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A Structural Damage Detection Technique Using Measured Flexibility Matrix

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Abstract
Most of methods developed for detecting damage in a structure using measured modal parameters require a correlated finite element model, or at least, modal data of the structure for the intact state as baseline. For beam-like structures, curvature potential techniques, e.g., mode shape curvature and flexibility curvature have been applied to localize damage. In this paper, an approximate flexibility matrix has been derived using a few number of vibration modes whose columns are referred as flexibility shapes. Next, a damage localization method based on changes in flexibility shapes as well as its curvature is developed. Differential Quadrature Method has been implemented to obtain the curvatures of flexibility shapes. It was shown that the method works well using damaged model. To reduce the error due to differentiation of experimental data, a moving curve fit method has been implemented on flexibility shapes. The method is demonstrated through numerical and experimental case studies.

Keywords: Modal Analysis, Modal Testing, Damage Detection

Introduction

Damage generally produces changes in the structure physical properties (i.e., stiffness, mass, and damping), and these changes are accompanied by changes in the modal parameters of the structure (i.e., natural frequencies, mode shapes, and modal damping). This phenomenon has been widely noted and used by structural engineers for structural damage detection.

Sohn et al. [1] provided an excellent review on research advances in this area over the last 30 years, and summarized this kind of technology as vibration-based damage identification (VBDI) methods.

According to the process to treat the measured modal parameters, the VBDI methods can be classified as model based and non-model based. The model-based methods identify damage by correlating an analytical model, which is usually based on the finite element theory, with experimental modal data of the damaged structure [2, 3]. Comparisons of the updated model with the original one provide an indication of damage and further information on the damage location and/or its extent. However, the construction of the finite element model usually gives rise to model errors from simplified assumptions. To detect the damage other than the artificial errors from the model construction, a high quality finite element model that could accurately predict the behavior of the intact structure is required but is often difficult to achieve.

In non-model-based VBDI methods, the changes of modal parameters between the intact and damaged states of the structure are directly used to develop the damage indicators for localizing damage in the structure. Early works of this methodology make use of the natural frequency and mode shape information. Shifts in the natural frequencies [4] and changes in the multiple damage location assurance criterion [5] between the intact and damaged structure are formulated as indicators to localize damage. Pandey et al. [6] further demonstrated that changes in mode shape curvature could be a good indicator of damage for beam-like structures.

During the last decade, some researchers found that the modal flexibility can be a more sensitive parameter than natural frequencies or mode shapes alone for structural damage detection. Raghavendrachar and Aktan [7] examined the application of modal flexibility for a three span concrete bridge. In their comparison with natural frequency and mode shapes, the modal flexibility is reported to be more sensitive and reliable for local damages. Zhao [8] presented a theoretical sensitivity study comparing the use of natural frequencies, mode shapes, and modal flexibility for structural damage detection. The results demonstrate that modal flexibility is more likely to indicate damage than the other two. Pandey [9] proposed a damage localization method based on directly examining the changes in the measured modal flexibility of a beam structure.

In this paper, a structural damage localization approach is proposed based on information about damaged structure. The technique does not need analytical model or intact state information. Some case studies were selected to demonstrate the capability of the method for both simulated and experimental data.

Theory

The basic premise of VBDI methods is that the damage alters the mass, stiffness and damping properties of a structure which in turn affects structural modal parameters. Stiffness change due to damage is a promising factor in damage localization, because any change in elemental stiffness throughout the stiffness matrix can lead to structural damage location. Unfortunately, derivation of fairly accurate stiffness matrix based on a few lower frequency measured modal data is very difficult to achieve. However its inverse, i.e., flexibility matrix can be approximated using first few measured modal data due to inverse relationship to the square of modal frequencies [10].

With the mass normalized modes ϕ_i, the stiffness matrix K and the flexibility matrix F can be expressed by mode expansion

$$K = \Phi \Omega \Phi^T = \sum_{i=1}^{n} \omega_i^2 \phi_i \phi_i^T \quad (1)$$

$$F = \Phi \Omega^{-1} \Phi^T = \sum_{i=1}^{n} \frac{1}{\omega_i^2} \phi_i \phi_i^T \quad (2)$$

Where $\Phi = [\phi_1\ \phi_2 \ldots \phi_n]$ is an $m \times n$ dimensional mode shape matrix, m is the number of measurement points and n is the number of extracted modes. $\Omega = diag\left(\omega_1^2, \ldots, \omega_n^2\right)$ is the eigenvalue matrix and ω_i is the i th natural frequency. As it is evident from equations (1) and (2), the modal contribution to the stiffness matrix increases with the increase of natural frequency. Adversely, the modal contribution to the flexibility matrix decreases as the frequency increases, so it converges using a few lower modes. In this paper, 5 modes are used in numerical case study and 3 in experimental study to construct the flexibility matrix.

Each column of the flexibility matrix represents the displacement pattern of the structure associated with a unit force applied at the associated DOF. In the damaged vicinity of uniform beam structures, an abnormality occurs in the aforementioned displacement shape due to sudden stiffness change. This abnormality can also be seen in mode shapes, however flexibility shapes seems to exhibit the damage better, because they make use of mass and frequency information as well as mode shapes. Besides, available mode shapes are usually few and if the damage location coincides nodal points, the corresponding mode shape can not be useful in damage localization. But in flexibility method, the number of shapes used in the damage localization equals to the number of measurement grid points.

Two methods will be introduced to discover the sharp points of structural damage position in flexibility shapes, i.e., shape curvature and gapped smoothing methods.

Differential Quadrature Method (DQM) has been applied to calculate the shape curvatures. The DQM is an efficient and accurate numerical method and is frequently used for solving non-linear partial differential equations. The main feature of DQM is that the partial derivatives of a function can be numerically evaluated by multiplication of a weighting function. If the function $f(x_i)$ is given, the derivatives of it with respect to a spatial variable at any discrete point can be mathematically expressed as [11]

$$f^{(n)}(x_i) = \sum_{r=1}^{m} C_{ir}^{(n)} f(x_r) \quad (3)$$

Where m is the number of measurement grid points, $C_{ir}^{(n)}$ are the weighting coefficients associated with the n th order derivatives of $f(x)$ with respect to x at point x_i. The weighting coefficients can be obtained using following recurrence formulae

$$C_{ir}^{n} = n\left(C_{ii}^{(n-1)} C_{ir}^{(1)} - \frac{C_{ir}^{(n-1)}}{x_i - x_r} \right) \quad (4)$$

$$i, r = 1,2,\ldots,m\ , r \neq i\ , n = 2,3,\ldots,m-1$$

$$C_{ii}^{(n)} = -\sum_{\substack{r=1 \\ r\neq i}}^{m} C_{ir}^{(n)} \quad i = 1,2,\ldots,m\ \ , n = 2,3,\ldots,m-1 \quad (5)$$

$$C_{ir}^{(1)} = \frac{M^{(1)}(x_i)}{(x_i - x_r) M^{(1)}(x_r)} \quad i, r = 1,2,\ldots,m\ , r \neq i \quad (6)$$

Where

$$M^{(1)}(x_i) = \prod_{\substack{r=1 \\ r\neq i}}^{m} (x_i - x_r)$$

Another technique called Gapped Smoothing Method (GSM) locates nodal variations in flexibility shapes. In this method, a gapped polynomial is fitted to the flexibility shape. An irregularity factor is then calculated as the difference between the available nodal shape and approximated value by the polynomial. Figure (1) depicts calculation of the irregularity factor δ_i, using the GSM. The continuous line represents the flexibility shape, and the dotted line shows the polynomial function at the i th grid point.

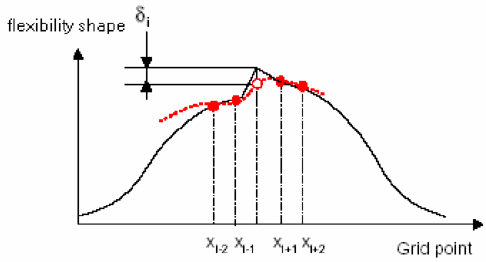

Figure 1: calculation of irregularity factor

In this paper, cubic polynomials are implemented as curve fitting functions. Compared to linear, higher order polynomials result in more reliable factors but there are two drawbacks with them. Firstly, irregularity factors at the boundaries are based on extrapolation which has less accuracy. Secondly, indicated damaged region is wider using higher order polynomials.

Numerical case study

The aforementioned method was applied to a cantilever aluminum beam of rectangular cross section of 8×40 millimeters and length of 400 millimeter. The beam has been modeled by 40 2D beam elements each has 6 DOFs. Damage is artificially modeled in four cases through reducing the thickness of an element as shown in table (1). Thickness reduction for cases A, B and C is 4mm and for the case D is 2mm.

Table 1: damage cases

Case	Damaged element
A	6
B	11
C	36
D	11

Figures (2) to (4) depict some mode shapes of the structure. As can be seen from figures, there is no considerable difference found between mode shapes of damaged and undamaged cases, especially in lower modes.

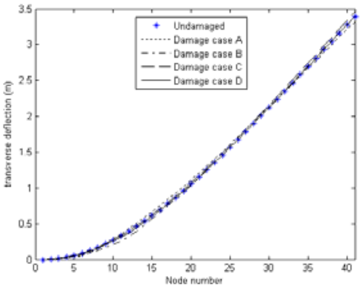

Figure 2: first mode shape of intact and damaged structures

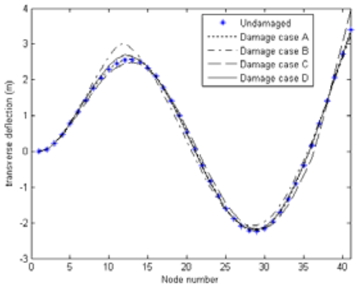

Figure 3: third mode shape of intact and damaged structures

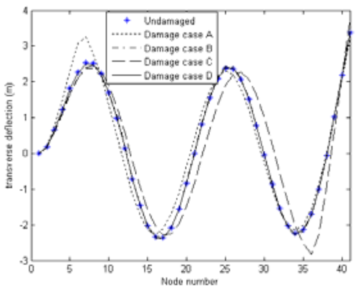

Figure 4: fifth mode shape of intact and damaged structures

Next, curvature of flexibility shapes was calculated using DQM for damage cases. Figures (5) to (8) show the absolute curvatures. It is seen from figures that the damage region is well detectable using flexibility matrix of damaged structure except for the case C. The results will be more obvious if the curvature difference between intact and damaged structure is considered.

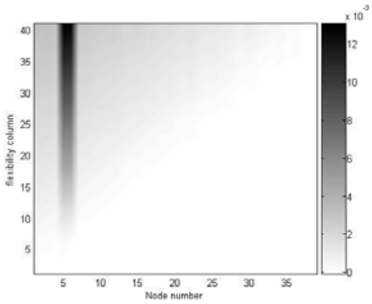

Figure 5: curvature of flexibility shapes: case A

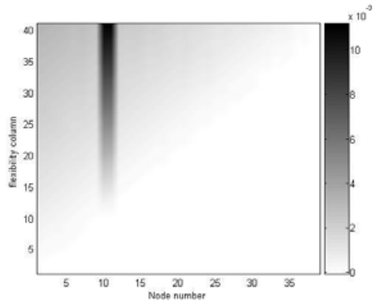

Figure 6: curvature of flexibility shapes: case B

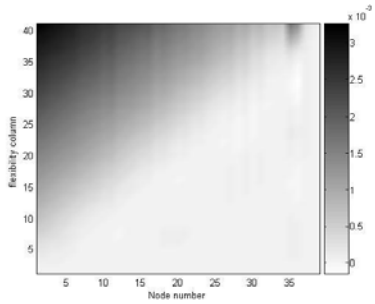

Figure 7: curvature of flexibility shapes: case C

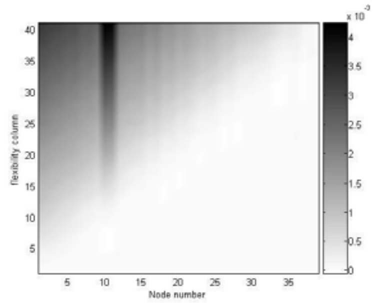

Figure 8: curvature of flexibility shapes: case D

The GSM approach was then implemented on damaged state flexibility shapes. Figures (9) to (12) show the results of GSM for damage cases.

As can bee seen, except for case C, damage region have been well localized using linear GSM. Comparing cases B and D, it can be concluded that in the more severe damage, the method works better.

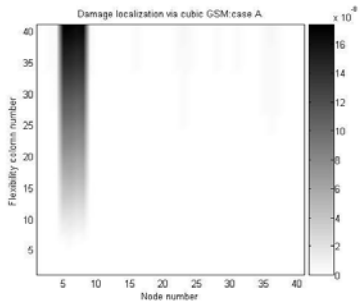

Figure 9: Damage localization via GSM: case A

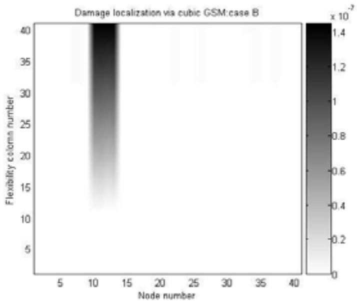

Figure 10: Damage localization via GSM: case B

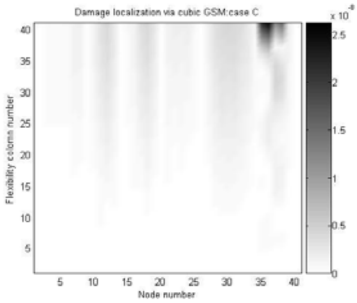

Figure 11: Damage localization via GSM: case C

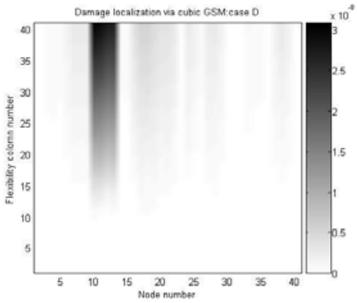

Figure 12: Damage localization via GSM: case D

To study the sensitivity of damage indication method to damage position, 4mm thickness reduction was moved along the beam. Curvature of first five mode shapes was studied instead of flexibility shapes. Due to linear dependence of flexibility shapes to mode shapes, the sensitivity conclusion about modes will be also valid for flexibility shapes. Figures (13) to (15) depict some mode shapes, their curvature in intact state and curvature difference between intact and each moving damage case. Curvatures have been scaled for convenient inspection. As it is evident, irregularity amplitude in each damaged position for all modes is proportional to absolute curvature in that location. In other words, if the damage has been occurred in a region of large amplitude curvature of all modes, the flexibility shape damage indication method works better. That is why the damaged region of case C in the previous example could not be recognized accurately.

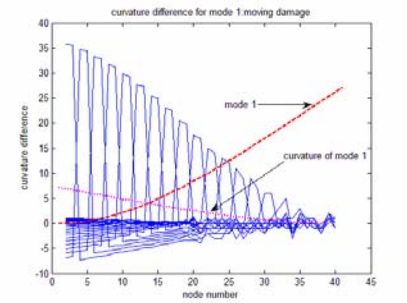

Figure 13: First mode shape, its curvature and curvature change for various damage positions

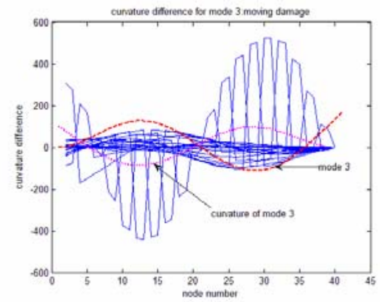

Figure 14: Third mode shape, its curvature and curvature change for various damage positions

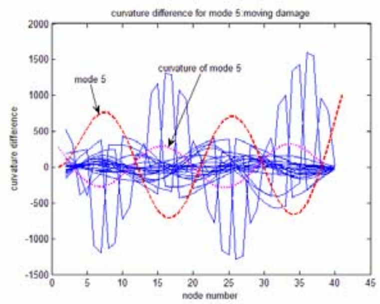

Figure 15: Fifth mode shape, its curvature and curvature change for various damage positions

Experimental case study

To verify the proposed method in practical point of view, modal test and analysis were carried out on a rectangular cross section aluminum beam. Test has been carried out in free-free condition by suspending the test object by some elastic bungees. The beam is of cross section 50×25 millimeters and 800 millimeters length. Seventeen equally spaced measurement points were marked on the beam. The test was carried out in the frequency range of 0 to 1600 Hz. A B&K 8202 hammer with metal tip was used to excite the structure and acceleration response was measured by a PCB 356B08 accelerometer. An FFT analyzer and modal analysis software used to carry out and process the frequency response functions were B&K Portable PULSE 3560D and ICATS respectively. Figure (16) shows the test setup and instrumentations.

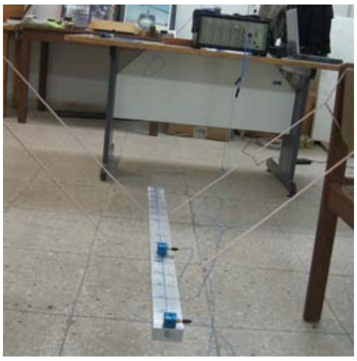

Figure 16: experimental test setup

Table (2) describes the damage cases considered. All of the damage cuts are on node 7 with 8mm width.

Table (2): Modal test cases

Case	Depth of damage cut (mm)
A0	No damage
A1	10
A2	14
A3	18

Multi-FRF Global-M method has been used to extract modal parameters. To avoid errors due to material and geometry discrepancies, all tests were made on the same beam. The first three elastic extracted modes from experimental data are depicted in figures (17) to (19).

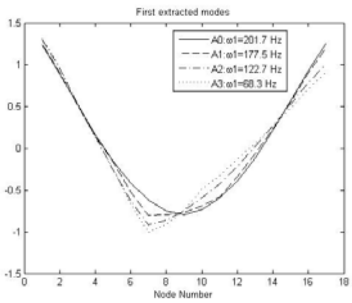

Figure 17: First modes of experimental cases

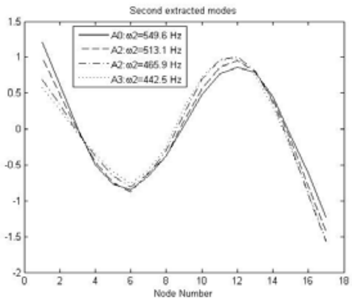

Figure 18: Second modes of experimental cases

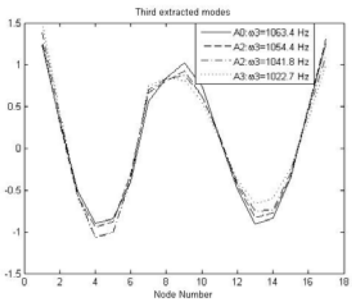

Figure 19: Third modes of experimental cases

Flexibility matrix for each damaged case was calculated using equation (2) and then curvature and GSM approaches implemented on them. Figures (20) to (22) show the results of curvature method.

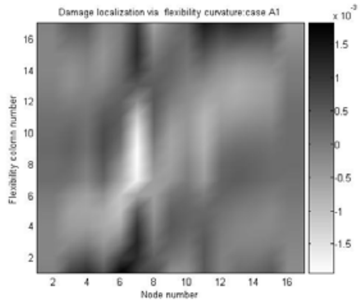

Figure 20: curvature of flexibility shapes: case A1

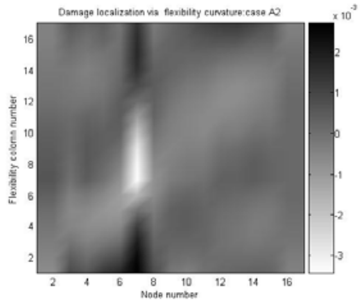

Figure 21: curvature of flexibility shapes: case A2

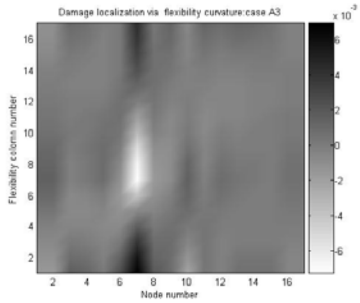

Figure 22: curvature of flexibility shapes: case A3

GSM was implemented on experimental flexibility shapes. Figures (23) to (25) show the results.

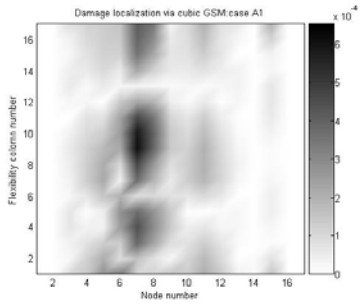

Figure 23: Damage localization via GSM: case A1

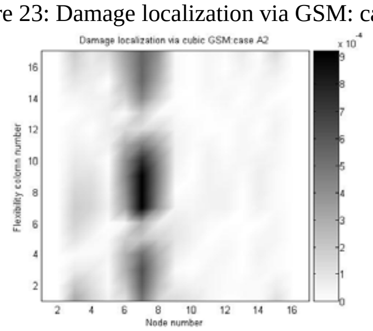

Figure 24: Damage localization via GSM: case A2

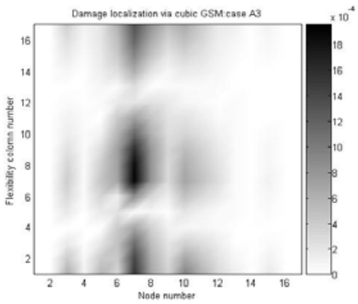

Figure 25: Damage localization via GSM: case A3

The results show that the damage location has been well localized by both methods. GSM seems to work better especially for the less severe damage case A1. As can be seen from figures (20) to (25), flexibility columns 4, 5, 13, 14 are less damage indicative. The reason explained in the numerical case study in terms of sensitivity of damage indication to location of damage. Calculating curvatures of three extracted modes in healthy case reveals that damaged site, node 7, have a curvature near zero for third mode. Figure (27) shows dependency factors of flexibility shapes to mode shapes in terms of modal assurance criterion formulation [12]:

$$MAC\left(F(:,i),\varphi_j\right)=\frac{\left|\{F(:,i)\}^T\{\varphi_j\}\right|^2}{\left(\{F(:,i)\}^T\{F(:,i)\}\right)\left(\{\varphi_j\}^T\{\varphi_j\}\right)} \qquad (7)$$

Where $F(:,i)$ and φ_j are i th flexibility shape and j th mode shape respectively. Higher MAC between a flexibility shape and a mode shape can be concluded as major contribution of that mode shape in the flexibility shape.

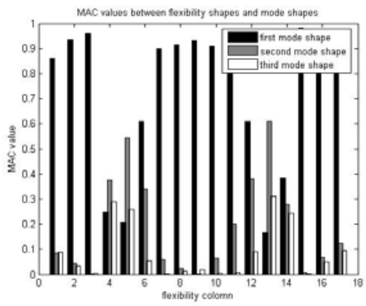

Figure 27: MAC values between flexibility shapes and mode shapes

As expected, flexibility shape numbers 4,5,13 and 14 have high contribution of third mode whose curvature is near zero at damaged zone. As a result, aforementioned flexibility shapes are less indicative of damage near node 7.

Conclusions

It was shown that the damage can be localized using a few number of measured mode shapes and natural frequencies using the proposed method and no more theoretical model and baseline information about intact state of the structure is required. The DQM has been shown to increase the differentiation accuracy in calculating curvatures. Two kinds of linear and cubic GSM methods have also been implemented to locate the irregularity caused by damage in structural flexibility shapes. Although cubic GSM is of higher accuracy, linear GSM restricts the damage site to a narrower spatial interval. The method has been verified via numerical as well as experimental case studies. In both cases, the results are satisfactory. The sensitivity of the method to damage location is also investigated numerically. It was found that if damage falls in regions of higher curvature value of measured modes, the damage localization method would be more efficient. The proposed method can be applied for damage detection in real life uniform and homogenous beam-like structures.

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