Natural frequency analysis of cracked plates using Singular Finite Element Method

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Abstract. A numerical model is presented for free vibration of a thin square isotropic plate containing a crack located at the center of the plate. The procedure used is singular finite element method through MATLAB software. In this regard, an eigenvalue analysis is performed to obtain the natural frequencies of the cracked plate by considering different boundary conditions. The first two modal shapes are formed for different ratios of crack length to plate width. The results are validated by comparing with those in other articles.

1 Introduction

Plate as a basic structural element is widely at risk of cracking, so analysis of cracked plates has been the subject of intensive investigations during recent decades. In fact, the presence of a crack in a plate causes changes in stiffness of the plate and affecting its static and dynamic characteristics. One of these characteristics is natural frequency that is analyzed in this study. Natural frequency of plates has already been analyzed by various methods, such as decomposition method [1], Ritz method [2] or generalized Rayleigh-Ritz method [3], Galerkin's method [4], finite element method [5-10] or generalized differential quadrature finite element method [11], extended finite element method [12], and also extended cell-based smoothed discrete shear gap method [13].

As known in theory of fracture mechanics, the stresses at the crack tips reach to infinity so that a phenomenon known as singularity occurs. This phenomenon is usually resulted in increasing of computing time of finite element analysis due to need to small mesh sizes around the crack tips. Thus, it is desirable to use a method to overcome this problem. This paper uses singular finite element method to frequency analysis of plates.

2 Singular finite elements

Singular elements are particular elements used around the crack tip to present the singularity of the crack. In fact, the exclusivity of them is their compatibility with singularity behavior. Owing to the fact that out-of-plane analysis of a cracked plate is affected by the in-plane stress distribution, to get more accurate results, an in-plane analysis should be first done. The singular element used for in-plane analysis has five nodes with two degrees of freedom at each node (u, v) shown in Fig. 1. More details of this element can be found in ref. [14].

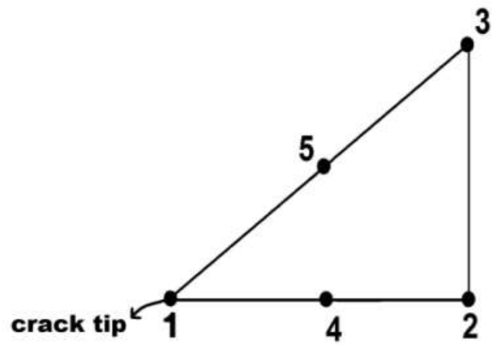


Figure 1: In-plane five-node singular triangular element [14]

The out-of-plane singular element used here has three nodes with three degrees of freedom at each node; including a transverse displacement and two rotations. The geometric of this element is contemporary shown in a Cartesian and polar coordinate system in Fig. 2. The transverse displacement, w, can be expressed in polar system as follow [15]:

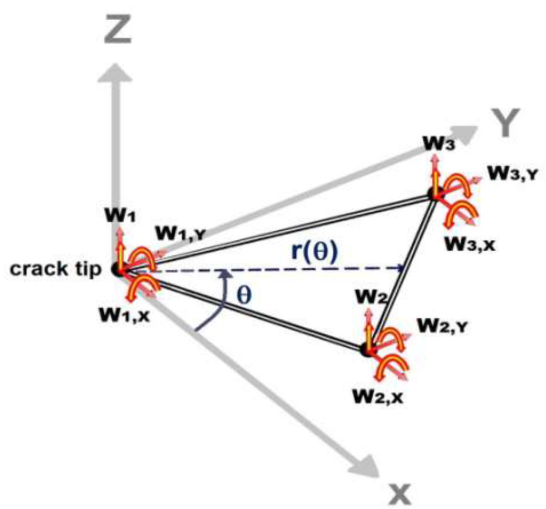


Figure 2: Out-of-plane three-node Singular triangular element [15]

$$\begin{aligned} w(r,\theta) \quad = \quad & \alpha_1 + \alpha_2 r\cos\theta + \alpha_3 r\sin\theta + \alpha_4 r^{\frac{1}{2}}\cos\frac{\theta}{2} + \alpha_5 r^{\frac{1}{2}}\sin\frac{\theta}{2} + \\ & \alpha_6 r^{\frac{1}{2}}\left[\cos^3\frac{\theta}{2} + \sin^3\frac{\theta}{2}\right] + \alpha_7 r^2\cos\theta\sin\theta + \alpha_8 r^2\cos^2\theta + \alpha_9 r^2\sin^2\theta \end{aligned}$$

and in the matrix form displays as below:

$$w = [\varphi]\cdot[\alpha]\,.$$

The relation between parameters α and nodal degrees of freedom is expressed as:

$$[W] = [C] \cdot [\alpha] \,,$$

where $[C]$ is the corresponding transformation matrix. The well-known form of transverse displacement in finite element method is as:

$$w = [N] \cdot [W] \,,$$

where $[N]$ are the element shape functions can be derived based on interpolation functions as following:

$$[N] = [\varphi] \cdot [C]^{-1} \,.$$

3 Finite element formulations

Free vibration of plates can be modeled mathematically by algebraic equations based on Energy theory as following:

$$\Pi = U - T,$$

where U is the total potential energy derived by:

$$U = U_b + U_g$$

U_b is the strain energy due to bending and U_g is the effect of in-plane forces on the transverse deflection.

$$\begin{aligned} U_b &= \int \frac{Et^3}{12(1-\nu^2)} \left[w_{,xx}^2 + 2w_{,xx}w_{,yy} + w_{,yy}^2 + 2(1-\nu)w_{,yy}^2 \right] dA \\ U_g &= \int \left[N_{xx}w_{,x}^2 + 2N_{XY}w_{,X}w_{,X} + N_{yy}w_{,y}^2 \right] dA \end{aligned}$$

and T is the kinetic energy obtained by:

$$T = \frac{\rho\, t\omega^2}{2} \int w^2 \, dA.$$

In above equations w is the transverse displacement and comma indicates partial differentiation with respect to the next subscribed variable, ω is the natural frequency, t is the plate thickness, ρ is the density of the plate material, E is the Young's modulus and ν is the Poisson's ratio. N_{xx}, N_{yy} and N_{XY} are in-plane stress resultants.

Based on principle of minimum total energy ($\delta\Pi = 0$), the eigen-equations of free vibration of the plate are obtained as below:

$$[(K_S + K_G) - \lambda \, M] \, [W] = 0.$$

Then, the dimensionless natural frequency λ is expressed as:

$$\lambda = \omega \, L^2 \sqrt{\frac{\rho \, t}{D}}$$

where L is the plate width.

4 Model descriptions

The MATLAB software is utilized for the modeling and vibration analysis of the considered cracked plate in this study. The geometric of the cracked plate is shown in Fig. 3.

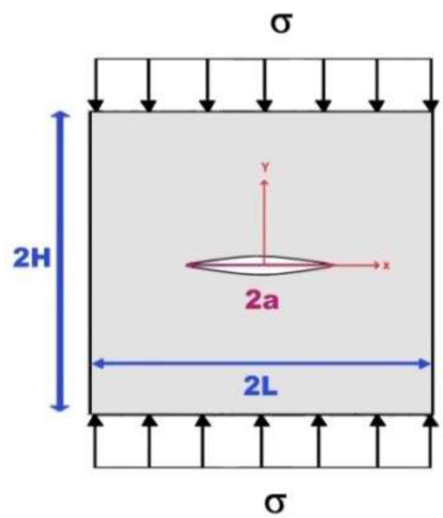


Figure 3: Specimen geometry

Two sides of the plate parallel to the crack line have in-plane restrictions subjected to uniform pressure. So two in-plane and out of plane models are coded for it. In both models, the crack was presumed to be through thickness since thin plate is used and having no friction else and no propagation was allowed. Three types of boundary conditions are considered for it, once has four simply supported sides (SSSS), other has two simply supports in its opposite sides and two clamped supports in its other sides (CSCS) and the last has four clamped sides (CCCC). The considered geometric parameters are: plate height and width $2H = 2L = 1.2\,m$, plate's thickness $t = 0.01\,m$, and relative crack's length $\frac{a}{L} = 0.0, 0.2, 0.4, 0.6, 0.8$.

The plate material is supposed to be linear elastic and isotropic with Young's modulus as: $E = 204\,GN/m^2$, Poisson's ratio $\nu = 0.3$ and density $\rho = 7860\,kg/m^3$.

In both in-plane and out of plane models, two kinds of singular and regular elements are used in this way, 8 singular triangular elements are located around each crack tip and a number of regular quadrature elements depend on the mesh sizes are used in other parts of plate. Regular elements have four nodes and the singular elements as previously explained have five nodes through in-plane and three nodes through out of plane models. Different mesh sizes are also used to get the sufficient convergence. The assembling samples of two models elements are shown in Figs. 4-5.

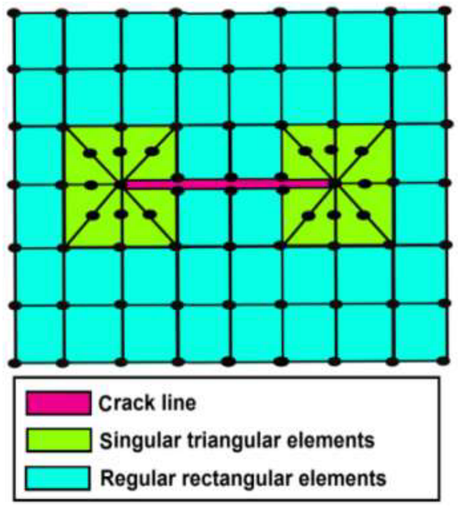


Figure 4: Assembling sample of in-plane model elements

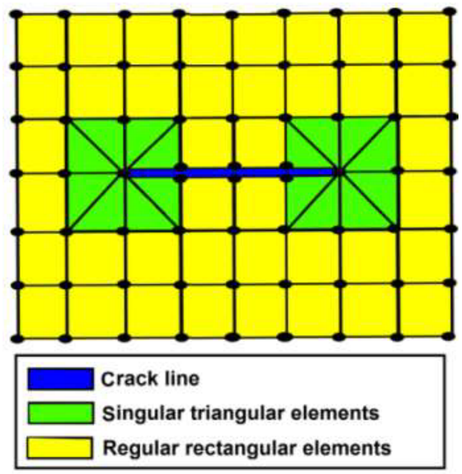
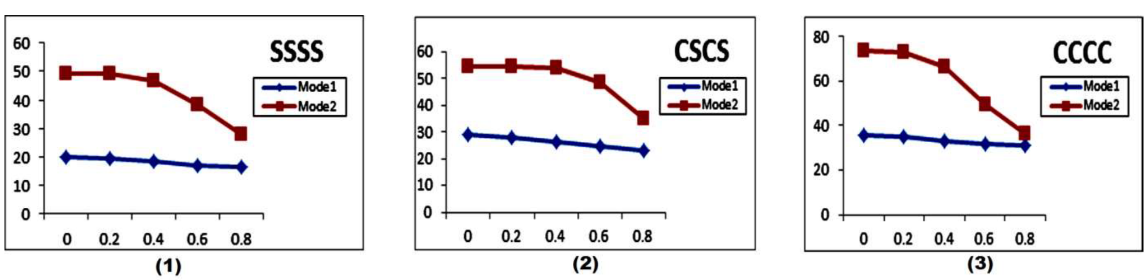


Figure 5: Assembling sample of out of plane model elements

5 Numerical results

In this section, natural frequencies are obtained for a square plate (aspect ratio=1). The results are comparing with other researchers studies just found for simply supported plate, [1, 16, 12, 13]. Table 1 shows the two lowest modes of non-dimensional frequency parameter λ compared to those in other articles. The results of the current analysis show good agreement.

It is important to observe how the frequency parameter changes with different crack lengths and types of supports. Consequently, diagrams 1, 2, 3 indicate the values of the first two non-dimensional frequency parameter λ versus different relative crack lengths $\frac{a}{L}$ for three types of supports.



Diagrams 1, 2, 3: changes of two first modes of non-dimensional frequency parameter λ versus different relative crack lengths for three types of supports

The mode shapes of obtained frequencies are also plotted in Fig. 6.

Aspect ratio=1				
Supports	Crack ration	Articles	Mode 1	Mode 2
SSSS	0.0	Liew et al. [1]	19.740	49.350
		Bachene et al.[12]	19.739	49.348
		T. Nguyen-Thoi et al. [13]	19.730	49.404
		Present study	19.730	49.323
	0.2	Liew et al. [1]	19.380	49.160
		Bachene et al.[12]	19.305	49.181
		Huang et al.[16]	19.330	49.190
		Present study	19.266	49.169
	0.4	Liew et al. [1]	18.440	46.440
		Bachene et al.[12]	18.278	46.635
		Huang et al.[16]	18.290	46.650
		Present study	18.261	46.709
	0.6	Liew et al. [1]	17.330	37.750
		Bachene et al.[12]	17.180	37.987
		Huang et al.[16]	17.190	37.990
		Present study	17.183	38.168
	0.8	Liew et al. [1]	16.470	27.430
		Bachene et al.[12]	16.406	27.753
		Huang et al.[16]	16.410	27.770
		Present study	16.416	27.917
CSCS	0.0	Present study	28.937	54.699
	0.2	Present study	28.087	54.609
	0.4	Present study	26.284	54.057
	0.6	Present study	24.469	48.302
	0.8	Present study	23.247	34.963
CCCC	0.0	Present study	35.962	73.331
	0.2	Present study	34.989	72.929
	0.4	Present study	33.168	66.430
	0.6	Present study	31.717	49.331
	0.8	Present study	31.145	36.226

Table 1: The first two modes of non-dimensional frequency parameter λ for different relative crack lengths $\frac{a}{L} = 0.0, 0.2, 0.4, 0.6, 0.8$ and three types of supports (simple, simple-clamped, clamped)

6 Conclusions

In the present paper, a numerical model based on singular finite element method (SFEM) has been developed for natural frequency of central cracked, square plates. In this procedure, the obtained eigen-equations have been implemented based on principle of minimum total energy $\delta\Pi = 0$, using MATLAB software and the effects of the crack length and different types of supports on the natural frequencies and the corresponding mode shapes

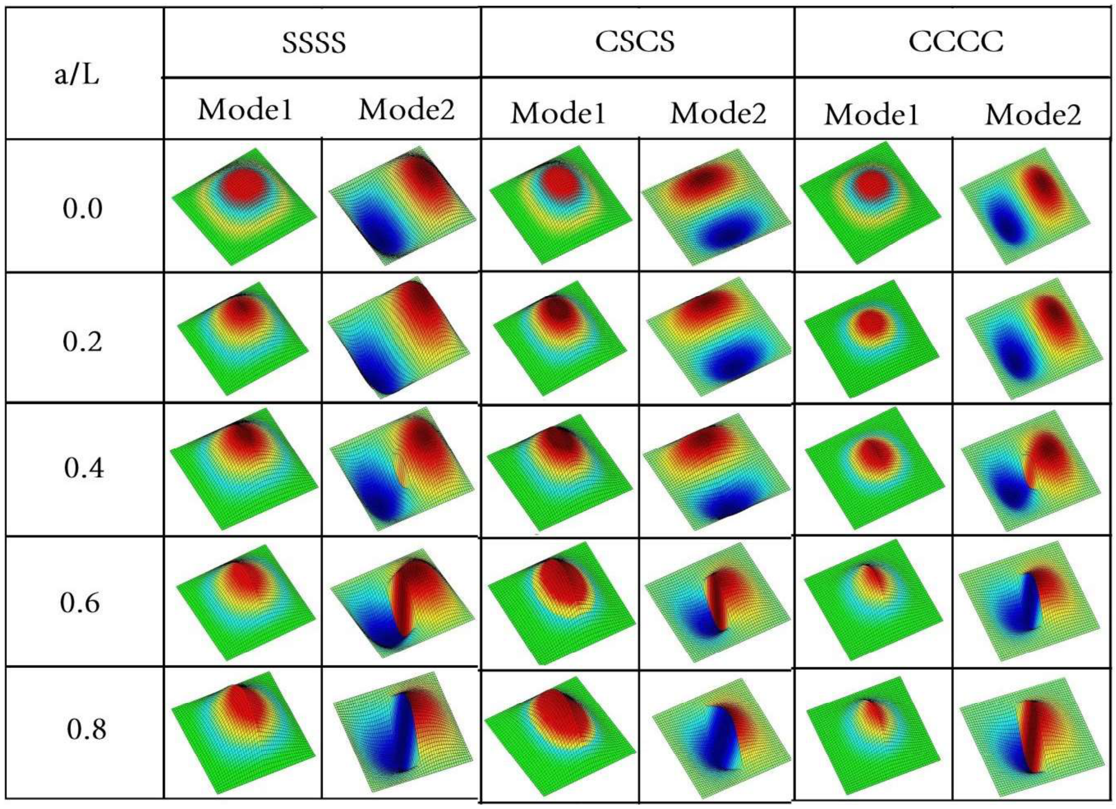


Figure 6: Mode shapes of non-dimensional frequency parameter λ for different relative crack lengths and three types of supports

have been investigated. On the basis of the achieved results the following conclusions can be stated:

1. The numerical simulations show that the frequency decreases as the crack length increases. This is due to the reduction in stiffness of the material structure.

2. The change in frequencies due to the presence of a crack is a function of the crack parameters and it also depends upon the mode shapes of the plate.

3. Existence of clamped supports causes higher increment of frequency in comparing with simple supports. The increase in stiffness is the cause for increase in frequency when the boundary condition is changed from SS to CS or CC.

The present results are in very good agreement with the numerical results reported in the literature so it can be concluded that the SFEM is an efficient method for the vibration analysis of cracked plates.

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