Morphological Image Processing

Chapter 9

Instructor: **Hossein Pourghassem**

Morphological Image Processing

- Morphology deals with form and structure
- Mathematical morphology is a tool for extracting image components useful in:
  - representation and description of region shape (e.g. boundaries)
  - pre- or post-processing (filtering, thinning, etc.)
- Based on set theory

Morphology

- ❑ Sets represent objects in images
- ❑ Sets in binary images → (x,y)
- ❑ Sets in gray scale images → (x,y,g)
- ❑ Some morphological operations:

Dilation & Erosion
Opening & Closing
Hit-or-Miss Transform
Basic Algorithms

Basic Concepts of Set Theory

- ❑ Assume A is a set in Z^2
- ➢ If a=(a1,a2) an element of A, then we write $a \in A$
- ➢ If not, then $a \notin A$
- ➢ $\varnothing$: null (empty) set
- ➢ Typical set specification: $C=\{w | w=-d, \text{ for } d \notin D\}$
- ➢ If A is the subset of B then we write $A \subseteq B$
- ➢ Union of A and B: $C=A \cup B$
- ➢ Intersection of A and B: $D=A \cap B$

Basic Concepts of Set Theory

- Disjoint sets: $A \cap B = \varnothing$
- Complement of A:

$$A^c = \{w \mid w \notin A\}$$

- Difference of A and B:

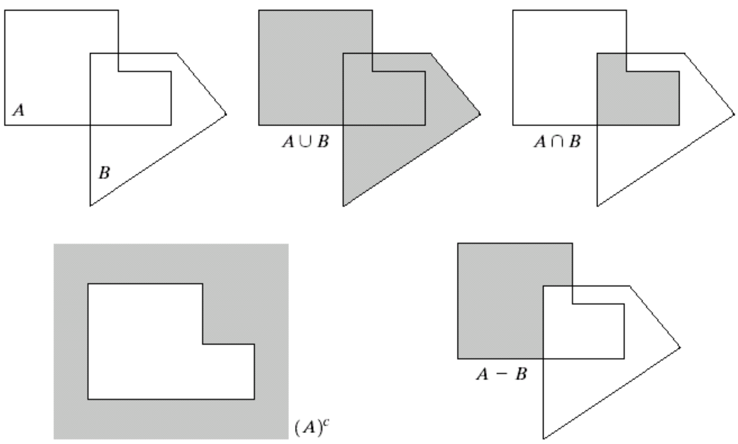
  $A\text{-}B=\{w|w \in A,\ w \notin B\}=\ A \cap B^c$
- Reflection of B:

$$\hat{B} = \{w \mid w = -b, b \in B\}$$

- Translation of A by $z=(z_1,z_2)$:

$$(A)_z = \{c \mid c = a + z, a \in A\}$$

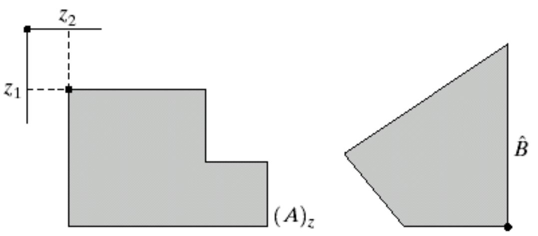
Morphological Image Processing



a b c
d e

FIGURE 9.1
(a) Two sets *A* and *B*. (b) The union of *A* and *B*. (c) The intersection of *A* and *B*. (d) The complement of *A*. (e) The difference between *A* and *B*.

Morphological Image Processing



a b

FIGURE 9.2 (a) Translation of A by z. (b) Reflection of B. The sets A and B are from Fig. 9.1.

Logical Operation

TABLE 9.1 The three basic logical operations.

p	q	p AND q (also $p \cdot q$)	p OR q (also $p + q$)	NOT (p) (also $\bar{p}$)
0	0	0	0	1
0	1	0	1	1
1	0	0	1	0
1	1	1	1	0

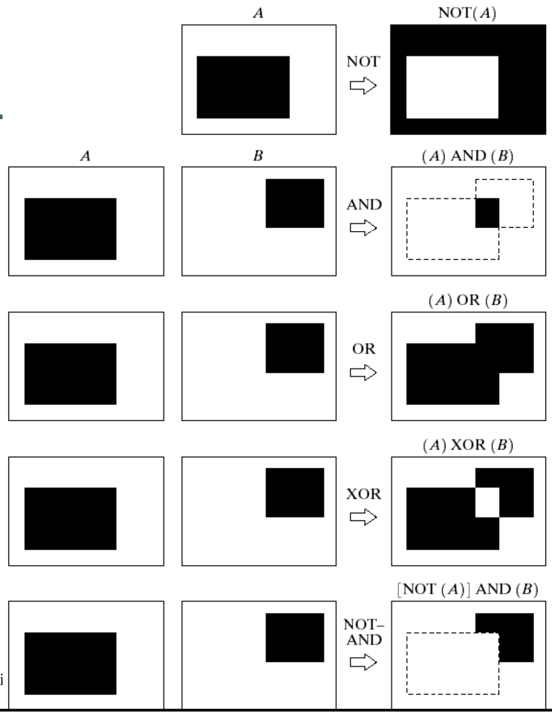


FIGURE 9.3 Some logic operations between binary images. Black represents binary 1s and white binary 0s in this example.

Dilation & Erosion

❑ Dilation:

- ∅: empty set; A,B: sets in Z^2
- Dilation of A by B:

$$A \oplus B = \{ z \mid (\hat{B})_z \cap A \neq \varnothing \}$$

Dilation & Erosion

- Dilation:
  - Obtaining the reflection of B about its origin and then shifting this reflection by z
  - The dilation of A by B then is the set of all z displacements such that $\hat{B}$ and A overlap by at least one nonzero element…

Dilation & Erosion

- Dilation:

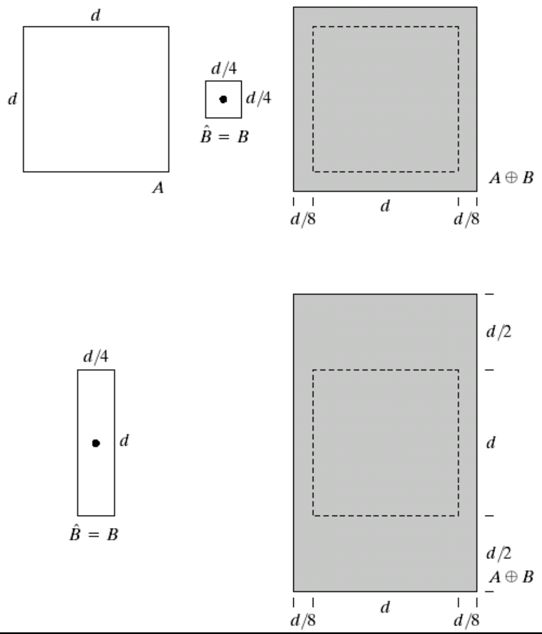
$$A \oplus B = \{x \mid [(\hat{B})_x \cap A] \subseteq A\}$$

B is the structuring element in dilation.

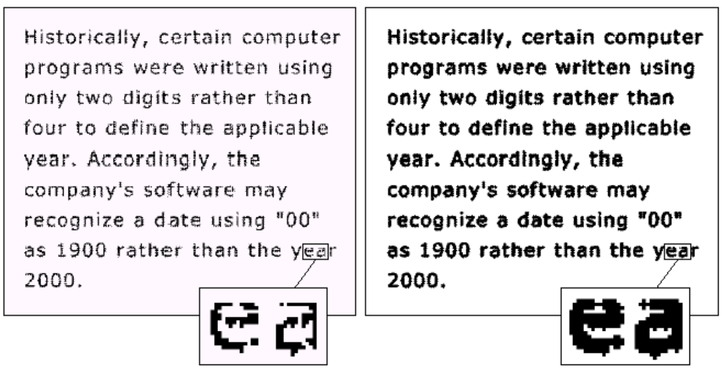
Morphological Image Processing

a b c
d e

FIGURE 9.4
(a) Set A.
(b) Square structuring element (dot is the center).
(c) Dilation of A by B, shown shaded.
(d) Elongated structuring element.
(e) Dilation of A using this element.



Morphological Image Processing



0	1	0
1	1	1
0	1	0

a c
b

FIGURE 9.5
(a) Sample text of poor resolution with broken characters (magnified view).
(b) Structuring element.
(c) Dilation of (a) by (b). Broken segments were joined.

Dilation & Erosion

- Erosion:

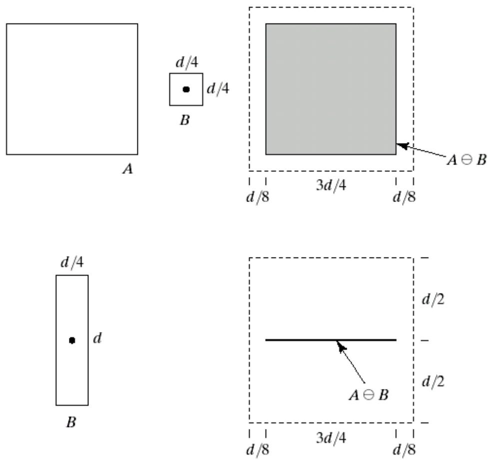
$$A \ominus B = \{x \mid (B)_x \subseteq A\}$$

i.e. the erosion of A by B is the set of all points x such that B, translated by x, is contained in A.

In general:

$$(A \ominus B)^c = A^c \oplus \hat{B}$$

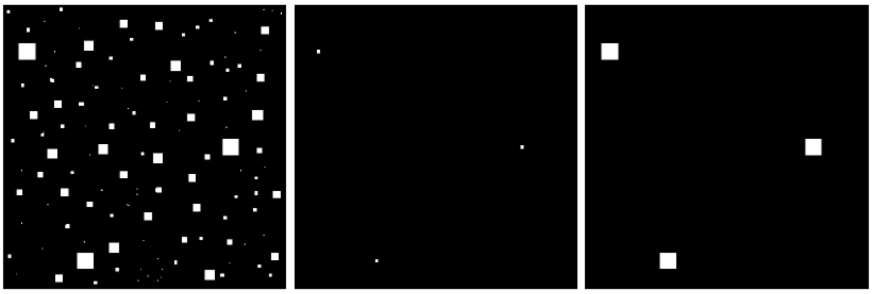
Dilation & Erosion



a b c
d e

FIGURE 9.6 (a) Set A. (b) Square structuring element. (c) Erosion of *A* by *B*, shown shaded. (d) Elongated structuring element. (e) Erosion of *A* using this element.

Morphological Image Processing



a b c

FIGURE 9.7 (a) Image of squares of size 1, 3, 5, 7, 9, and 15 pixels on the side. (b) Erosion of (a) with a square structuring element of 1's, 13 pixels on the side. (c) Dilation of (b) with the same structuring element.

Opening & Closing

- In essence, dilation expands an image and erosion shrinks it.
- Opening:
  - generally smoothes the contour of an image, breaks isthmuses, eliminates protrusions.
- Closing:
  - smoothes sections of contours, but it generally fuses breaks, holes, gaps, etc.

Opening & Closing

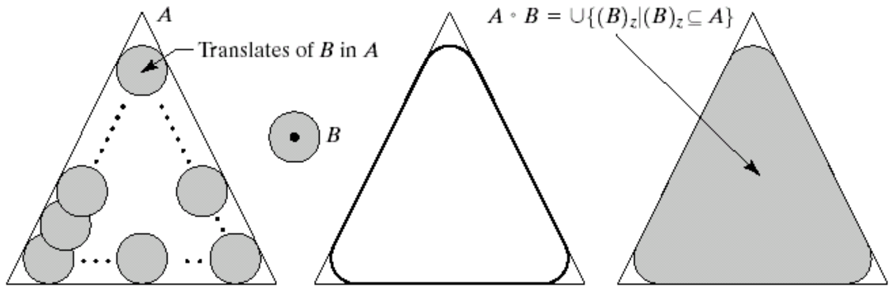
- Opening of A by structuring element B:

$$A \circ B = (A \ominus B) \oplus B$$

- Closing:

$$A \bullet B = (A \oplus B) \ominus B$$

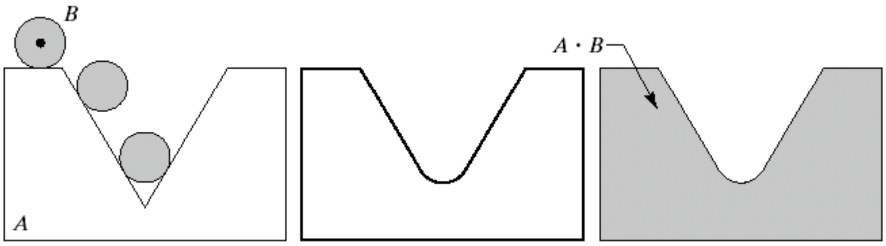
Morphological Image Processing



a b c d

FIGURE 9.8 (a) Structuring element B "rolling" along the inner boundary of A (the dot indicates the origin of B). (c) The heavy line is the outer boundary of the opening. (d) Complete opening (shaded).

Morphological Image Processing



a b c

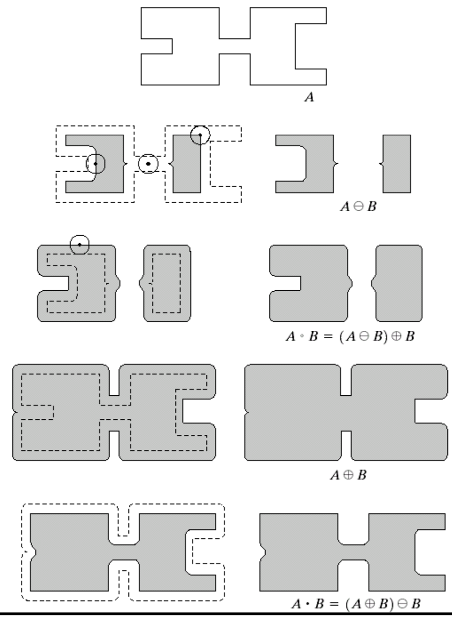
FIGURE 9.9 (a) Structuring element B "rolling" on the outer boundary of set A. (b) Heavy line is the outer boundary of the closing. (c) Complete closing (shaded).

$$(A \bullet B)^c = (A^c \circ \hat{B}).$$

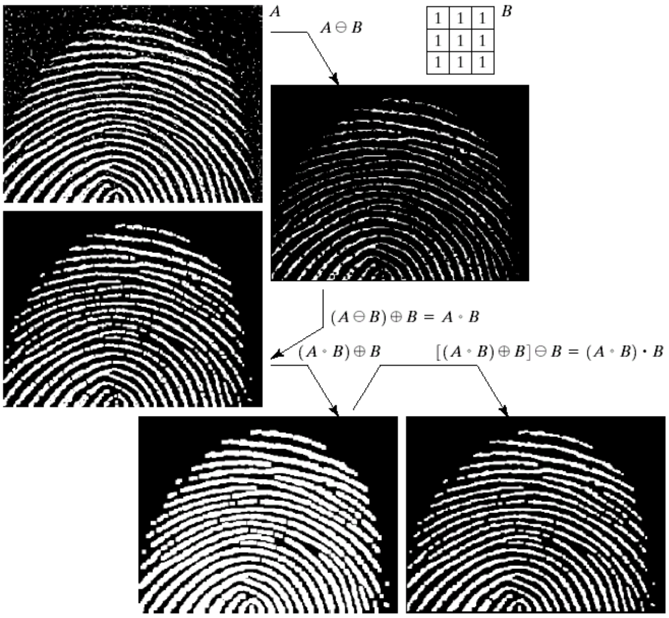
Morphological Image Processing

a
b c
d e
f g
h i

FIGURE 9.10 Morphological opening and closing. The structuring element is the small circle shown in various positions in (b). The dark dot is the center of the structuring element.



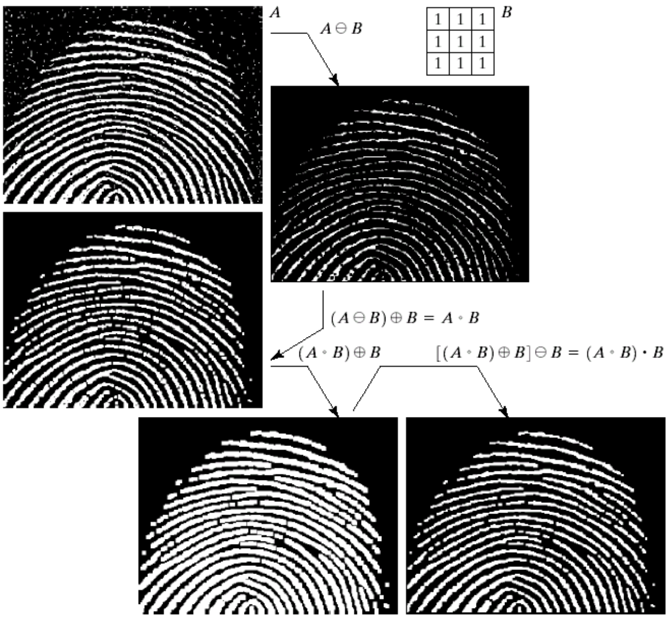
Morphological Image Processing



a b
d c
e f

FIGURE 9.11
(a) Noisy image. (c) Eroded image. (d) Opening of A. (d) Dilation of the opening. (e) Closing of the opening. (Original image for this example courtesy of the National Institute of Standards and Technology.)

Morphological Image Processing



a b
d c
e f

FIGURE 9.11
(a) Noisy image. (c) Eroded image. (d) Opening of A. (d) Dilation of the opening. (e) Closing of the opening. (Original image for this example courtesy of the National Institute of Standards and Technology.)

Opening & Closing

$$A \circ B \subseteq A$$

$$if\ C \subseteq D \rightarrow C \circ B \subseteq D \circ B$$

$$(A \circ B) \circ B = A \circ B$$

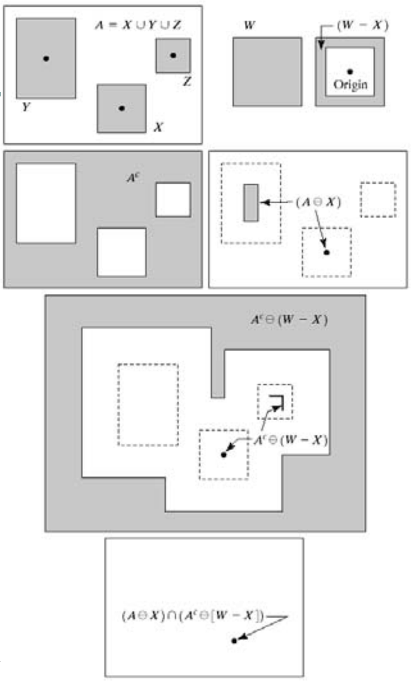
$$if\ C \subseteq D \rightarrow C \bullet B \subseteq D \bullet B$$

$$A \subseteq A \bullet B$$

$$(A \bullet B) \bullet B = A \bullet B$$

Hit-or-Miss Transform

- Morphological hit-or-miss transform is a basic tool for shape detection.
- Definitions:
  - B → (B_1,B_2)
    - B_1 is the set of elements of B associated with an object
    - B_2 is the set of elements of B associated with the corresponding background.



a b
c d
e
f

FIGURE 9.12 (a) Set A. (b) A window, W, and the local background of X with respect to W, $(W - X)$. (c) Complement of A. (d) Erosion of A by X. (e) Erosion of A^c by $(W - X)$. (f) Intersection of (d) and (e), showing the location of the origin of X, as desired.

Hit-or-Miss Transform

- $A \circledast B$
  - contains all the origin points at which, simultaneously:
    - B_1 found a match ("hit") in A and,
    - B_2 found a match in A^c.

$$B_1 = X \text{ and } \quad B_2 = (W - X)$$

$$A \circledast B = (A \ominus B_1) \cap (A^c \ominus B_2)$$

or

$$A \circledast B = (A \ominus B_1) - (A \oplus \hat{B}_2)$$

Example Basic Morphological Algorithms

- Purpose:
  - to extract image components that are useful in the representation and description of shape.
- Boundary Extraction:

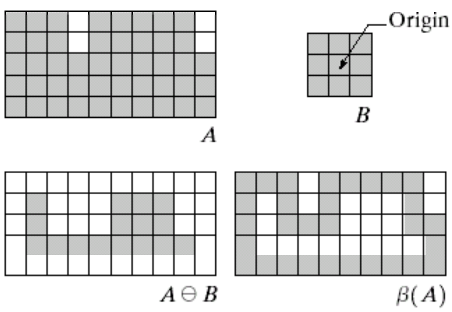
$$\beta(A) = A - (A \ominus B)$$

Morphological Image Processing

•**Boundary Extraction Example**

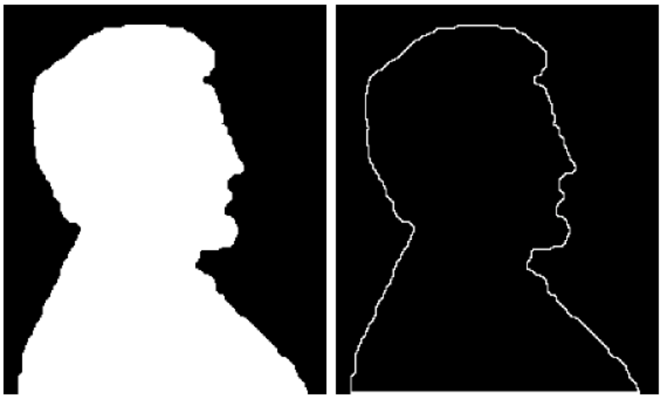
a b
c d

FIGURE 9.13 (a) Set A. (b) Structuring element B. (c) A eroded by B. (d) Boundary, given by the set difference between A and its erosion.



Morphological Image Processing

- **Boundary Extraction Example**



a b

FIGURE 9.14 (a) A simple binary image, with 1's represented in white. (b) Result of using Eq. (9.5-1) with the structuring element in Fig. 9.13(b).

Region Filling

❖Region filling based on set dilation, complementation and intersections.

❖Beginning with a point p inside the boundary, the objective is to fill the entire region with 1's.

$$X_k = (X_{k-1} \oplus B) \cap A^c$$

Where X_0=p →

when X_k=X_{k-1} the algorithm has converged.

Region Filling

$$X_k = (X_{k-1} \oplus B) \cap A^c$$

a b c
d e f
g h i

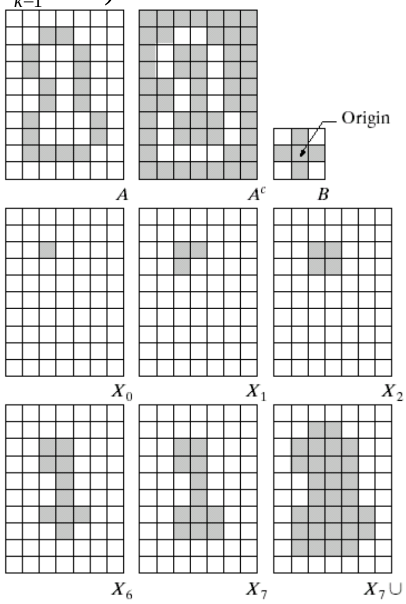
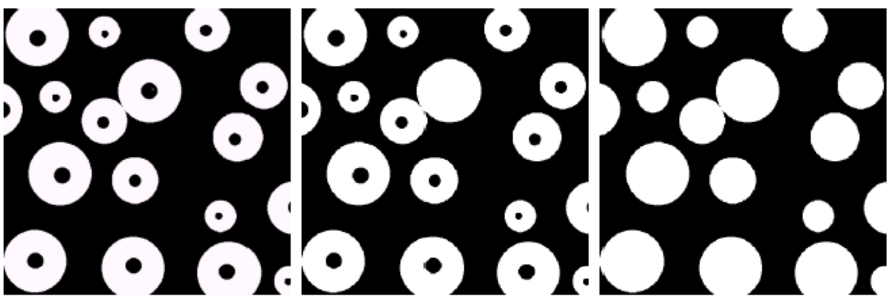


FIGURE 9.15 Region filling. (a) Set *A*. (b) Complement of *A*. (c) Structuring element *B*. (d) Initial point inside the boundary. (e)–(h) Various steps of Eq. (9.5-2). (i) Final result [union of (a) and (h)].

Morphological Image Processing

- **Example of Region filling**



a b c

FIGURE 9.16 (a) Binary image (the white dot inside one of the regions is the starting point for the region-filling algorithm). (b) Result of filling that region (c) Result of filling all regions.

Basic Morphological Algorithms

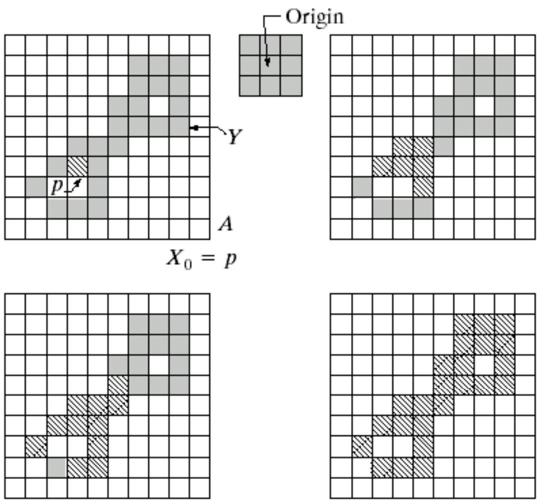
- Extraction of Connected Components:

$$X_k = (X_{k-1} \oplus B) \cap A \qquad k=1,2,3,\ldots$$

Where $X_0=p$ →

when $X_k=X_{k-1}$ the algorithm has converged.

Morphological Image Processing



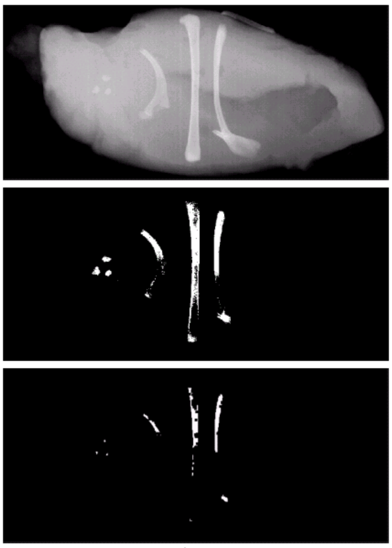
a b c
d e

FIGURE 9.17 (a) Set A showing initial point p (all shaded points are valued 1, but are shown different from p to indicate that they have not yet been found by the algorithm). (b) Structuring element. (c) Result of first iterative step. (d) Result of second step. (e) Final result.

Example of Extraction of Connected Components

a
b
c d

FIGURE 9.18 (a) X-ray image of chicken filet with bone fragments. (b) Thresholded image. (c) Image eroded with a 5 × 5 structuring element of 1's. (d) Number of pixels in the connected components of (c). (Image courtesy of NTB Elektronische Geraete GmbH, Diepholz, Germany, www.ntbxray.com.)



Connected component	No. of pixels in connected comp
01	11
02	9
03	9
04	39
05	133
06	1
07	1
08	743
09	7
10	11
11	11
12	9
13	9
14	674
15	85

Convex Hull

- *Convex Hull* H of an arbitrary set S is the smallest convex containing S.
- Convex deficiency: *H-S. these are useful for object description.*
- *The procedure consists of iteratively applying the hit-miss transform to A with B.*

$$X_k^i = (X_{k-1}^i \circledast B^i) \cup A \quad i = 1,2,3,4 \;\; and \;\; k = 12,3,...$$
$$X_0^i = A$$
$$if \;\; X_k^i = X_{k-1}^i \Rightarrow D^i = X_{conv}^i = X_k^i$$
$$C(A) = \bigcup_{i=1}^{4} D^i$$

Convex Hull

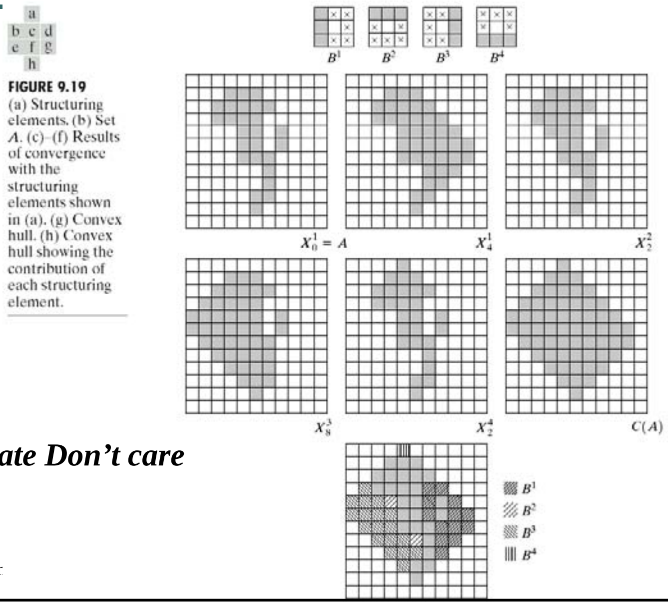


FIGURE 9.19 (a) Structuring elements. (b) Set A. (c)–(f) Results of convergence with the structuring elements shown in (a). (g) Convex hull. (h) Convex hull showing the contribution of each structuring element.

✗ ***indicate Don't care***

Thinning

$$A \otimes B = A - A * B$$
$$= A \bigcap (A * B)^C$$

$$\{B\} = \{B^1, B^2, B^3, \ldots, B^n\}$$

$$A \otimes \{B\} = ((\ldots((A \otimes B^1) \otimes B^2)\ldots) \otimes B^n)$$

$where\ B^i\ is\ a\ rotated\ version\ of\ B^{i-1}$

Algorithm:

1- thin with B^1 to B^n

2- go to step 1 until no further change is obtained

Thinning

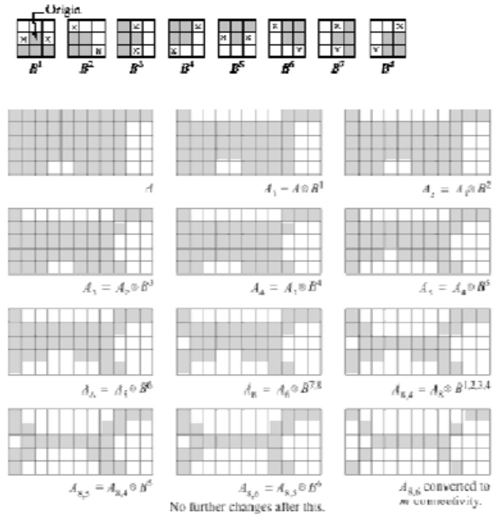


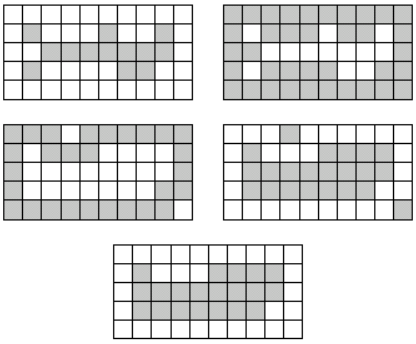
FIGURE 9.21 (a) Sequence of rotated structuring elements used for thinning. (b) Set A. (c) Result of thinning with the first element. (d)–(i) Results of thinning with the next seven elements (there was no change between the seventh and eighth elements). (j) Result of using the first element again (there were no changes for the next two elements). (k) Result after convergence. (l) Conversion to m-connectivity.

Thickening

$$A \odot B = A \cup (A \circledast B)$$
$$A \odot \{B\} = ((\ldots((A \odot B^1) \odot B^2)\ldots) \odot B^n)$$

- Structuring elements B in the above equation is the Not of B in thinning operator.
- Thickening can be performed by thinning A^c
- Connectivity check may be performed after thickening.

Thickening



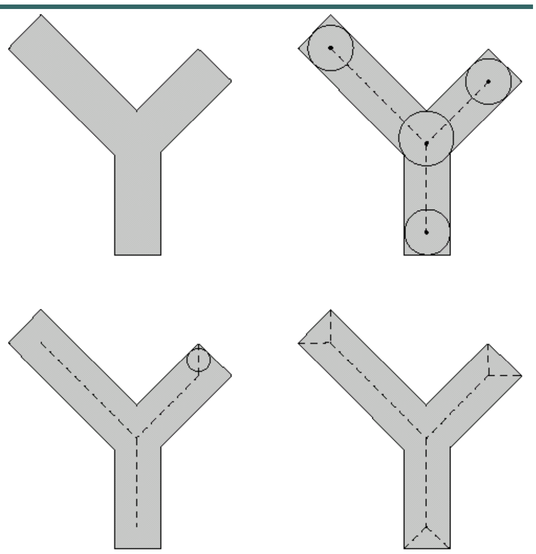
a b
c d
e

FIGURE 9.22 (a) Set A. (b) Complement of A. (c) Result of thinning the complement of A. (d) Thickened set obtained by complementing (c). (e) Final result, with no disconnected points.

Skeletons

a b
c d

FIGURE 9.23
(a) Set A.
(b) Various positions of maximum disks with centers on the skeleton of A.
(c) Another maximum disk on a different segment of the skeleton of A.
(d) Complete skeleton.



Skeletons

(a) If z is a point of $S(A)$ and $(D)_z$ is the largest disk centered at z and contained in A, one cannot find a larger disk (not necessarily centered at z) containing $(D)_z$ and included in A. The disk $(D)_z$ is called a *maximum disk*.
(b) The disk $(D)_z$ touches the boundary of A at two or more different places.

Skeleton

$$S(A) = \bigcup_{k=0}^{K} S_k(A)$$

$$S_k(A) = (A \ominus kB) - (A \ominus kB) \circ B$$

$$(A \ominus kB) = (\ldots((A \ominus B) \ominus B) \ldots) \ominus B$$

$$K = \max\{k \mid (A \ominus B) \neq \varnothing\}$$

$$A = \bigcup_{k=0}^{K} (S_k(A) \oplus kB)$$

$$(A \oplus kB) = (\ldots((A \oplus B) \oplus B) \ldots) \oplus B$$

Skeletons

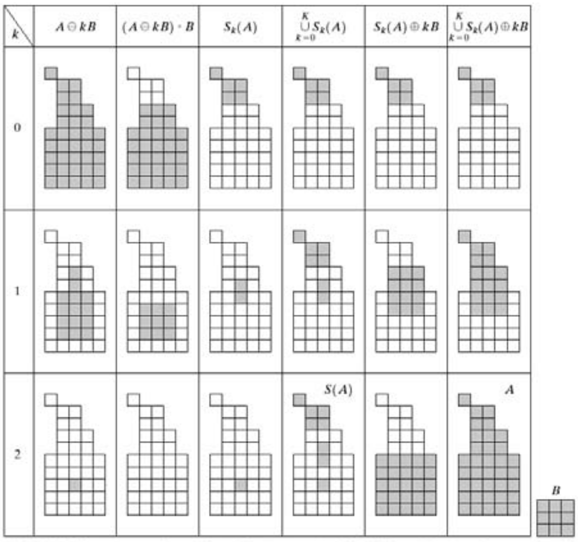


FIGURE 9.24 Implementation of Eqs. (9.5-11) through (9.5-15). The original set is at the top left, and its morphological skeleton is at the bottom of the fourth column. The reconstructed set is at the bottom of the sixth column.

Pruning

- Post processing after thinning or and skeletonizing to cleanup parasitic components.
- Method
  - Thin with proper structuring element n times

$$X_1 = A \otimes \{B\}$$

  - Find end points

$$X_2 = \bigcup_{k=1}^{8} (X_1 \circledast B^k)$$

  - Dilate end points with proper structuring element n times

$$X_3 = (X_2 \oplus H) \cap A$$

$$X_4 = X_1 \cup X_3$$

Pruning

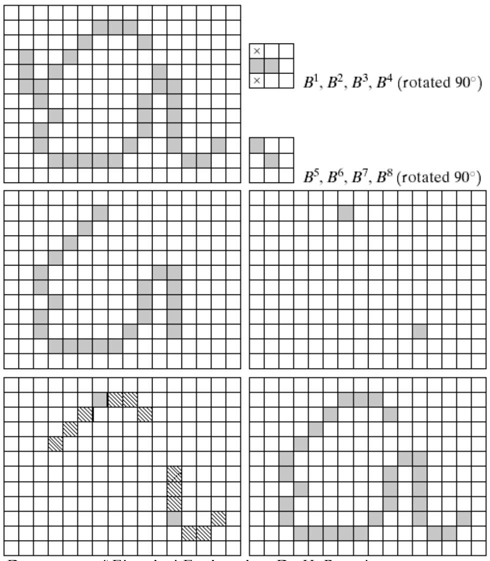


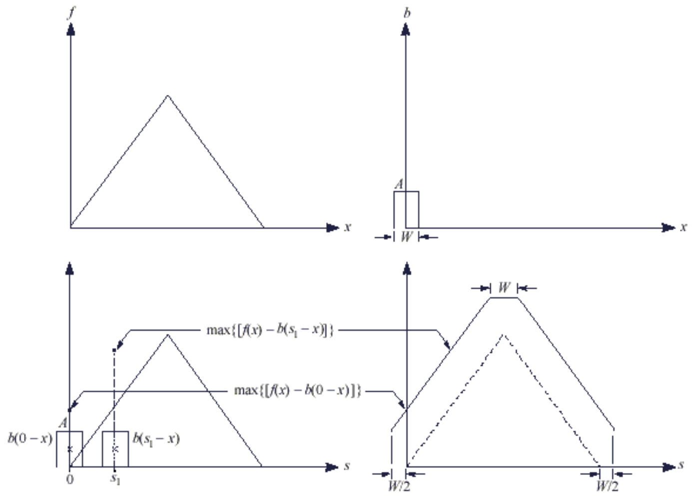
FIGURE 9.25 (a) Original image. (b) and (c) Structuring elements used for deleting end points. (d) Result of three cycles of thinning. (e) End points of (d). (f) Dilation of end points conditioned on (a). (g) Pruned image.

Gray Scale Dilation

$$(f \oplus b)(s,t) =$$
$$\max\{f(s+x,t+y)+b(x,y) \mid (s+x),(t+y) \in D_f ; (x,y) \in D_b\}$$

$$(f \oplus b)(s) =$$
$$\max\{f(s+x)+b(x) \mid (s+x) \in D_f ; x \in D_b\}$$

Gray Scale Dilation



Gray Scale Erosion

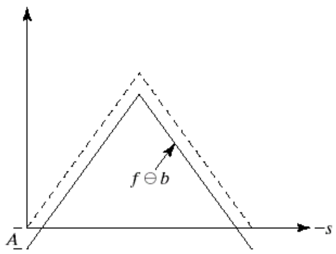
$$(f \ominus b)(s,t) =$$
$$\min\{\, f(s+x, t+y) - b(x,y) \mid (s+x),(t+y) \in D_f; (x,y) \in D_b \}$$

$$(f \ominus b)(s) =$$
$$\min\{\, f(s+x) - b(x) \mid (s+x) \in D_f; x \in D_b \}$$

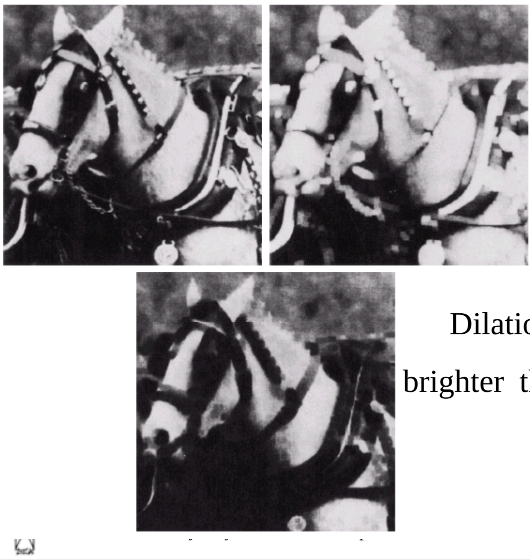
$$(f \ominus b)(s,t) = (f^c \oplus \hat{b})(s,t)$$
$$\hat{b}(x,y) = b(-x,-y)$$
$$f^c(x,y) = -f(x,y)$$

Gray Scale Erosion

FIGURE 9.28
Erosion of the function shown in Fig. 9.27(a) by the structuring element shown in Fig. 9.27(b).



Gray scale Erosion and Dilation



a b
c

FIGURE 9.29
(a) Original image. (b) Result of dilation. (c) Result of erosion. (Courtesy of Mr. A. Morris, Leica Cambridge, Ltd.)

Dilation produces the image with

brighter than and dark details have been reduced

Gray Scale Opening and Closing

$$f \circ b = (f \ominus b) \oplus b$$

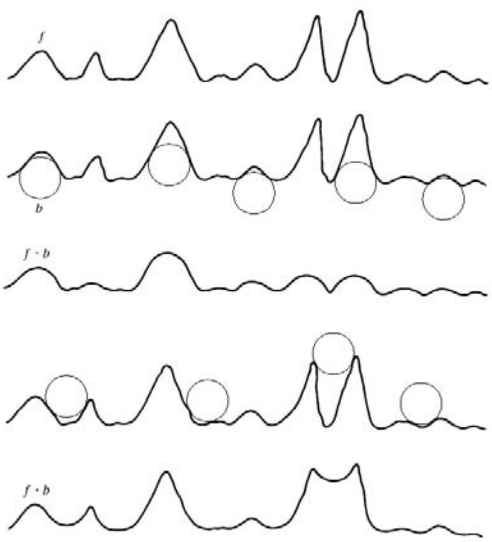
$$f \bullet b = (f \oplus b) \ominus b$$

$$(f \bullet b)^c = f^c \circ \hat{b}$$

$$f^c = -f(x, y)$$

$$-(f \bullet b) = (-f \circ \hat{b})$$

Gray Scale Opening and Closing



a
b
c
d
e

FIGURE 9.30 (a) A gray-scale scan line. (b) Positions of rolling ball for opening. (c) Result of opening. (d) Positions of rolling ball for closing. (e) Result of closing.

Gray Scale Opening and Closing

a b

FIGURE 9.31 (a) Opening and (b) closing of Fig. 9.29(a). (Courtesy of Mr. A. Morris, Leica Cambridge, Ltd.)