Homework 1
Theory Bayesian
Chapter 2

Problem 1.

Conditional densities for a one-dimensional feature of a two-class problem have a Cauchy distribution

$$p(x \mid \omega_i) = \frac{1}{\pi b} \cdot \frac{1}{1 + (\frac{x - a_i}{b})^2}, \quad i = 1, 2, \quad a_2 > a_1$$

(a) Design the Bayes min-error classifier in terms of a_i and b if $P(\omega_1) = P(\omega_2)$. Simplify the rule as much as possible.

(b) What is the equation for decision boundary in part (a).

(c) What is the Bayes min-error in part (a).

(d) Do parts (a) and (b) if $P(\omega_1) = \frac{1}{3}$, $P(\omega_2) = \frac{2}{3}$.

(e) Design the Bayes min-risk classifier if $P(\omega_1) = \frac{3}{4}$, $P(\omega_2) = \frac{1}{4}$

$$\lambda_{11} = \lambda_{22} = 0, \quad \lambda_{12} = 1, \quad \lambda_{21} = 2.$$

(f) What are conditional risks $R(\alpha_1 \mid x)$ and $R(\alpha_2 \mid x)$ in part (e) if $a_2 = 4$, $a_1 = 1$ and $b = 1$.

Problem 2.

In a two-category classification, instead of assigning a pattern x to one of the two classes, we may reject it as being unrecognizable. Let $\lambda_{11} = \lambda_{22} = 0$, $\lambda_{12} = \lambda_{21} = a$, and $\lambda_{31} = \lambda_{32} = b$. In other words, a is the loss incurred for making a substitution error and b is the loss for rejection.

(a) Find $R(\alpha_i \mid x)$, $i = 1, 2, 3$ and use them to find Bayes min-risk rule. Find $R(\alpha_i \mid x)$ in terms of conditions on $P(\omega_i \mid x)$, a, and b.

(b) Using the result of (a) find appropriate discriminant functions $g_i(x)$, $i = 1, 2, 3$ in terms of a, b, $p(x \mid \omega_i)$ and $P(\omega_i)$.

Problem 3.

Class conditional densities of two classes are given in the figure. A priori probabilities are $P(\omega_1) = .2$, $P(\omega_2) = .8$

(a) Design the Bayes min-error classifier and find the total probability of error associated with it.

(b) Design the Bayes min-risk classifier if $\lambda_{11} = \lambda_{22} = 0$ $\lambda_{12} = 1$, $\lambda_{21} = 2$

(c) Find the conditional risks $R(\alpha_i \mid x)$, $i = 1, 2$ and the total Bayes risk R in part (b).

$R = \epsilon_1 P(w_1) + \epsilon_2 P(w_2)$

$R = \int R(\alpha \mid x) p(x)\, dx = \int_{R_1} R(\alpha_1 \mid x) p(x)\, dx + \int_{R_2} R(\alpha_2 \mid x) p(x)\, dx$