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Research Article

A Game Theory Approach for Supply Chain Coordination Model with Incentive Mechanisms of Discount and Delay in Payments

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The uncertain nature of supply chains is one of the key challenges managers, and researchers encounter in decision-making. Accordingly, this paper proposes a three-echelon supply chain in which demands are uncertain. The proposed supply chain has three participants, including supplier, manufacturer, and retailer, while three decentralized, centralized, and coordination models have been formulated to maximize participants' profits. In the decentralized model, both the manufacturer and retailer independently determine the level of investment and order quantity regarding scenario-based demands. The centralized model determines the optimal order quantity and investment amounts for the whole network. However, these amounts may be different from optimal values for all participants. As such, using game theory, a bilevel adjustable contract based on wholesale price has been proposed as an incentive for players to participate in the coordination plan. Results show that the coordination model outperforms others by reducing the network's costs and increasing profits simultaneously.

1. Introduction

In today's global economy, supply chain management (SCM) plays a crucial role in any enterprise's major success [1]. In this respect, planning is the foundation of sourcing, production, and logistics, which can affect the supply chain's overall cost, productivity, and quality [2].

Supply chain planning is concerned with coordinating and synchronizing of several activities of different functions from the very beginning, e.g., procurement of raw materials for the final process and distribution of finished products, which may require different coordination mechanisms due to centralized or decentralized supply chain operations. In a centralized supply chain, a single decision-maker optimizes the entire system's performance by having access to all the required information. On the other hand, in a decentralized system with multiple players, there is no unique planner; instead, the members, who may have conflicting objectives, decide according to their own interests [3].

Since supply chain management incorporates all activities related to material flow and product transition from the raw state (extraction) to the final state (consumption), as well as the flow of surrounding information, improving the interaction between chain loops will give companies a reliable and sustainable competitive edge [4].

As intermediaries between manufacturers and consumers, retailers are continuously exposed to uncertain risks. Apart from the uncertain demand of consumers, retailers may also face uncertain yields. Uncertain performance in manufacturing and logistics could lead to increased risks caused by uncertainty in the chain. To deal with this, the concept of supply chain coordination has been developed to benefit all chain members, especially in times of uncertainty [5].

With the growing focus on sustainable supply chain management, firms find out that outputs across the entire supply chain can be more efficiently managed through greater cooperation and better coordination. Coordination in supply chain management is based on centralized and decentralized decision-making. The supply chain coordination aims to minimize the total supply chain cost. In a decentralized supply chain, each player tries to maximize his own performance.

In centralized decision-making, coordination allows supply chain players to work closely to streamline their decision-making, aiming to maximize the entire supply chain performance. Therefore, supply chain coordination must involve incentive plans to attract customers [6].

In many real-world situations, it is difficult to accurately estimate key parameters, including market demand, supply time, production volume, which are major sources of uncertainty in the supply chain. The uncertainty in these parameters is an inevitable part of decision-making in the supply chain. In practice, the uncertain demand for many products can be modeled through a scenario-based approach. As a result, the coordination of chain members is an efficient means to improve the overall performance of the supply chain and coordinate various decisions [7].

Studies on supply chain coordination can be classified into three flows. In the first flow, a chain mechanism is coordinated only when it leads to the improvement of the whole supply chain. In the second flow, the chain mechanism is coordinated only when it leads to the improvement of the entire supply chain compared to the default solution, which lacks coordination. In the final flow, implementing the coordination mechanism could lead to a practical solution across the supply chain system [3].

This study investigates a three-echelon supply chain where the supplier, manufacturer, and retailer are each concerned about their own benefit in the game theory, and it seeks to increase the profits for each actor. In this respect, three centralized, decentralized, and coordinated models are employed to formulate the proposed supply chain under uncertain demand. The following sections discuss the results of applying these models to the petrochemical industry. The structure of the paper is as follows. Section 2 presents the literature review and specifies the research gap within the present research. The proposed models and the solution approaches are presented in Section 3. Section 4 is dedicated to the model validation and the final results of the models, followed by conclusions in Section 5.

2. Literature Review

Supply chain coordination has been the subject of many studies:

According to Christy et al. [8], the demand of a product is linearly dependent on the retail price and quality of the product. They address a closed-loop supply chain where the manufacturer manufactures products according to the demand and sells them through a retailer in the market. A third party collects the used products from costumers and sends to the manufacturer to increase the quality. If the products can retrieve the original quality, thus the process is called remanufacturing. Not every product can retrieve the original quality; thus, manufacturers refurbish these products with lower price. We construct four different scenarios–centralized and decentralized led by manufacturer, retailer, and third party. From the comparison of the result obtained in the numerical example, they conclude that the joint profit obtained under centralized, manufacturer-led, and retailer-led policies is higher than third party-led policy.

Taleizadeh et al. [9] examined the multiperiod sustainable planning of a closed-loop supply chain using a comprehensive model that considers the social and environmental impacts of supply chain decisions and measures their effects on social and environmental factors through the (global report) guide indices. The proposed planning model incorporates tactical decisions such as the product's price and logistic decisions. Using a discount for a returned product is an incentive policy designed to increase the tendency of customers to return the products. In addition, two recovery approaches have been considered, including remaking and recycling along with using high-quality returned products.

Xiao et al. [10] explored a two-echelon supply chain consisting of a manufacturer and many retailers whose demands lie in unreliability. This paper examines two practical issues through a collaborative game approach, where prominent coalition values are demonstrated with an expected favorable profit. We have also examined an advertisement collaboration issue of a manufacturer and retailer where the manufacturer plays the Stackelberg game by forming a coalition with the retailer.

In their research, Malekian [11] considered the increase in price and national advertisements of the manufacturer in a manufacturing supply chain with the effects of the consumers' reference price. The paper examines a centralized game, followed by two Stackelberg games named "Increasing consumer price" and "Increasing retailer and consumer price."

Li et al. [12] presented two novel contract types to achieve supply chain coordination. In this paper, the quality flexibility contract and the capacity reservation contract are proposed as mechanisms for encouraging manufacturers to increase capacity and improve overall supply chain performance. The proposed model proves that it is possible to coordinate the supply chain by adjusting the precise parameters of the two contracts. This will ensure that the total profit in the decentralized supply chain is the same as the one in the centralized system.

Ahmad Jauhari et al. [13] develop a closed-loop supply chain model consisting of a single manufacturer, single retailer, and single collector under various coordination scenarios. New products produced from the manufacturing and remanufacturing processes will be sold to the market at the same price. Used products collected by the collector are sorted so that products categorized as recoverable will be sold to the manufacturer. There are two recovery processes considered in this paper, namely remanufacturing and refurbishing. Used products below the minimum acceptable quality level of the manufacturer will be categorized as waste and will be disposed of. They assume that the manufacturing process is imperfect as it produces rework able defective products. A carbon cap-and-trade policy and investment in green technologies are applied in order to restrict the carbon emissions generated by the production stage of the system. The demand at the market place depends on the green technology level, the quality of the product, and the selling price. The proposed model is constructed under five different scenarios–centralized, decentralized, and three Stackelberg games led, respectively, by the manufacturer, retailer, and collector. A numerical example is provided to illustrate and compare the proposed model under each scenario and investigate the sensitivity of some of the model parameters on the optimal solutions. The results show that the centralized scenario performs better in maximizing the total profit compared to the decentralized one. However, the retailer-led Stackelberg model tends to give more equitable profit to all players when the selling price is set at the lower level as this will attract more demand.

Yan et al. [14] suggested a method for the supply chain coordination of new crops by considering the strategic behavior of the consumer. This paper introduces the optimal consumer performance by considering the characteristics of the supply chain of fresh crops. Afterward, this study focuses on the effect of consumer behavior on decision-making in the supply chain based on a centralized chain approach. Ultimately, it proposes two coordination contracts based on revenue sharing and wholesale price to decentralize decision-making in the supply chain of upcoming crops. Afterward, this study focuses on the effect of consumer behavior on decision-making in the supply chain based on a centralized chain approach. Finally, it suggests two coordination contracts for decentralized decision-making in the supply chain of upcoming crops based on revenue sharing and wholesale pricing.

Heydari et al. [15] analyzed the green channel coordination in a two-echelon supply chain in which demand is a function of pricing and the green quality of the product. In this model, the retailer makes pricing decisions, and the manufacturer adjusts the product's green quality. For channel coordination and a win-win result, the "green division of costs" and the "revenue-sharing" contract have been combined.

Toktaş-Palut [16] introduced a three-echelon forward and reverse green supply chain, where manufacturers invest in green production processes. This paper considers the coordination of this integrated green supply chain by introducing a mathematical model associated with fair bilateral contracts between the two sides. Accordingly, a two-part integrated tariff contract is created so that all the supply chain members act based on a rational centralized solution, where both sides can earn a profit.

Cao et al. [17] considered a green agricultural supply chain and examined the coordination and optimal decisions of all units within a decentralized and centralized system by using a game model. The paper proposes a revenue-sharing contract and a repurchase agreement to coordinate the decision-making for greening the whole supply chain and analyzes the effects of green standards at different stages.

Zhu et al. [18] introduced a decision-making model in which the conditional value at risk (CVaR) is used as the risk evaluation criteria. To coordinate a dual-channel supply chain, revenue-sharing and repurchase contracts are jointly used to make bilateral contracts for optimal decision-making in centralized and decentralized situations. This paper proposes joint contacts that could lead to Pareto efficiency for a dual-channel supply chain in which a risk-averse consumer is involved in performance and demand uncertainty.

Zhao et al. [19] studied the centralized and decentralized aspects of decision-making and optimal profit of supply chains. They illustrated that the uncertainty of manufacturing cost exaggerates the motivation for supply chain formation; however, it might increase the expected profit regarding centralized decision-making. Therefore, an incomplete contract has been designed that determines the wholesale price and order quantity in the first stage. Once the manufacturing cost is achieved, companies can renegotiate the contract in the second stage. Interestingly, these incomplete contracts can also coordinate the supply chain.

Ganji et al. [20] introduced a new coordination model in the supply chain, which can gain customers' satisfaction by planning a precise logistic system. This mathematical model accounts for all coordination among chain members and causes an increase in the members' profits by considering the integrated planning of the supply chain, determining the delivery time, the orders' schedules, determining the vehicle's function based on the freight capacity, and minimizing distribution costs, fixed fuel costs, variable fuel costs, carbon emissions, and the time required for cargo deliveries.

Sarada and Sangeetha [21] examined reverse supply chain coordination with a price- and warranty-dependent stochastic demand under collection uncertainty. This paper introduces two theoretical game models of one reverse supply chain (RSC) for the retail of one type of remanufactured product. The first model analyzes a two-stage RSC with one manufacturer and one retailer and considers the uncertainties in association with demand, collection quantity, and the system's performance. The second model studies a three-echelon RSC with one supplier, one manufacturer, and one retailer. Moreover, it considers the stochastic system's demand and performance with price- and warranty period-dependent demand. The manufacturer provides a warranty for remanufactured products that are produced from returned items retrieved from the customers. The policies of centralized, decentralized, and revenue-sharing contracts are imposed on both models. In the end, the revenue-sharing contract increases the profits of the supply chain members.

Putri Adam et al. [22] develop a coordination mechanism for a closed-loop supply chain, operating under several policies to control the carbon emission, namely a carbon tax regulation, government incentives policy, green technology investment, and energy-saving investment. The carbon tax regulation is implemented to lessen the emissions from the manufacturer's activities whereas, to encourage the manufacturer to cut down the emission as well as to increase the product return and energy savings, the government provides incentives based on a target level. The system operates under a variable market demand which is affected by the retailer's selling price, green technology, and energy-saving levels. The proposed problem is formulated under two different scenarios, which are the centralized model and the decentralized model. To improve the supply chain coordination, they also propose two different contracts, namely the green technology revenue-investment-sharing contract (GRIS contract) and the energy saving revenue-investment-sharing contract (ERIS contract). The system inflicted with two types of inspection error in classifying the returned products. The models are formulated mathematically and optimized using a proposed algorithm. The result shows that the centralized model performs better in maximizing the total profit compared to the decentralized model. The results also imply that the government incentives toward product returns, green technology, and energy-saving actually affect the optimal decision of the supply chain system. In addition, the proposed contracts are proven to provide win-win solutions and improve supply chain coordination.

Based on the problem statement and research gap analysis, it can be concluded that the simultaneous use of delays in payment and discounts considering the game theory deserves further study. Additionally, channel coordination is an efficient way to improve the supply chain's overall performance and various decisions. Many past studies on channel coordination have implemented the uncertain nature of market demand with a unique probability distribution, such as a normal distribution. However, in the real world, the market demand could only be estimated with a specific set of discrete scenarios. When market demand is unclear, determining the optimal order quantity can be challenging. A scenario-based approach must be used to model this issue and optimize decisions in such a situation. Nonetheless, to our knowledge, uncertainty through a scenario-based approach in past studies on supply chain coordination still needs further investigation. This research has been conducted through a case study in the real world's petrochemical industry, where a manufacturer sells its products to customers through a distributing channel (retailer). Simultaneously, by providing discounts and delaying payments, the manufacturer seeks to positively affect the customer's perception of its product, thus increasing its market demand. Hence, this study first investigates the problem under a single-scenario stochastic demand and suggests a basic model. Afterward, it considers the scenario-based stochastic demand through the development of the model and analyzes the problem under decentralized, centralized, and coordinated models (three solutions in the game theory).

3. The Proposed Method and the Mathematical Model

This study investigates a three-echelon competitive supply chain through a mathematical model using the game theory concept. The proposed model is an extension to the model presented by Hosseini-Motlagh et al. [7] to include the game theory for implementing coordination in the supply chain. Meanwhile, it considers the incentive mechanisms of discounts and delayed payments based on Aljazzar et al. [6].

3.1. The Research Questions

(i) How to coordinate and create effective communication in a three-level supply chain, taking into account incentive mechanisms (discount and delay in payment) in order to equalize benefits using game theory?

(ii) According to the proposed mathematical models, what is the profit of the chain members and which of the game theory scenarios do they prefer?

(iii) How can a scientific solution be presented for case study problems in the form of decision-making models?

(iv) How can channel coordination be achieved when random demand in the future can only be predicted with a set of discrete scenarios?

(v) How can contract parameters be adjusted in a three-echelon supply chain to induce all actors to participate in the coordination program in each demand scenario?

3.2. Assumptions

(i) The proposed supply chain has three echelons, including supplier, manufacturer, and retailer (buyer);

(ii) A single product is made of several raw materials;

(iii) The demand rate depends on the discounts;

(iv) The production quantity of the supplier exceeds the manufacturer's demand for raw materials, and the production rate of the final products in the manufacturer is faster than the retailer's demand;

(v) Shortage is not permissible;

(vi) The manufacturing policy of the manufacturer follows the Hill policy in which equal batches of production and same-size cargoes are made;

(vii) The holding (storing) cost consists of the two physical and monetary components;

(viii) The discounts and delay in payments are considered as decision-making variables;

(ix) While planning the permissible delays, the manufacturer and retailer invest in the remaining products collected;

(x) The manufacturer and retailer pay for their remaining products in single payments;

(xi) The maximum discount by the supplier, manufacturer, or retailer cannot exceed the profit margin.

3.3. Problem Statement. The supply chain system presented in this research constitutes a three-echelon supply chain (supplier, manufacturer-retailer). In this system, the retailer orders a great quantity of manufactured items Q for the annual demand rate D. The manufacturer manufactures the manufactured items with the annual per rate P, where $P > D$. The manufacturer orders the product quantity αQ from the supplier, where α is the number of raw material units required for the manufacturing of the final product. The manufacturer makes their payments according to the agreed-upon time with the supplier, within a period interest-free. If the manufacturer makes their payment at τ_m after the t_s, where $\tau_m > t_s$, the interest rate k_s is deducted by the time unit to create equilibrium $\tau_m - t_s$. Throughout the time period τ_m or t_s, the manufacturer pays the debt equilibrium to the supplier through the interest rate k_m. Consequently, the manufacturer allows the retailer to make their interest-free payment at t_m within a period of time. As a result, the retailer can postpone their payment to the manufacturer to after the manufacturer has received the interest rate k_m remaining from their account. Hence, the retailer pays their balance to the manufacturer with the interest rate k_r. This paper first considers the problem under the single-echelon stochastic demand and proposes basic models. Afterward, the models are developed to create scenario-based stochastic demand, and the problem is dissected under decentralized, centralized, and coordinated models. In the decentralized model under the scenario-based stochastic demand, by considering the scenario-based stochastic CSR-sensitive demand, the retailer and the manufacturer would separately make decisions on the order quantity and investment. The optimal order quantity and investment are then obtained under the scenario-based stochastic demand according to the whole chain. The optimal solutions under the centralized model may not be satisfactory to all the three agents (three echelons of the supply chain, i.e., supplier, manufacturer, and retailer). To persuade the three agents to enter the coordination plan, we proposed an adjustable bilevel wholesale price contract. According to the proposed contract, the wholesale price of the supplier and the manufacturer is assigned as the contract parameters, and it is determined in such a way that the proposed results of the contract would have a winning situation for all the agents of the supply chain.

This research utilizes the study of Hosseini-Motlagh et al. [7] to calculate the uncertainty in the demand rate while estimating the uncertain model within the supply chain. Accordingly, various scenarios are applied to the chain depending on the retailer's demand rate. A demand distribution function based on discounts and uncertain demand in each scenario is made. Based on the probability of each scenario, the total profit rate of the chain, which consists of the profit of all the three suppliers, manufacturers, and retailer echelons, is affected. As a result, the final goal is the optimization of the chain's total profit. Figure 1 demonstrates the overall schematic of the problem.

3.4. The Mathematical Model. In the development of the mathematical model, the paper uses the symbols as follows.

3.4.1. Parameters

i is determined corresponding to the chain members

s indicates the supplier

w indicates the raw materials of the manufacturer

m indicates the manufactured products of the manufacturer

r indicates the retailer

c indicates the customer

A_i indicates setup/ordering cost

C_i indicates production/purchase cost per unit

h_i indicates financial holding cost per unit

s_i indicates holding (storing) cost per unit

n_1 indicates number of transportations done by the supplier to the manufacturer per each period of the manufacturer's raw material

n_2 indicates number of transportations done by the manufacturer to the retailer per each retailer period

α indicates raw material quantity required for the manufacturing of a final product

t_i indicates permissible delay in payment

τ_i indicates settlement time

k_i indicates return on capital

P indicates manufacturer's annual production rate

d_i indicates discount in the monetary unit by factor i to its customer

S_i indicates demand scenarios index

$\rho(S_i)$ indicates probability of a scenario's occurrence S_i

D_{s_i} indicates annual demand rate under scenario S_i, $D_s < P$.

$f(D_{s_i})$ indicates probability distribution function of the demand in scenario S_i to discount

$f(d_{\mathrm{rs}})$ indicates annual demand of the retailer. In this case, it is assumed that this is a discount and stochastic demand function in various scenarios: $D_s + f(D_{s_i})d_r$

μ_{S_i} indicates average market demand before CSR in scenario S_i

σ_{s_i} indicates maximum increase in medium demand by investing in CSR in scenario S_i

T indicates length of the time period

ψ_i indicates annual profit for each factor i.

3.4.2. Decision Variables

Q indicates order quantity

η indicates cost of the paid unit by the manufacturer in CSR activities per each product

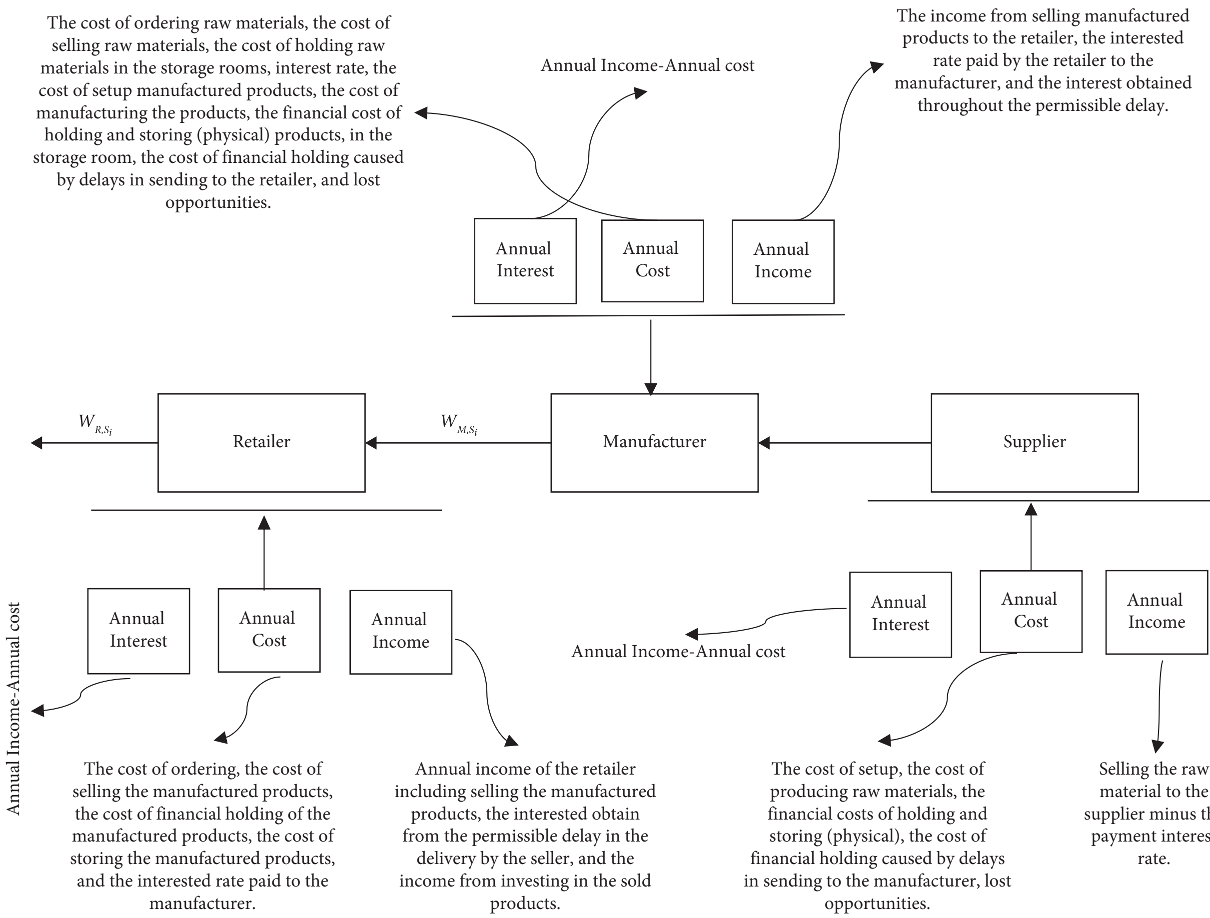

Figure 1: The overall schematic of the problem.

W_{M,S_i} indicates wholesale price of the manufacturer under the proposed contract in scenario S_i

W_{R,S_i} indicates wholesale price of the retailer under the proposed contract in scenario S_i.

3.5. Determining the Total Chain Profit Using the Game Theory

3.5.1. Decentralized Model. Figure 2 illustrates the schematics of this model.

Using stochastic scenario-based demand in this section, the decentralized basic model is developed. In the real world, in most cases, the market demand can be demonstrated with a discrete stochastic variable with merely three probable scenarios, i.e., (1) high demand, (2) medium demand, and (3) low demand. Therefore, in this problem, three different scenarios of S_1, S_2, and S_3 have been chosen and demonstrated as such. More precisely, the future demand of the retailer under each probable scenario of $S_i\,(i = 1, 2, 3)$ is mentioned by D_{S_i} in which D_{S_i} is the normal distribution $N(\mu_{S_i} + \alpha_{S_i} y(\eta), \sigma^2_{S_i})$, where $N = (\mu_0 + \alpha y(\eta), \sigma^2)$. Moreover, the occurrence probability of each scenario is illustrated with $\rho(S_i)$. It is noteworthy that the number of demand scenarios can easily be renewed with a few adjustments in the proposed models.

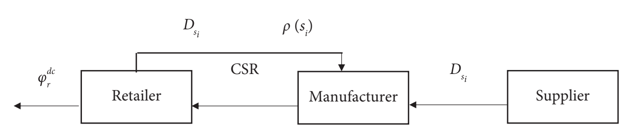

Figure 2: Configuration of the decentralized model.

Based on the decentralized model, the present problem is modeled as a manufacturer-Stackelberg game, where the manufacturer optimizes their decision by considering the most appropriate response of the retailer. The optimal solutions of the Stackelberg game can be determined through the backward induction method. In this method, the retailer's problem is first optimized, and the retailer's best response to any investment in CSR done by the manufacturer can be obtained. To maximize their profit performance in future demand scenarios, retailers must select their order quantity under scenario-based stochastic demand. Furthermore, the order quantity selected must ensure that the retailer does not incur losses under any demand scenario. Afterward, the manufacturer's problem is optimized by considering the best retailer's response to the manufacturer's profit performance, and the optimal investment in CSR is achieved. Ultimately, the retailer's optimal order quantity is obtained based on the manufacturer's investment in CSR.

Therefore, the retailer's profit function can be written as the function (1) as follows:

$$\begin{aligned}\varphi_r^{\mathrm{dc}}(Q,t_m,\tau_r,d_r,d_m) &= \rho(S_1)\times\varphi_{r,S_1}(Q,t_m,\tau_r,d_r,d_m)\\ &+\rho(S_2)\times\varphi_{r,S_2}(Q,t_m,\tau_r,d_r,d_m)\\ &+\rho(S_3)\times\varphi_{r,S_3}(Q,t_m,\tau_r,d_r,d_m),\end{aligned} \tag{1}$$

where

$$\begin{aligned}&\varphi_{r,S_1}(Q,t_m,\tau_r,d_r,d_m)\geq 0,\\ &\varphi_{r,S_2}(Q,t_m,\tau_r,d_r,d_m)\geq 0,\\ &\varphi_{r,S_3}(Q,t_m,\tau_r,d_r,d_m)\geq 0.\end{aligned} \tag{2}$$

Since the order quantity has been adjusted before the occurrence of each demand scenario, the profitability of the supplier and manufacturer is independent of the probable scenarios. Hence, the average profit of the supplier and the manufacturer in all the scenarios is illustrated in function (2) for the supplier and function (5) for the manufacturer, respectively.

$$\begin{aligned}\varphi_s(t_s,\tau_{m,w},n_1,n_2,d_s,d_r) &= \psi_s(t_s,\tau_{m.w_1},d_s,d_r)\\ &-\gamma_s(t_s,\tau_{m,w},n_1,n_2,d_r),\end{aligned} \tag{3}$$

where ψ_s denotes the supplier's annual income, and γ_s denotes the supplier's annual cost, illustrated in functions (3) and (4)

$$\begin{aligned}\psi_s(t_s,\tau_{m.w_1},d_s,d_r) &= (C_{m,w}-d_s)\rho(S_i)\alpha f(d_{\mathrm{rs}})\\ &+(C_{m,w}-d_s)\rho(S_i)\alpha f(d_{\mathrm{rs}})\left(e^{k_s(\tau_{m,w}-t_s)}-1\right).\end{aligned} \tag{4}$$

In the previous equation, for all the small amounts of the interest rate k_s, $e^{k_s\tau_{m,w}-t_s}-1$ has been approximated as $k_s(\tau_{m,w}-t_s)$.

$$\begin{aligned}\gamma_s(t_s,\tau_{m,w},n_1,n_2,d_r) &= \frac{A_s f(d_r)}{n_2 Q}+C_s\rho(S_i)\alpha f(d_{\mathrm{rs}})+\frac{(n_1-1)}{2}(h_s+s_s)\left(\frac{an_2A\rho(S_i)f(d_{\mathrm{rs}})}{Pn_1}\right)\\ &+h_s\tau_{m,w}\alpha\rho(S_i)f(d_r)+(C_{m,w}-d_s-C_s)\alpha\rho(S_i)f(d_{\mathrm{rs}})\left(e^{k_st_s}-1\right),\end{aligned} \tag{5}$$

where in this equation, when $k_s \ll 0$, we have:

$$e^{k_st_s}-1\approx k_st_s\varphi_m(Q,t_s,t_m,\tau_{m,w},\tau_r,n_1,n_2,d_m,d_s,d_r)=\psi_m(t_m,\tau_{m.w},\tau_r,d_m,d_s,d_r)-\gamma_m(Q,t_s,\tau_{m,w},n_1,n_2,d_s,d_m,d_r). \tag{6}$$

In equation (6), ψ_m denotes the manufacturer's annual income, and γ_m denotes the manufacturer's annual cost, which are illustrated in equations (7) and (9), respectively.

$$\begin{aligned}\psi_m(t_m,\tau_{m.w},\tau_r,d_m,d_s,d_r) &= (C_r-d_m)\rho(S_i)f(d_{\mathrm{rs}})+(C_r-d_m)\rho(S_i)f(d_{\mathrm{rs}})\left(e^{k_m(\tau_r-t_m)}-1\right)\\ &+(C_{m,w}-d_s)\alpha\rho(S_i)f(d_{\mathrm{rs}})\left(e^{k_m(\tau_{m,w})}-1\right).\end{aligned} \tag{7}$$

At $k_s \ll 0$, we have:

$$e^{k_m(\tau_r - t_m)} - 1 \approx k_m(\tau_r - t_m),$$
$$e^{k_m(\tau_{m,w})} - 1 \approx k_m \tau_{m,w}, \tag{8}$$

$$\begin{aligned}\gamma_m\left(Q, t_s, \tau_{m,w}, n_1, n_2, d_s, d_m, d_r\right) = {} & \frac{\left(n_1 A_{m,w} + A_m\right)\rho(S_i) f(d_{\mathrm{rs}})}{n_2 Q} + \left(C_{m,w} - d_s\right)\alpha\rho(S_i) f(d_{rs}) \\ & + \frac{\left(s_{m,w} a n_2 Q \rho(S_i) f(d_{\mathrm{rs}})\right)}{2Pn_1} + h_{m,w}\left(Q, t_s, \tau_{m,w}\right) + \left(C_{m,w} - d_s\right)\alpha\rho(S_i) f(d_{\mathrm{rs}})\left(e^{k_s(\tau_{m,w} - t_s)} - 1\right) \\ & + C_m f(d_{\mathrm{rs}}) + (h_m + S_m)\left[\frac{Q\rho(S_i)(2f(d_r) + (P - f(d_r))n_2 - P)}{2P}\right] \\ & + h_m \tau_r \rho(S_i) f(d_{rs}) + (C_r - d_m - C_m)\rho(S_i) f(d_{rs})\left(e^{k_s t_m} - 1\right).\end{aligned} \tag{9}$$

At $k_s, k_m \ll 0$, we have:

$$e^{k_s(\tau_{m,w} - t_s)} - 1 \approx k_s\left(\tau_{m,w} - t_s\right),$$
$$e^{k_m t_m} - 1 \approx k_m t_m. \tag{10}$$

In the scenario-based decentralized decision-making model, the manufacturer first optimizes their investment unit in CSR i.e. (η). Afterward, the retailer chooses the optimal order quantity. Since the optimal order quantity and the investment unit in CSR are not directly calculated through the equations (2), (5), and (1), a solution to obtain the optimal order quantity and investment in CSR in the scenario-based decentralized model is illustrated below. In other words, since equations (2), (5), and (1) are associated, the iterative search method suggested below can be used to find the desirable values of decision-making variables.

3.5.2. Decentralized Iterative Solution

Step 1: **(Initial Amounts)** We write $K_1 = 1$ and $\eta_{K_1} = 0$

Step 2: **(Optimal Order Values)** We write $K_2 = 1$, $Q_{K_2} = 0$, $Q^*_{K_1} = 0$

Step 2.1: We write $K_2 = K_2 + 1$, $Q_{K_2} = Q_{K_2} + \varepsilon_2$ (where ε_2 is a small value) and we calculate the retailer's interest rate through $\varphi_{r,S_1}(Q_K, t_m, \tau_r, d_r, d_m, \eta_{K_1})$ in (1).

Step 2.2: If $(Q_K, t_m, \tau_r, d_r, d_m, \eta_{K_1})$ satisfies all the restrictions of (1), we move to Step 2.3, otherwise we move to Step 2.4.

Step 2.3: If $\varphi_{r,S_1}(Q_K, t_m, \tau_r, d_r, d_m, \eta_{K_1}) > \varphi_{r,S_1}(Q^*_K, t_m, \tau_r, d_r, d_m, \eta_{K_1})$, then we write $Q^*_{K_1} = Q_{K_2}$, and move to Step 2.4.

Step 2.4: If $Q_{K_2} \leq \mu_3 + 3\sigma_3$, we move to Step 2.1., otherwise we go to step 3.

Step 3: **(Optimal Investment in CSR)** We calculate the manufacturer's profit $\varphi_m(Q^*_K, t_s, t_m, \tau_{m,w}, \tau_r, n_1, n_2, d_m, d_s, d_r, \eta_k)$ from equation (5), and move to step 3.1.

Step 3.1: If $\eta_{k_1} + \varepsilon_1 \leq \eta_{\max}$, we write $K_2 = K_2 + 1$, $\eta_{K_1} = \eta_{K_1} + \varepsilon_1$ (where ε_1 is a small value), and move to step 2 to calculate the optimal order quantity, otherwise, we move to step 4.

Step 4: **(Optimal Values)** We obtain the maximum value for $\varphi_m(Q^*_{K_1}, t_s, t_m, \tau_{m,w}, \tau_r, n_1, n_2, d_m, d_s, d_r, \eta_{k_1})$ by citing the parameters $Q^*_{K_1}, \eta_{k_1}$, and putting $Q^{*\mathrm{dc}} = Q_{k_1}$ and $\eta^{*\mathrm{dc}} = \eta_{k_1}$. Afterward, we obtain the optimal values for $Q^{*\mathrm{dc}}, \eta^{*\mathrm{dc}}$ and end the iterative method.

3.5.3. The Centralized Model. Figure 3 demonstrates the schematics of the centralized model.

Supply chain agents operate as an integrated vertical supply chain with a unique decision-maker under the centralized structure. The unique decision-maker of the supply chain, which is facing the scenario-based stochastic demand, must adjust Q and η before the start of the selling season to give the chain's maximum total profit expected. Similar to the methodology implemented in the decentralized model, the scenario-based centralized model can be described as follows:

$$\begin{aligned}\varphi^c_{SC}\left(Q, t_s, t_m, \tau_{m,w}, \tau_r, n_1, n_2, d_s, d_m, d_r, \eta\right) = {} & \rho(S_1) \times \varphi^c_s\left(t_s, \tau_{m,w}, n_1, n_2, d_s, d_r, \eta\right) + \rho(S_1) \\ & \times \varphi^c_m\left(Q, t_s, t_m, \tau_{m,w}, \tau_r, n_1, n_2, d_m, d_s, d_r, \eta\right) + \rho(S_1) \times \varphi^c_r(Q, t_m, \tau_r, d_r, d_m, \eta),\end{aligned} \tag{11}$$

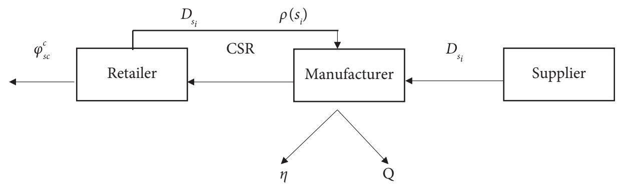

Figure 3: Configuration of the centralized model.

where:

$$\begin{aligned}&\varphi_{\mathrm{SC},S_1}(Q,t_m,\tau_r,d_r,d_m,\eta)\geq\varphi^{*\mathrm{dc}}_{\mathrm{SC},S_1},\\&\varphi_{\mathrm{SC},S_2}(Q,t_m,\tau_r,d_r,d_m,\eta)\geq\varphi^{*\mathrm{dc}}_{\mathrm{SC},S_2},\\&\varphi_{\mathrm{SC},S_3}(Q,t_m,\tau_r,d_r,d_m,\eta)\geq\varphi^{*\mathrm{dc}}_{\mathrm{SC},S_3}.\end{aligned}\tag{12}$$

In the scenario-based stochastic model in the centralized system, the mathematical model is able to maximize the expected total profit of the chain by considering various demand scenarios.

The restrictions of the model above guarantee that the total profit of the supply chain in the centralized adjustments will exceed that of decentralized adjustments in all the scenarios. If the expected profit for the whole supply chain under the centralized mode in a scenario (S_i) is lower than that of the decentralized model, the obtained optimal results will not be desirable. In other words, equations $[\varphi^c_{\mathrm{SC},S_i}(Q,\eta)\geq\varphi^{*\mathrm{dc}}_{\mathrm{SC},S_i}\,\forall S_i]$ must be satisfied. In the equations above,

$$\varphi^{*\mathrm{dc}}_{\mathrm{SC},S_i}=\varphi^{*\mathrm{dc}}_{r,S_1}\left(Q^{*\mathrm{dc}},t_m,\tau_r,d_r,d_m,\eta^{*\mathrm{dc}}\right)+\varphi^{*\mathrm{dc}}_{P,S_1}\left(Q^{*\mathrm{dc}},t_m,\tau_r,d_r,d_m,\eta^{*\mathrm{dc}}\right)+\varphi^{*\mathrm{dc}}_{D,S_1}\left(Q^{*\mathrm{dc}},t_m,\tau_r,d_r,d_m\right).\tag{13}$$

The optimal values for the two decision-making variables (for instance, the order quantity and the investment unit (CSR)) cannot be directly calculated through equation (11). Below, an iterative approach has been devised to calculate the optimal values for (Q^{*c},η^{*c}) in the adjustment of the centralized scenario:

3.5.4. Centralized Iterative Solution

Step 1: **(Initial Values)** We write $K_1=1$ and $\eta_{K_1}=0$

Step 2: If $\eta_{K_1}\leq\eta^{\max}$, then $K_2=1$, $Q_{K_2}=0$, and we move to step 3. Otherwise, we move to step 5.

Step 3: If $Q_{K_2}\leq\mu_3+3\sigma_3$, then we move to step 3.1. Otherwise, we move to step 4.

Step 3.1: we obtain φ_{SC} $(Q_{K_2},t_s,t_m,\tau_{m,w},\tau_r,n_1,n_2,d_s,d_m,d_r,\eta_{K_2})$ from equation (11).

Step 3.2: If (Q_{K_2},η_{K_1}) cannot satisfy at least one of the centralized model's restriction, then $\varphi_{\mathrm{SC}}(Q_{K_2},t_s,t_m,\tau_{m,w},\tau_r,n_1,n_2,d_s,d_m,d_r,\eta_{K_2})=-\infty$. We move to Step 3.3.

Step 3.3: We write $K_2=K_2+1$, $Q_{K_2}=Q_{K_2}+\varepsilon_2$ (where ε_2 is a small value), and move to Step 3.

Step 4: We write $K_1=K_1+1$, $\eta_{K_2}=\eta_{K_2}+\varepsilon_2$ (where ε_2 is a small value), and move to step 2.

Step 5: We determine the maximum value of $\varphi^c_{\mathrm{SC}}(Q_{K_2},t_s,t_m,\tau_{m,w},\tau_r,n_1,n_2,d_s,d_m,d_r,\eta_{K_1})$ by citing the parameters Q_{K_2},η_{K_1}, and putting $Q^{*c}=Q_{k_2}$ and $\eta^{*c}=\eta_{k_1}$. Afterward, we obtain the optimal values for Q^{*c},η^{*c}, and end the iterative method.

Although the centralized model increases the total profit of the supply chain compared to the decentralized one, the centralized solution does not necessarily increase the profit of all the agents of each echelon in the supply chain. Since the independent agents of the supply chain intend to maximize their profit, the chain agents who incur losses will not be interested in choosing centralized decisions. We will propose a new wholesale price-based coordinated contract to resolve the channel conflict and achieve a coordinated system under scenario-based stochastic demand in the next section.

3.5.5. The Coordinated Model. Figure 4 demonstrates the schematics of the coordinated model.

The optimal decision-making variables derived from a centralized model may reduce the retailer or manufacturer's profit compared to one derived from a decentralized model. The centralized method, however, is far more beneficial than its decentralized counterpart. In supply chain management, coordinated contracts have been used to encourage the decentralized SC agents for decentralized decision-making. Various coordination plans have been provided for supply chain networks, such as wholesale prices, delayed payments, repayments, and quantitative degrees of flexibility. Previous contracts for coordinated models are implemented when future market demands can be forecast using a single scenario. However, in most cases, an uncertain demand can be determined through a set of

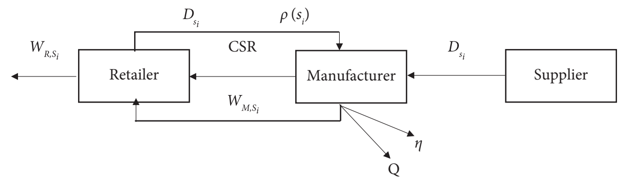

Figure 4: Configuration of the coordinated model.

separate scenarios with relevant probabilities. In this section, we propose a new coordinated contract known as the adjustable bilevel wholesale price contract to achieve channel coordination among the next supply chain's three agents i.e. the manufacturer, supplier, and retailer, under a scenario-based stochastic demand. The proposed contract has been designed in a way that will benefit all the chain's agents regardless of the demand scenario in the future. This can be acknowledged as the unique quality of the proposed contract. Literature on past coordinated contracts can only apply if the future market demand is forecast through a scenario. In the problem under question, the profit of the agents directly depends on the demand scenario, which will occur in the future. Consequently, a new coordinated contract is needed to provide a win-win situation regardless of the scenario. To solve the problem, in the proposed contract of this study, the contract parameters have been designed by considering probable demand scenarios in the future.

In the proposed contract, when the real demand in scenario S_i is determined, the manufacturer presents the wholesale price W^b_{M,S_i} to the retailer. As a result, the retailer presents a certain wholesale price W^b_{R,S_i} to the customer. Hence, the parameters of the proposed contract of each scenario can be states as W^b_{M,S_i}, W^b_{R,S_i}. These two parameters must be determined in a way that the profit of all the agents will rise after participation in the coordinated model. It is noteworthy that the contract parameters W^b_{M,S_i}, W^b_{R,S_i} in each scenario S_i are independent from another scenario.

The proposed coordinated contract must be acceptable to all three agents i.e. the supplier, manufacturer, and retailer. Otherwise, they will refuse to participate in the coordinated model. Hence, at minimum, the necessities of all three agents must be taken into account for participation. The term "bilevel" in the name of the proposed contract refers to the fact that not only will the proposed contract determine the wholesale price of the manufacturer, but it will also determine the wholesale price of the retailer at the same time so that under various demand scenarios all three parties profit from the coordinated model. These two contract parameters must be simultaneously adjusted so that all three agents participate in the coordinated model. As a result, we consider the steps below as the steps to the coordinated model in the supply chain:

(i) Step 1: Before the start of selling season (i.e. before demand fulfillment), the retailer introduces a centralized-based order.

(ii) Step 2: While investing in the CSR of each product according to the occurrence probability of each demand scenario, the manufacturer produces the retailer's order quantity based on the manufacturing restrictions and investment in centralized CSR.

(iii) Step 3: Depending on the needs of the manufacturer, the retailer provides the manufacturer with the raw materials needed to produce the product.

(iv) Step 4: After the onset of the selling season (i.e. after determining the demand quantity), depending on the agreement between different levels of the chain and contract parameters W^b_{M,S_i}, W^b_{R,S_i}, a certain time is considered for the delivery and settlement of the services between two sides, based on which various scenarios are considered for delays and late payments.

(v) Step 5: The customer pays W^b_{R,S_i} to the retailer and the retailer pays the W^b_{M,S_i} to the manufacturer.

According to the coordinated model, before achieving the demand scenario, the members of the supply chain (i.e. the supplier, manufacturer, and retailer) work together to discuss the order quantity, CSR investment, and contract parameters. As a result, the order quantity of the manufacturer must be commensurate with the optimal order quantity of the retailer under the centralized model, meaning equation (11) is applicable in this case. As can be seen, under the coordinated model, the retailer and manufacturer choose centralized solutions to order quantity and CSR investment, respectively. Moreover, the wholesale price is reevaluated by the manufacturer and supplier. The contract parameters must be adjusted in a way that the profit of all the SC agents increases by choosing the centralized solution under any scenario compared to the decentralized model. If not, they will refuse to cooperate in the structure of coordinated decision-making. Hence, the contract parameters $(W^b_{M,S_i}, W^b_{R,S_i})$ must satisfy the restrictions below:

$$\begin{aligned}
&\varphi^{\text{co}}_{R,S_i} \geq \varphi^{*\text{dc}}_{R,S_i},\\
&\varphi^{\text{co}}_{S,S_i} \geq \varphi^{*\text{dc}}_{S,S_i},\\
&\varphi^{\text{co}}_{M,S_i} \geq \varphi^{*\text{dc}}_{M,S._i}.
\end{aligned} \tag{14}$$

By applying mathematics, we will achieve the equation below for the model:

$$P\left[\left(\mu_{S_i}+\alpha_{S_i}y\left(\eta^{*c}\right)-Q^{*c}\right)F_z\left(\frac{Q^{*c}-\mu_{S_i}-\alpha_{S_i}y\left(\eta^{*c}\right)}{\sigma_{S_i}}\right)\right]$$

$$+\left(P-W^{b}_{R,S_i}\right)Q^{*c}\geq\varphi^{*\mathrm{dc}}_{R,S_i}, \tag{15}$$

$$\left(W^{b}_{R,S_i}-W^{b}_{M,S_i}\right)Q^{*c}\geq\varphi^{*\mathrm{dc}}_{S,S_i}, \tag{16}$$

$$\left(W^{b}_{M,S_i}--C-\eta^{*c}\right)Q^{*c}\geq\varphi^{*\mathrm{dc}}_{M,S_i}. \tag{17}$$

Both contract parameters ($W^{b}_{M,S_i}, W^{b}_{R,S_i}$), which satisfy equation (15), will also increase the profit of all the members (agents) of the chain under the coordinated model compared to the decentralized model. Therefore, the new wholesale prices (contract parameters) under the proposed contract will be satisfactory for all chain agents and will enable them to benefit from the coordinated model simultaneously.

In the proposed game model, the objective functions equal the multiplication of all the agents' profit functions after participation in the coordinated model. The proposed game model can be adjusted as below:

$$\operatorname{Max}\left\{\left[\left(\mu_{S_i}+\alpha_{S_i}y\left(\eta^{*c}\right)-Q^{*c}\right)F_z\left(\frac{Q^{*c}-\mu_{S_i}-\alpha_{S_i}y\left(\eta^{*c}\right)}{\sigma_{S_i}}\right)-\sigma_{S_i}f_z\left(\frac{Q^{*c}-\mu_{S_i}-\alpha_{S_i}y\left(\eta^{*c}\right)}{\sigma_{S_i}}\right)\right]+\left(P-W^{b}_{R,S_i}\right)Q^{*c}-\varphi^{*\mathrm{dc}}_{R,S_i}\right\}$$
$$\times\left\{\left(W^{b}_{R,S_i}-W^{b}_{M,S_i}\right)Q^{*c}-\varphi^{*\mathrm{dc}}_{S,S_i}\right\}\times\left\{\left(W^{b}_{M,S_i}--C-\eta^{*c}\right)Q^{*c}-\varphi^{*\mathrm{dc}}_{M,S_i}\right\}. \tag{18}$$

For this model, the restrictions are as follows:

$$W^{b}_{D,S_i}\leq\frac{P\left[\left(\mu_{S_i}+\alpha_{S_i}y\left(\eta^{*c}\right)-Q^{*c}\right)F_z\left(Q^{*c}-\mu_{S_i}-\alpha_{S_i}y\left(\eta^{*c}\right)/\sigma_{S_i}\right)-\sigma_{S_i}f_z\left(Q^{*c}-\mu_{S_i}-\alpha_{S_i}y\left(\eta^{*c}\right)/\sigma_{S_i}\right)\right]+PQ^{*c}-\varphi^{*\mathrm{dc}}_{R,S_i}}{Q^{*c}},$$
$$\left(W^{b}_{R,S_i}-W^{b}_{M,S_i}\right)\geq\frac{\varphi^{*\mathrm{dc}}_{S,S_i}}{Q^{*c}}, \tag{19}$$
$$\left(W^{b}_{M,S_i}\right)\geq\frac{\varphi^{*\mathrm{dc}}_{M,S_i}}{Q^{*c}}+\left(C+\eta^{*c}\right).$$

By solving the model above, the precise contract parameter acceptable to all three agents is determined. By achieving channel coordination in all the probable demand scenarios, the proposed Nash equilibrium model will offer a win-win situation for all three agents.

4. Results

4.1. Model Verification. To verify the proposed model of this study, the model presented by Aljazzar et al. [6] is used to analyze the total chain profit under various circumstances, and compare them with the results of this study. For this purpose, the initial values used by Aljazzar et al. [6] are presented in Table 1.

Now, based on the modeling proposed in Aljazzar et al. and by determining the annual income ψ, annual cost γ, and annual profit ϕ of each agent of the chain i.e. the supplier, manufacturer, and retailer, we will determined the final profit of the chain, which is equal to the total profit of each agent. Subsequently, we will compare the results in different modes with the paper in question. Table 2 demonstrates this matter.

Based on Table 2, the difference between the chain's total profit obtained here and obtained by Aljazzar et al. is less than 5% in different cases, indicating the high precision of the modeling and confirming the verification. Meanwhile, the paired samples t-test is used for comparison of two population means (Table 3).

$$T=\frac{\overline{d}-D_0}{S_d/\sqrt{n}}, \tag{20}$$

where there are n pairs, $\overline{d}$ is the mean and S_d is the standard deviation of their differences. the test statistic has Student's t-distribution with $n-1$ degrees of freedom.

$$t=\frac{\sum di}{\sqrt{n\left(\sum d_i^2\right)-\left(\sum d_i\right)2/n-1}}. \tag{21}$$

If $|t|>t_{\alpha/2}$, $n-1$ the null hypothesis is rejected.

Table 1: The initial values required for verification (according to the quantities presented in Aljazzar et al. [6]).

Parameter	Description	Quantity	Unit
A_m	Setup/ordering cost of manufactured products	125	\$/order
A_w	Manufacturer's setup/ordering cost	355	\$/order
A_r	Retailer's setup/ordering cost	242	\$/order
A_s	Supplier's setup/ordering cost	264	\$/order
C_s	Supplier's production/purchase cost	20	\$/unit
C_w	Manufacturer's production/purchase cost	28	\$/unit
C_m	Production/purchase cost of manufactured products	50	\$/unit
C_r	Retailer's production/purchase cost	70	\$/unit
C_c	Customer's production/purchase cost	98	\$/unit
h_s	Supplier's financial holding cost	3	\$/unit/year
h_w	Manufacturer's financial holding cost	4.2	\$/unit/year
h_r	Supplier's financial holding cost	10.5	\$/unit/year
h_m	Financial holding cost of manufactured products	7.5	\$/unit/year
s_s	Supplier's holding (storing) cost	2	\$/unit/year
s_w	Manufacturer's holding (storing) cost	2.8	\$/unit/year
s_r	Retailer's holding (storing) cost	7	\$/unit/year
s_m	Physical holding (storing) cost of manufactured products	5	\$/unit/year
α	Quantity of initial raw materials	1	—
k_s	Supplier's return on capital	10	%
k_m	Manufacturer's return on capital	8	%
k_r	Retailer's return on capital	5	%
P	Manufacturer's annual production rate	300	unit/year
D_0	Retailers annual demand rate	1000	unit/year
D_1	Elasticity of demand to a discount	70	—

Table 2: Comparing the final profit obtained with the final profit from Aljazzar et al. [6].

(d_s, d_m, d_r)	Total profit of the chain obtained	Total profit of the chain obtained in the paper	Deficit (%)
(0, 0, 0)	21295.49	21039.88	1.215
(0, 0, 1)	23885.16	22769.67	4.899
(0, 1, 0)	21968.31	21039.88	4.413
(0, 1, 1)	23004.76	22769.67	1.032
(1, 0, 0)	22142.13	21409.54	3.422
(1, 0, 1)	23847.39	22769.67	4.733
(1, 1, 0)	22007.63	21039.88	4.600
(1, 1, 1)	23134.75	22769.67	1.603

Table 3: Comparison of two population means - paired samples t-test.

(d_s, d_m, d_r)	Total profit of the chain obtained	Total profit of the chain obtained in the paper	Difference (D_i)	D_i^2
(0, 0, 0)	21295.49	21039.88	255.61	65336.47
(0, 0, 1)	23885.16	22769.67	1115.49	1244318
(0, 1, 0)	21968.31	21039.88	928.43	861982.3
(0, 1, 1)	23004.76	22769.67	235.09	55267.31
(1, 0, 0)	22142.13	21409.54	732.59	536688.1
(1, 0, 1)	23847.39	22769.67	1077.72	1161480
(1, 1, 0)	22007.63	21039.88	967.75	936540.1
(1, 1, 1)	23134.75	22769.67	365.08	133283.4

$$t = \frac{5677.76}{\sqrt{8(4994896) - 32236959/7}} = 0.82,$$
$$t_{0.025,7} = 2.365. \tag{22}$$

Therefore, since $0.82 < 2.365$, the null hypothesis is not rejected and the model is valid. By verifying the model, we will later analyze the results obtained from the case study for the three coordinated, centralized, and decentralized models.

4.2. Case Study (Shahid Tondgooyan Petrochemical Co.). Founded on 1998-04-26 at Imam Khomeini port, in site IV of the petrochemical Special Economic Zone, Shahid Tondgooyan Petrochemical Co. is located in the northeast of the Persian Gulf Coast in Khuzestan province. It is a 34-hectare area for the production of PET (Polyethylene terephthalate) and PTA (Pure terephthalic acid). In order to meet part of its obligations under the second five-year plan of the Islamic Republic of Iran and in light of the expanding market for the consumption of these products both domestically and abroad, the National Petrochemical Company implements this plan according to these objectives:

(i) Providing raw materials to downstream units of the country's textile and packaging industries, and saving currency.

(ii) Reaching global markets and creating Foreign Exchange Earnings.

(iii) Moving toward self-dependence and transferring advanced technology.

(iv) Sharing knowledge and training professional human resources.

(v) Creating jobs in the downstream industries with the economic and social development of the country.

One of the most important raw materials for the production of PET, whose production and consumption process has been of significant importance all over the world, is PTA. PET is also one of the most important polyesters used in the production of synthetic fiber and cotton in the textile industry, all types of bottles, cans, and packages used as food, medicine, and health packaging, as well as in the production of plastic films, which is increasing in production and consumption. Moreover, this company produces two significant products: polyester yarn (POY) and polyester fiber (STAPLE).

Table 4 demonstrates the data collected from the company to determine the results of the paper's main model.

Moreover, Tables 5–8 illustrate the costs of the setup of related units:

Table 9 demonstrates the annual production rate for 4 types of products i.e. PTA, BG, TG, and POU.

Table 10 illustrates the volume of raw materials needed to manufacture products.

Following the presentation of values and the calculation of other parameters in Sensor Cplex in GAMS, Table 11 below shows the results derived from solving the model based on the time spent on solving, as well as the objective function value for the supplier.

4.3. The Results of the Models. After verifying the model, we use game theory-based models to determine the optimal results of the three centralized, decentralized, and coordinated models. Figures 5–7 demonstrate the convergence diagram of the profit rate obtained from each model, respectively:

As can be seen in Figures 4–7, the uncoordinated model quickly achieves convergence, and it dwindles steadily. Accordingly, the total profit of the chain for this model equals 560935.05 in financial units. The centralized model also reaches convergence after 152 repetitions and remains consistent. The coordinated model reaches convergence after 60 times of repetition, the optimal value for the highest profit rate of all the members of the chain. Additionally, its downward trend stops. Table 12 illustrates the results of the values obtained from each model and the time spent on solving the model using them. The table can provide adequate comparison of the performance of each model to determine the chain's total profit.

5. Sensitivity Analysis

Sensitivity analysis determines how much the dependent variable will change if the value of an independent variable is changed in a specific and defined situation and assumes that other variables are constant. The values presented in Table 4 are changed here to determine the effect each increase or decrease will have on the final chain's profit chain's profit. The sensitivity analysis is investigated here using all three models.

Based on Table 13, with the increase in the number of suppliers, the chain's profit has increased in all three models. In general, by doubling the number of suppliers, about 139.25% has been added to the overall profit of the chain (based on the analysis of the coordinated model, which performed best among other models). This is because, for example, by increasing the number of suppliers, the number of raw materials required for production increases, and the number of exchanges between the supplier and the manufacturer decreases, contributing to reducing the costs of the entire chain and increasing profits.

From Table 14, it can be observed that the average profit of the entire chain has increased with the increase in the number of raw materials. In other words, by doubling the number of raw materials, the profit of the entire chain has doubled by about 231.31. Because with the increase of the raw amount, in addition to the transfer costs, construction costs, and side costs have decreased, all of which indicate a reduction in the overall costs of the chain for all actors and an increase in the chain's profit by doubling the number of w.

Based on Table 15, with the increase in the number of manufactured products, the chain's profit also increases. In other words, by doubling the number of products (m), the chain's profit has increased by 88.40%. In this case, the chain costs and the actors' profits have increased. In addition to the reasons mentioned concerning the increase in the value of w, this is due to the fact that the costs related to the decrease in the movement and the number of customers, and consequently, the agreement between the actors have increased, which results to more profit to the chain.

From Table 16 for the sensitivity analysis of the number of retailers, it can be observed that with the increase in the number of retailers, the total cost of the network has decreased, and its profit has increased. In other words, by doubling the number of retailers, about 38.03% has been added to the overall profit of the chain.

Table 4: The initial values collected from The Shahid Tondgooyan Petrochemical Co.

Parameters	Description	Value
i	It is determined, corresponding to the supply chain members	3
s	Indicating the supplier	4
w	Indicating the manufacturer's raw materials	4
m	Indicating the manufacturer's manufactured products	4
r	Indicates the retailer	10
n_1	Number of transportations from the supplier to the manufacturer per period of the manufacturer's raw materials	30
n_2	Number of transportations from the manufacturer to the retailer per period of the retailer	156
α	The raw material needed to manufacture the final product	4
t_i	Permissible delay in payment from factor i	3
τ_i	Settlement time for each factor i	1
d_i	Currency discount from factor i to their customer	2-3%
D_{s_i}	Annual demand rate under scenario S_i, $D_s < P$	350000 ton
T	Length of time period	12
ψ_i	Annual profit for each factor i	3–5%

Table 5: Setup/order costs A_i.

Name of the center	Setup/order costs (currency)
Supplier	3500
Manufacturer	8600
Distributor	2890

Table 6: Production/purchasing costs C_i.

Name of the center	Production/purchasing costs (currency)
Supplier	350
Manufacturer	745
Distributor	935

Table 7: Financial holding costs of each unit h_i.

Name of the center	Financial holding costs (currency)
Supplier	180
Manufacturer	210
Distributor	250

Table 8: Holding (storing) costs of each unit S_i.

Name of the center	Holding (storing) costs (currency)
Supplier	100
Manufacturer	113
Distributor	152

Table 9: Annual production rate by the manufacturer for each product (ton) (P).

Product name	Annual production rate of each product (ton)
Pta	315000
Bg	280000
Tg	340000
Poy	47000

Table 10: The volume of raw materials required for manufacturing.

Material name	Required volume (annual) (ton)
Acid	37000
Meg	242000
Deg	720
Px	420000

Table 11: Results of the supplier's income, costs, and annual profit.

Annual income ψ_s (currency)	$1.7941e+18$
Annual cost γ_s (currency)	$5.3899e+15$
Annual profit φ_s (currency)	$1.7887e+18$

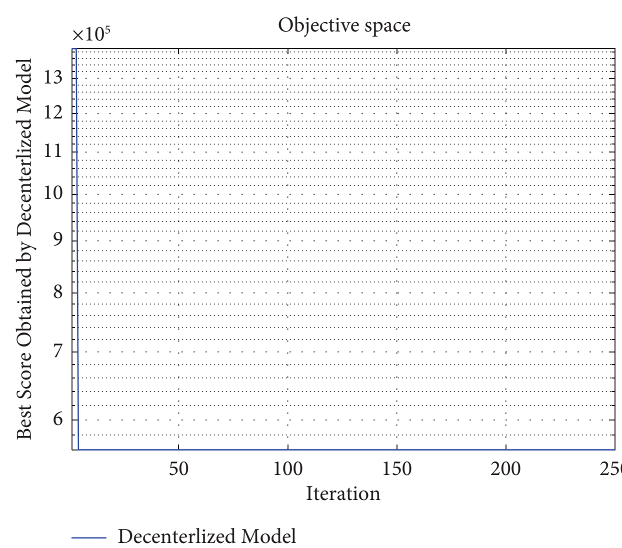

Figure 5: The convergence diagram of the decentralized model.

Based on Table 17, the increase in the allowed delay has caused an increase in the overall chain's profit because this delay increases the amount of production, and as a result, the income from the sale of more products increases. This increase is smaller than the other factors mentioned above. Specifically,

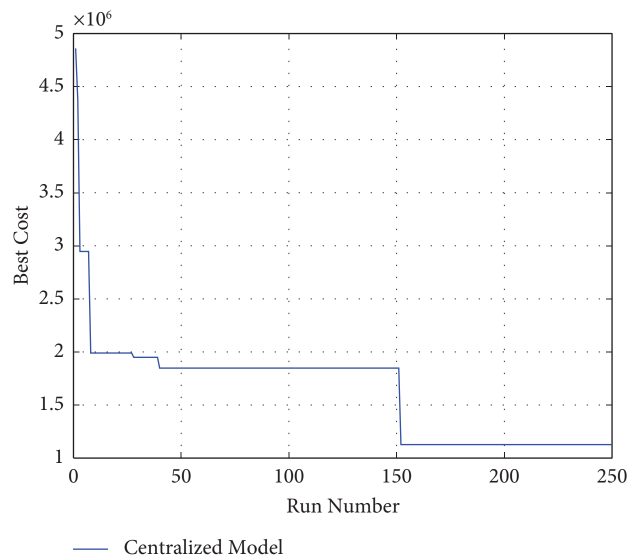

Figure 6: The convergence diagram of the centralized model.

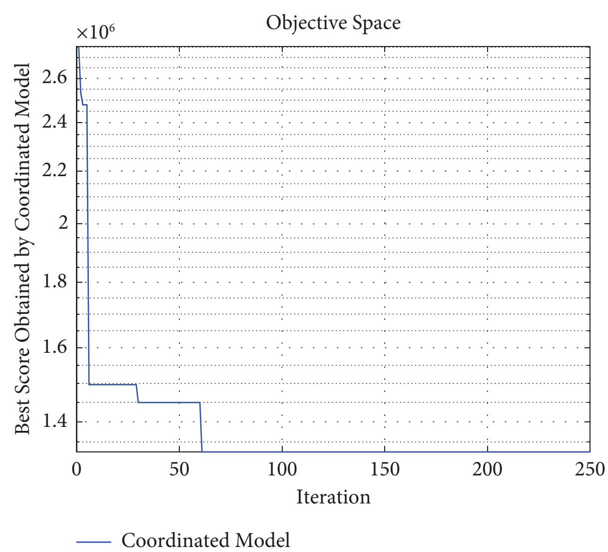

Figure 7: The convergence diagram of the coordinated model.

Table 12: Evaluation of the profit rate obtained from the whole chain and the time spent on each model.

Model	Profit obtained from the whole chain (currency)	Time spent on solving the model
Decentralized	560935.05	134.853878
Centralized	1127169.225	34.094986
Coordinated	1325962.21	4.380576

Table 13: Sensitivity analysis on the number of suppliers with other parameters remain the same.

Row	Number of suppliers (s)	Total profit of the chain (in monetary units)		
		Coordinated model	Centralized model	Decentralized model
1	6	1828036.11	1455246.112	854421.12
2	8	3172376.195	2504619.771	968626.34
3	10	3725926.165	2891553.27	1120113.28
4	12	4889133.145	3105179.553	1332077.45
Main model	4	1325962.21	1127169.225	560935.05

Table 14: Sensitivity analysis on the number of raw materials with other parameters remain the same.

Row	Number of raw materials (w)	Profit of the entire chain (in monetary units)		
		Coordinated model	Centralized model	Decentralized model
1	6	2036312.17	1698579.63	859612.55
2	8	4154491.05	3421515.79	1004625.12
3	10	4384239.155	3686213.24	1386475.98
4	12	7301725.12	5993416.774	1694162.5
Main model	4	1325962.21	1127169.225	560935.05

Table 15: Sensitivity analysis on the number of products with other parameters remain the same.

Row	Number of products (m)	Profit of the entire chain (in monetary units)		
		Coordinated model	Centralized model	Decentralized model
1	6	2047536.13	1363597.14	774512.41
2	8	2498194.295	1798544.27	994016.55
3	10	2626378.31	1939641.08	1252549.76
4	12	2812802.38	2107544.39	1504773.21
Main model	4	1325962.21	1127169.225	560935.05

Table 16: Sensitivity analysis on the number of retailers with other parameters remain the same.

Row	Number of retailers (r)	Profit of the entire chain (in monetary units)		
		Coordinated model	Centralized model	Decentralized model
1	15	1727628.175	1352416.89	652124.86
2	20	1830273.17	1398587.16	721628.96
3	30	1993012.5	1436419.25	807546.35
4	40	2046201.22	1495748.31	896321.47
Main model	10	1325962.21	1127169.225	560935.05

Table 17: Sensitivity analysis on the allowed delay in payment with other parameters remain the same.

Row	Permissible delay (t_i)	Profit of the entire chain (in monetary units)		
		Coordinated model	Centralized model	Decentralized model
1	5	1481770.105	1284623.14	567485.12
2	6	1582223.425	1396745.52	604248.77
3	12	1693421.714	1483229.75	625353.46
4	15	1727754.01	1557993.24	699741.03
Main model	3	1325962.21	1127169.225	560935.05

Table 18: Sensitivity analysis on settlement time with other parameters remain the same.

Row	Settlement time (τ_i)	Profit of the entire chain (in monetary units)		
		Coordinated model	Centralized model	Decentralized model
1	2	1384633.19	1135956.14	606023.12
2	4	1389535.79	1136221.17	634985.25
3	5	1402122.34	1139657.46	657941.16
4	6	1406321.77	1153028.74	689972.14
Main model	1	1325962.21	1127169.225	560935.05

by doubling the allowed delay time from 3 to 6 months, the profit of the entire chain has increased by 19.32%.

Table 18 investigates the sensitivity analysis of the settlement time. It can be observed that with the increase in the settlement time, the total profit of the network has increased by 4.42%.

Based on the analysis done in Tables 19–22, by doubling the setup/ordering cost, production/purchasing cost, financial holding cost per unit, and holding (storing) cost per unit, the total chain's profit increase by 4.07%, 0.89%, 0.048%, and 0.27%, respectively.

Table 19: Sensitivity analysis on setup/ordering cost with other parameters remain the same.

Row	Setup/ordering cost	Profit of the entire chain (in monetary units)		
		Coordinated model	Centralized model	Decentralized model
1	2 to double	1355942.21	1130253.57	569241.17
2	3 to double	1379982.21	1197524.16	571462.55
3	5 to double	1403288.007	1241552.49	575463.29
4	7 to double	1597243.556	1285749.33	579663.74
Main model	—	1325962.21	1127169.225	560935.05

Table 20: Sensitivity analysis on production/purchasing cost per unit with other parameters remain the same.

Row	Production/purchasing cost per unit	Profit of the entire chain (in monetary units)		
		Coordinated model	Centralized model	Decentralized model
1	2 to double	1331782.21	1135526.79	565316.75
2	3 to double	1337767.21	1138546.04	569448.75
3	5 to double	1340052.16	1139934.25	572163.745
4	7 to double	1358580.46	1142413.69	575363.87
Main model	—	1325962.21	1127169.225	560935.05

Table 21: Sensitivity analysis on the financial holding cost per unit with other parameters remain the same.

Row	Financial holding cost per unit	Profit of the entire chain (in monetary units)		
		Coordinated model	Centralized model	Decentralized model
1	2 to double	1326602.21	1127548.32	561253.26
2	3 to double	1327242.21	1127996.34	562524.17
3	5 to double	1328562.21	1128104.16	563596.38
4	7 to double	1329586.21	1128693.85	565968.74
Main model	—	1325962.21	1127169.225	560935.05

Table 22: Sensitivity analysis on the holding (storing) cost per unit with other parameters remain the same.

Row	Holding (storing) cost per unit	Profit of the entire chain (in monetary units)		
		Coordinated model	Centralized model	Decentralized model
1	2 to double	1329645.32	1129968.32	561857.74
2	3 to double	1331414.79	1130421.74	561883.31
3	5 to double	1341409.1	1130625.68	561902.47
4	7 to double	1345530.43	1131253.79	561905.79
Main model	—	1325962.21	1127169.225	560935.05

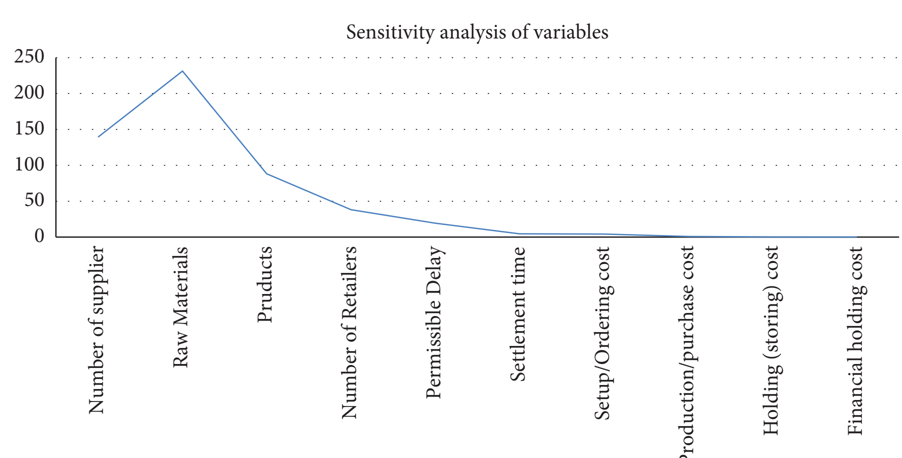

Figure 8: The total profit of the chain based on the changes in different variables.

As shown in Figure 8, the number of raw materials has the greatest impact on the overall costs of the supply chain and the profit of actors.

6. Conclusion

Nowadays, the gas and oil industry has caused fundamental changes in regional growth and development. Moreover, today's need for oil products and their various functions in different fields, and existing potentials have led to significant investments in this area. Regarding this, the first step is to provide raw materials for the production of oil derivatives by conducting the necessary operations. If these materials can be provided in a shorter time and with adequate quality, the supply chain will be more effective and the organization's objectives can be met. The main objective of this paper was to increase the effectiveness of the supply chain plan and create coordination between the components. A three-level mathematical model was used to decrease the chain's total costs and increase its total profit. After verifying the model through GAMS to examine the results of each chain echelon's cost, income, and profit, three centralized, decentralized, and coordinated models based on game theory were used to solve the model. Comparing the results, it was found that the coordinated model has better performance in reducing the chain's costs and boosting profits than the other models. In particular, it can be stated that the coordinated model outperforms the decentralized model by 136.38% and the centralized model by 17.63%, while it increases the chain profit too. Meanwhile, it was observed that the decentralized model spent more time solving the model, than the other two. The coordinated model has the advantage of assessing the computation solution space for the agents with greater precision, and it can present a higher profit for each of the echelons in the chain. These models can be extended in several possible directions. First, consider a specific system of transportation or distribution to move the products from the centers to the final destination (customer). Second, create a backup for bottle distribution centers that, if the manufactured product is unavailable, receive the requested product with compensation from other distribution centers. Third add other supply chain echelons to the model.

Data Availability

The data used to support the findings of this study are available from the corresponding author upon request.

Conflicts of Interest

The authors certify that there is no conflict of interests (considering both financial and nonfinancial gains) with any organization regarding the material discussed in the paper. This study did not receive any funding in any form.

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