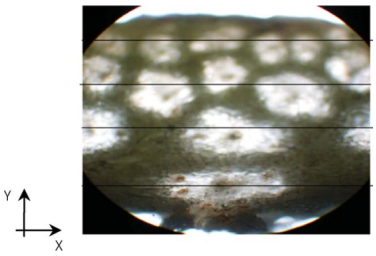

Fig.18
magnified cross-section of a banana peel [21].

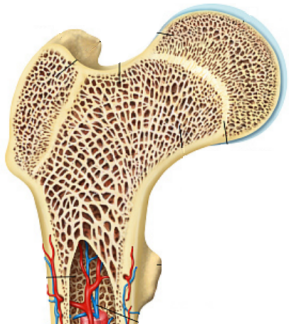

Fig.19
The foam structure of a bone [22].

The aim of an optimization problem is determining the minimum or maximum point of the f(x) function under boundaries such as equality or none quality. Today, some of the common methods used for solving these complex problems are the methods based on swarm intelligence including genetic algorithm.

8 OPTIMIZING THE ENERGY ABSORPTION BEHAVIOR OF METAL FOAMS

As mentioned in section 3, the area under the stress – strain graph is a measure for the absorbed energy of an energy absorbent structure. In other words, if we have the force – displacement graph of the absorbent during the process, it will be possible to calculate the amount of absorbed energy using equation (20) [3].

$$EA = \int_0^l F(x)dx \tag{20}$$

Where l is the compression length and F is the force applied to the support.

8.1 Specific energy absorption measure

During the design of an energy absorbent, its weight is also important. In other words, an ideal energy absorbent is one with highest absorbed energy to mass ratio.

By dividing the amount of absorbed energy to absorbent's mass, it is possible to calculate specific energy absorption (SEA) of the structure as shown in equation (21) which shows the amount of absorbed energy for each unit of mass [1].

$$SEA = \frac{EA}{m} \tag{21}$$

Highest SEA values show higher absorbent efficiency.

During behavior optimization of an absorbent with graded structure, first we need to determine its specific energy function. The process then aims to maximize the value of this function for a foam sample.

If an open-cell foam with three cells in considered, using equation (4), energy absorption capacity of the first cell (U_1) is calculated using equation (22).

$$U_1 = E_1 V_{foam} \tag{22}$$

The mass of the first cell is equal to:

$$m_1 = V_{members} * \rho_s \tag{23}$$

Where ρ_s is the density of the base metal used in the foam. The same equations can also be written for the second and third cells in order to determine U_2, U_3, m_2 and m_3 values. Therefore, the energy absorbed by this foam can be calculated using equation (24).

$$u = U_1 + U_2 + U_3 \tag{24}$$

While its mass is presented by equation (25):

$$m = m_1 + m_2 + m_3 \tag{25}$$

Based on the definition of SEA, the value for u/m should be maximized. In other words, if the cost function is the inverse of the above function, then the value for m/u should be minimized. In the next section, the optimization section of MATLAB software is used to minimize m/u value.

We assume that cell dimensions and base thickness have initial and final values within a possible range. Therefore, software can use optimization algorithm to suggest the optimal dimensions for reaching minimum m/u values.

The highest and lowest bounds of the variables are shown by ub and lb, respectively.

lb = [0.04 0.04 0.04 0.002 0.04 0.04 0.04 0.0025 0.04 0.04 0.04 0.003]
ub = [0.05 0.05 0.05 0.0025 0.05 0.05 0.05 0.003 0.05 0.05 0.05 0.0035]

8.2 Boundaries of optimization problem

In order to determine the boundaries of the problem, it is necessary to pay attention to the densification mechanism of Gibson – Ashby cells. The relative density of each foam is determined by dividing foam density with the density of the base metal as shown in equation (26).

$$\rho_{rel} = \frac{\rho^*}{\rho_s} \tag{26}$$

In an open-cell foam, if the relative density is larger than 0.3, then foam will have squat and thick bases and yield occurs as compression or tensile yield instead of bending. This means that in this case, it is better to consider the structure as a bulk metal structure with a hole in the middle instead of a foam. Furthermore, in very low relative densities, elastic collapse is more important than plastic collapse which is also undesirable. This means that it is necessary to set two boundaries in order to create proper plastic behavior in the foam.

1) Relative density of the foam should be lower than 0.3 as shown in inequality (27):

$$\left(\frac{\rho^*}{\rho_s}\right) < 0.3 \tag{27}$$

2) Relative density of the foam should not be lower than a lower limit (as shown in inequality (28) in order to observe plastic behavior instead of elastic behavior).

$$\sigma^*_{pl} < \sigma^*_{el} \tag{28}$$

This inequality can be rewritten as follows based on equation (2) and using plateau stress equation (when elastic collapse mechanism is active) [3]:

$$\left(\frac{\rho^*}{\rho_s}\right) > 36\left(\frac{\sigma_{ys}}{E_s}\right)^2 \tag{29}$$

Since the foam is made from three cells, then ρ^* value in equations (28) and (29) should be calculated as the average density of three cells.

$$\rho^*_{total} = \frac{\rho^*_1 + \rho^*_2 + \rho^*_3}{3} \tag{30}$$

The boundaries should in equations (28) and (29) are both considered during optimization problem of foam energy absorption.

8.3 The results of optimization problem

Using two methods of genetic algorithms and SQP (Sequential Quadratic Programming), the problem can be solved. The parameters resulting from these two algorithms are presented in table 2.

Table 2
The parameters resulting from two genetic algorithms and SQP

Design variable	Initial value (m)	Optimised value by GA (m)	Optimised value by SQP (m)
a_1	0.045	0.0401	0.04
b_1	0.045	0.0402	0.04
c_1	0.045	0.04	0.04
t_1	0.0025	0.0025	0.0025
a_2	0.045	0.0404	0.04
b_2	0.045	0.0402	0.04
c_2	0.045	0.0402	0.04
t_2	0.003	0.003	0.003
a_3	0.04	0.04	0.04
b_3	0.045	0.0403	0.04
c_3	0.045	0.04	0.04
t_3	0.0035	0.0035	0.0035

If the design parameters obtained from genetic algorithm are considered as the final answer, then open-cell foam can absorb a total of 78.6896 (J) of energy. However, if the answer of the Sequential Quadratic Programming algorithm is considered, total energy absorption will be 78.5785 (J).

This means that the final answer of genetic algorithm can be considered as the final answer.

In order to ensure the accuracy of the optimization results provided by MATLAB, the convergence graph for genetic algorithm is drawn and presented in figure 20.

This graph shows that with increase in the number of generations, the value of the cost function decreases and converges to a constant value which is the optimal value. The optimization method can be used to design multi cell three dimensional structures [19].

8.4 Simulation of optimum graded foam in ABAQUS environment

The graded foam presented in the previous function should be able to absorb 78.57(J) of energy based on equation (8) and total values for three cells. Using equation (31), it can be calculated that this amount of energy is equal to an impact by a 5 (Kg) weight at the speed of 5.6 (m/s).

$$v = \sqrt{\frac{2K}{m}} = \sqrt{\frac{2 \times 78.57}{5}} = 5.6 \ (\mathrm{m/s}) \tag{30}$$

In order to simulate the graded foam optimized in the previous section, first, its geometrical model is created in CATIA software with a plane at the top to act as the striker plane (in contact with thicker cell) and a plane under the structure to act as support (in contact with the weaker cell). The joints between cells in the geometrical model is considered to be continuous in order to prevent more contacts in finite element model. This model is then imported into ABAQUS environment and after defining material properties, the effects of impact from a 5 (Kg) weight is modeled. The contact between cell bases and striker and support planes are defined similar to the single cell conditions. In the loading and boundary conditions, an initial velocity of 5.6m/s is applied to the weight placed in the striker plane. The finite element model of the open-cell foam is presented in figure 21.

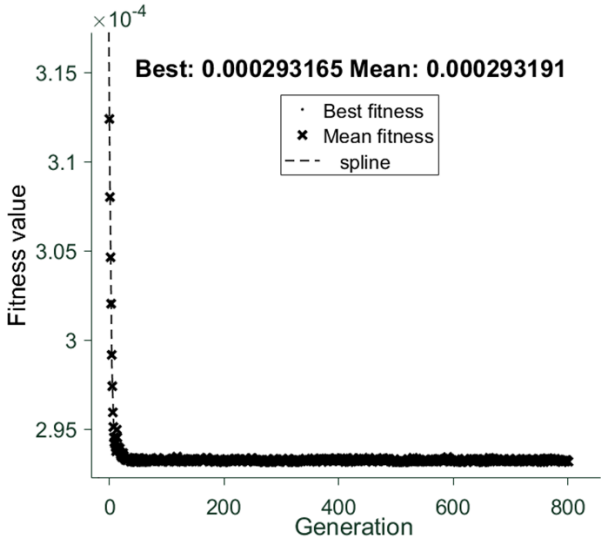

Fig.20
The Convergence graph for the genetic algorithm.

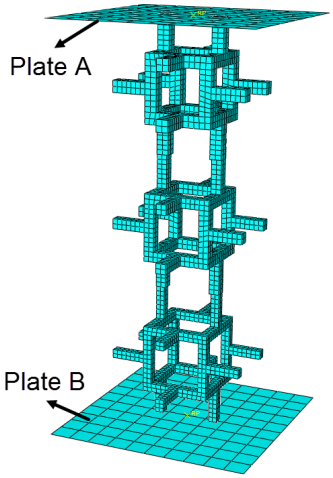

Fig.21
Finite element model of open-cell foam with three cells.

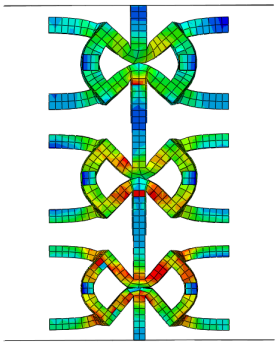

Fig.22
Foam deformation under impact loading.

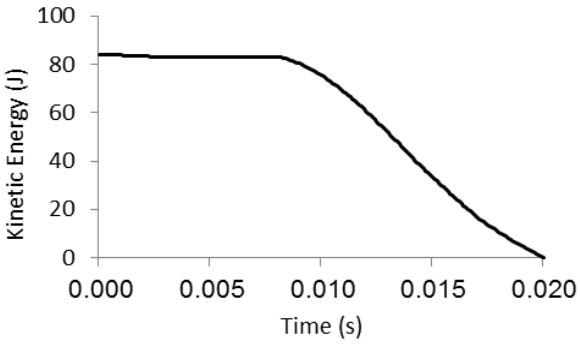

Fig.23
The absorption of kinetic energy by graded foam over time.

As mentioned before, it is expected that the modeled graded foam can absorb the energy resulting from an impact of a 5 (Kg) weight at the speed of 5.6 (m/s). However, the simulated graded foam can absorb the energy of an impact at the velocity of 5.8 (m/s) which indicates 3% error. Figure 22 shows the deformed cross section of the foam at the final time step of the analysis.

Figure 23 shows the decrease in the kinetic energy of the striker until reaching zero.

9 CONCLUSION

By considering elastic perfectly plastic model for the materials, the equation for absorbed energy in a foam with Gibson – Ashby cell was extracted. The energy calculated using this equation was fully compatible with the results of numerical simulation in ABAQUS software. In order to ensure the accuracy of the numerical simulation, the results provided for simulation of a five-cell open-cell foam in another study which had been confirmed using empirical tests were used for comparison. In order to determine the sensitivity of finite element results to element type, simulation results were compared to the results obtained using solid and beam elements. Furthermore, the results for impact loading of a Gibson – Ashby cell was compared to the results of quasi-static loading. This comparison showed that the force transferred to the support in impact loading is higher than the transferred force during quasi-static loading. The results of numerical simulations also indicated that the effective cross-section area of Gibson – Ashby open-cell foam is different from traditional effective cross-section area of absorbent structures and the reason for this difference was stated. Next, since effective cross-section area of the foam is important in calculating the force transferred to the support, a logical equation was presented to determine the effective cross-section area of Gibson – Ashby cells using base metal density, foam density, cell dimensions and base thickness of the foam. Using this equation and plateau stress equations in open-cell Gibson – Ashby cell, an equation was presented to predict the force transferred to the support in these cells. The possibility of using graded foam was also investigated using a three-cell open-cell structure. By considering a range for cell dimensions and base thickness, two methods including genetic algorithm and SQP (Sequential Quadratic Programming) were used to create a graded foam structure with lowest mass to absorbed energy ratio. The foam created using genetic algorithm was capable of absorbing 0.1 (J) more energy compared to the one optimized using SQP. This method can be used to measure the capacity of energy absorption and design of open, multi cell and three dimensional foam structures.

10 Nomenclature

E^*	Young modulus of the foam
G^*	Shear modulus of the foam
I	2nd moment of area
E_s	Young modulus of base metal
A_C	Effective cross-section area of the foam in absorbent structures
A_{CG}	Effective cross-section area of the open-cell Gibson – Ashby foam
F_{TG}	Reaction force
ρ^*	foam density
ρ_s	Base metal density
ρ_{rel}	Relative density of the foam
v^*	Poisson ratio of the foam
σ^*_{el}	Plateau stress resulting from elastic collapse mechanism
σ^*_{pl}	Plateau stress resulting from plastic collapse mechanism
σ_{ys}	Yield strength of base metal
ε_D	Densification strain

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