

## Modeling the Electric Field Emission of Switch Mode Power Supplies Using Magnetic Near-Field Measurement and Method of Moments

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### ABSTRACT

Electromagnetic Compatibility (EMC) is one of the basic considerations in SMPS design. In this paper, we propose a combined method of moments – magnetic near field measurement model for efficient determination of radiated electric field due to SMPSs mounted on printed circuit boards (PCBs). The proposed model involves four steps. First, the magnetic field distribution on an arbitrary rectangular plan in the vicinity of the board is measured. Then we calculate normal electric field base on the measured magnetic fields. In the third step calculated electric field used to identify the equivalent source currents on the surface of the radiating board, using the method of moments. Having determined the radiating sources, electric field distributions are then estimated at an arbitrary distance from the PCB. The main feature of the proposed technique is its relatively high computation speed for inverting near-field data to unknown equivalent magnetic current sources as compared to the conventional method where time-consuming heuristic search algorithms are used. The validity of the proposed technique is demonstrated by comparing the actual and predicted electric far-field radiations due to a small electrical dipole simulating conduction paths and heat sinks mounted on a typical PCB with SMPS.

**KEYWORDS:** Electromagnetic Compatibility; Switch Mode Power Supply; Near field Measurement; Method of Moments.

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### INTRODUCTION

Electromagnetic interference due switching mode power supplies (SMPSs) is a great concern for the designers of electrical and electronic equipment. With the increase of switching frequency of electronic switches (MOSFET, IGBT and etc.), SMPSs are found to be more and more prone to electromagnetic radiated emission. Electromagnetic compatibility (EMC) regulations applied to SMPSs are aimed to reduce conducted and radiated emissions in the design stage. SMPSs are commonly tested in a semi-anechoic chamber (SAC) or open area test site (OATS) for assessment of their compatibility with the EMC standards. If the EMC tests fail, designers must modify their design and repeat the test. This process is very time-consuming and expensive, and hence, designers seek more efficient methods to predict radiated emission form SMPSs and their related components in the early design stages. It is worth noting that SMPSs require high frequency conduction paths and heat sinks, radiating strong electric fields.

In the recent years, several approaches have been adopted to solve the problem addressed above for magnetic field radiation [1-2], but there are a few studies about electric field radiation. In [3], a finite-difference time-domain method is adopted to examine the electromagnetic resonant effects of various types of heat sinks which are commonly used in SMPSs. Recommendations are proposed for optimal selection of heat sinks and the placement of components to mitigate potential EMC effects. This approach is based on the use of numerical methods. Identification of voltage excitation and modeling all SMPS components in this approach is relatively complicated and cumbersome, making it unattractive for treating a complex SMPS. In an alternative method, the measured near-field (NF) data are directly transformed to the far field (FF) radiations using modal expansions [4]. While this method is very useful in antenna design, its application to radiated emission modeling is limited due to the lack of appropriate representation of the radiating sources. Another method involves determination of equivalent magnetic current [5] or electric current [6] sources from the NF scanning measurements. The radiated fields in this method can be calculated directly from the equivalent sources in free space. Pairs of electric and magnetic dipoles have also been used as equivalent sources that create the same NF components as the original sources [7-8]. Once the equivalent dipole set is found, the FF radiations can be easily computed analytically. The main drawback in this method is the need for a time-consuming heuristic search algorithm based on evolution strategies.

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In this paper, we propose an efficient algorithm to replace the radiating SMPS with equivalent magnetic current sources, using the NF data. For this purpose, we use the method of moments to create a set of linear equation which relates the NF data to the equivalent magnetic current sources. As opposed to the conventional heuristic search algorithm, the proposed method should drastically reduce the computation time required for determination of the equivalent sources. Having determined the equivalent current sources, the FF electromagnetic radiations can be readily obtained. To avoid difficulties in the measurement of near electric field, we describe a numerical method that produces the required electric field data from the respective measured magnetic field data.

The manuscript is organized as follows. First, the theory and basic formulation of the proposed method are presented. Next, the governing integral equation are derived, relating the near electric field data to the equivalent magnetic current sources. The method of moments will be then used for computation of the unknown equivalent magnetic currents, followed by a numerical method that produces the required electric field data from the respective measured magnetic field data. Finally, various simulations are presented to examine the validity of the proposed technique.

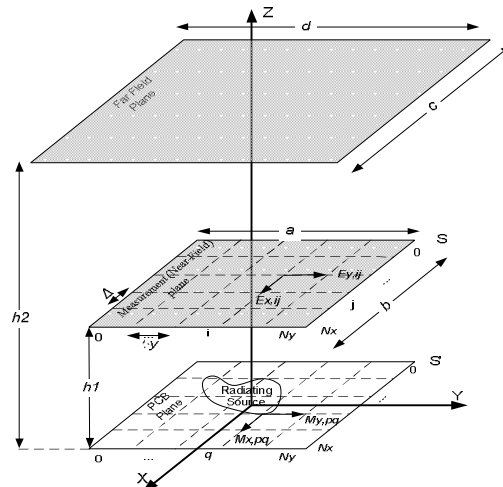
### Theoretical Basics

The geometry of the problem is shown in Fig. 1. As shown in this figure, a typical PCB of dimensions  $a \times b$  is represented by radiating current sources located at  $z = 0$ . It is assumed that the PCB contains a SMPS together with related components, including several conduction paths and heat sinks among other electronic devices. The SMPS is operating at frequency  $f$  large enough to cause related components to radiate. The measurement (near field) plane is a rectangular plane of dimensions  $a \times b$  parallel to PCB at distance  $h_1$ . We aim to predict radiations on far-field plane which is a rectangular plane of dimensions  $c \times d$  parallel to PCB at distance  $h_2$ .

To determine the far electric field ( $z=h_2$  in Fig. 1) distribution in the problem posed above, we adopt the electromagnetic equivalence principle [10].

According to the equivalence principle, a given set of sources bounded within a closed surface  $S$  can be characterized by equivalent electric currents ( $J$ ) and magnetic currents ( $M$ ) distributed on the surface  $S$  that encloses the original sources such that the generated fields outside the surface containing the sources are the same in both the original and the equivalent problem. We can use a rectangular plane above radiating source to measure electric and magnetic fields instead of closed surface but this surface must be large enough to ensure that most of the radiated energy is going through the measurement plane, Fig.1.

A flow chart of the proposed method is shown in Fig. 2. As seen in this figure, the measured tangential magnetic field components on the NF plane are first used to compute the respective normal component of the electric field. The computed electric field data are then used to obtain the equivalent magnetic current sources at the PCB plane, using the method-of-moments (MoM). Having determined the equivalent current sources, the FF radiations in free space are readily predicted in a common manner.



**Fig. 1** Geometry of the problem, comprising the plane of radiating current sources at the location of PCB ( $z=0$ ), the measurement plane in the near field zone ( $z=h_1$ ) and the far-field plane ( $z=h_2$ ) with unknown magnetic field distribution.

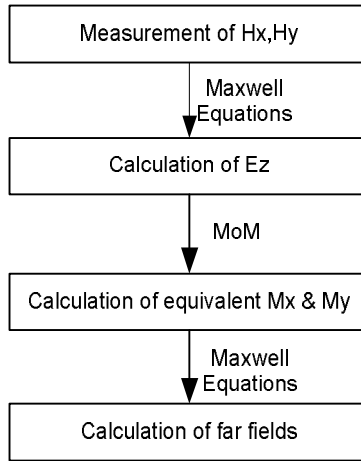


Fig. 2 A flowchart of the proposed method.

**Derivation of the Integral Equation**

In this section the mathematical formulation for calculation of equivalent current based on the normal electric near fields is presented. First, we develop the electric integral equation which relates electric near field with equivalent magnetic currents. Then we use MOM to translate the integral equation statement into system of linear equations.

The total electric field generated in free space by an arbitrary distribution of electric currents is given as follow [9]:

$$\vec{E} = -\frac{1}{\epsilon_0} \nabla \times \vec{F} \tag{1}$$

Where  $\epsilon_0$  is the permittivity of free space and  $\vec{F}$  is the auxiliary electric vector potential,

$$\vec{F} = \frac{\epsilon_0}{4\pi} \iint_{S'} \vec{M}_{eq} \frac{e^{-jkR}}{R} ds' \tag{2}$$

Here,  $R = \sqrt{(x - x')^2 + (y - y')^2 + (z - z')^2}$  is the distance between the source point  $(x',y',z')$  and the observation point  $(x,y,z)$ ,  $k$  is the propagation constant, and  $S'$  is the source surface.

By expansion of (1) and (2) in the Cartesian coordinates, the values of normal electric field on the near-field plane,  $E_z$ , can be determined as follow:

$$E_z = -\frac{1}{4\pi} \iint_{S'} [(y - y')M_x - (x - x')M_y] \frac{1 + jkR}{R^3} e^{-jkR} dx' dy' \tag{3}$$

where  $M_x$  and  $M_y$  represent, respectively, the x- and y- components of the unknown equivalent current density ( $M_{eq}$ ) on the PCB plane,  $S'$ .

**Solution by the Method of Moments**

To determine  $M_x$  and  $M_y$  in (3), we use the method-of-moments [10]. This is done by expanding  $M_{eq}$  in terms of appropriate basis functions in subdomains formed by discretizing  $S'$  in  $N_x$  and  $N_y$  segments along the x- and y-axes, respectively. The use of pulse basis function with constant amplitude and phase, we have

$$\vec{M}_{eq} = \sum_{p=1}^{N_x} \sum_{q=1}^{N_y} (M_{pqx} \hat{x} + M_{pqy} \hat{y}) \Pi(x - x_p, y - y_q) \tag{4}$$

where

$$\Pi(x, y) = \begin{cases} 1 & |x| < \frac{\Delta x}{2}, |y| < \frac{\Delta y}{2} \\ 0 & |x| > \frac{\Delta x}{2}, |y| > \frac{\Delta y}{2} \end{cases}$$

$$x_p = p \Delta x - \frac{\Delta x}{2}$$

$$y_q = q \Delta y - \frac{\Delta y}{2}$$

and  $M_{pqx}$  and  $M_{pqy}$  are, respectively, the unknown coefficients associated with the x- and y- components of  $M_{eq}$  on subdomain pq.

Substituting (4) into, the integral equations relating the electric fields to their equivalent magnetic currents lead to a system of linear equations where the number of equations is equal to the number of measurement points, i.e.,

$$E_{zij} = \sum_{p=0}^{N_x} \sum_{k=0}^{N_y} -\frac{1}{4\pi} [(y - y') M_{xpq} - (x - x') M_{ypq}] \frac{1 + jkR_{pq}}{R_{pq}^3} e^{-j\beta R_{pq}} \Delta x \Delta y \tag{5}$$

Or, in matrix form,

$$[E_z] = [Y_{E_z, M_x} \quad Y_{E_z, M_y}] \begin{bmatrix} M_x \\ M_y \end{bmatrix} \tag{6}$$

where

$$Y_{E_z, M_x}(i, j) = -\frac{1}{4\pi} (y - y') \frac{1 + jkR_{ij}}{R_{ij}^3} e^{-jkR_{ij}} \Delta x \Delta y$$

and

$$Y_{E_z, M_y}(i, j) = \frac{1}{4\pi} (x - x') \frac{1 + jkR_{ij}}{R_{ij}^3} e^{-jkR_{ij}} \Delta x \Delta y$$

Since  $Y_{E_z, M_x}$  and  $Y_{E_z, M_y}$  are large and sparse, the factorization methods are generally not efficient for solving (6).

Instead, we choose to use an iterative method such as the least square residual (LSQR) method to treat the problem [11].

Having determined the unknown current sources on the PCB, the radiated far-field electric field can be readily obtained.

**Calculation of Electric Field from the Measured Magnetic field**

Because of difficulty and inaccuracy in measurement of tangential electric field, we prefer to measure magnetic field instead of electric field. In this section, we proposed a method to calculate the normal component if the electric field from the measured tangential magnetic field components. For calculation of tangential electric field, we use the Maxwell equations as follow:

$$E_z = \frac{1}{j\omega\epsilon_0} \left( \frac{\partial H_x}{\partial y} - \frac{\partial H_y}{\partial x} \right) \tag{7}$$

Discrete form of above equation is as follow:

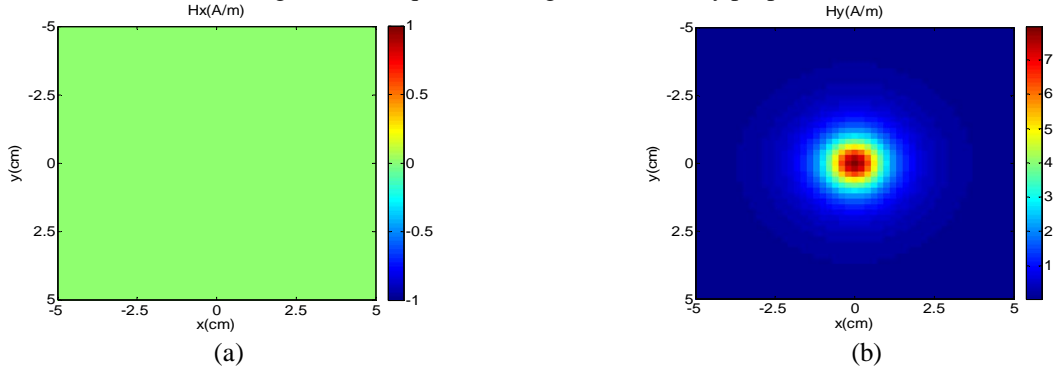
$$E_z(i, j, k) = \frac{1}{j\omega\epsilon_0} [(H_x(i, j + 1, k) - H_x(i, j, k)) - (H_y(i + 1, j, k) - H_y(i, j, k))] \tag{8}$$

According to these equations, normal electric field can be calculated by measurement of magnetic field on the measurement plane (See Fig.1).

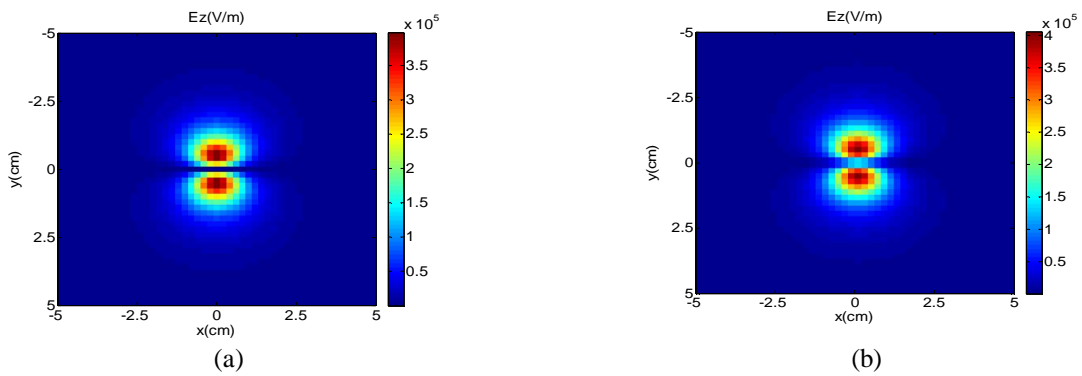
**Numerical Simulations**

In order to demonstrate the robustness of the model, simulated results are presented. In these simulations, the source is a small electrical dipole with length  $l=10\text{mm}$  placed at the location of the PCB ( $h=0$  in Fig. 1) as a radiating source. The dipole is excited with a 1A sinusoidal current source of frequency 30MHz.

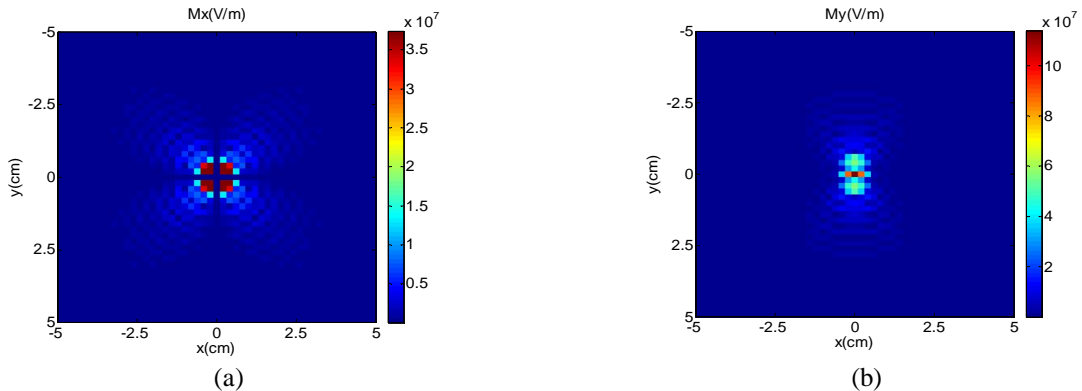
Fig. 3 shows variations of the magnetic field ( $H_x$  and  $H_y$ ) produced by the small electrical dipole on the near-field plane 1 ( $h_f=10\text{mm}$ ,  $a=100\text{mm}$  and  $b=100\text{mm}$  in Fig.1.) [8]. Field calculation are performed on  $N=N_x \times N_y=2601$  points at an equally space  $\Delta x = \Delta y = 2\text{mm}$ . Fig. 4 compare the actual value of  $E_z$  and calculated value of  $E_z$  by discret form of Maxwell evacuations. Fig. 5 shows equivalent magnetic current by proposed method.



**Fig. 3** Variations of magnetic field data on the near field plane (Fig. 1). (a)  $H_x$  and (b)  $H_y$ .



**Fig. 4** Variations of the normal electric field on the near field plane (Fig. 1) extracted from based on the respective tangential magnetic field data ; (a) actual values of  $E_z$  and (b) calculated values of  $E_z$ .



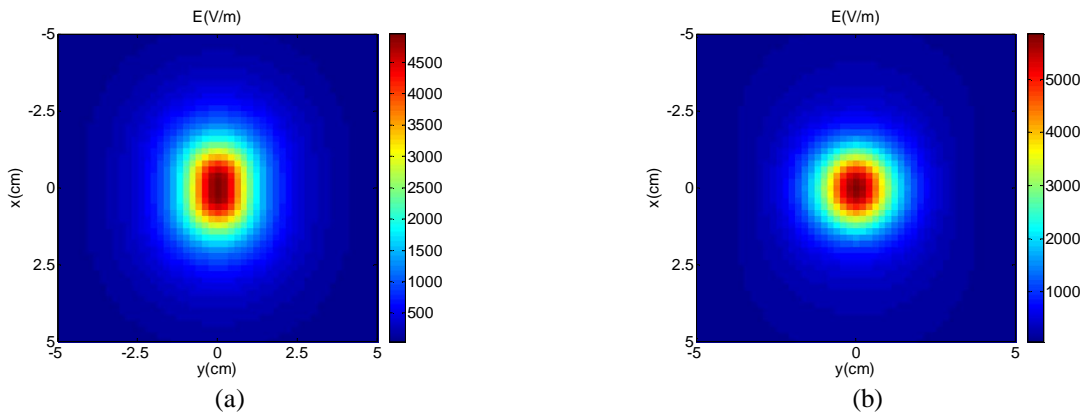
**Fig. 5** Distribution of the predicted equivalent magnetic currents on the PCB plane (Fig. 1); (a)  $M_x$  and (b)  $M_y$ .

To show the effect of measurement noise on the simulated data, the values of  $H_x, H_y$  on the near-field plane are superposed by Gaussian noise with various signal-to-noise ratios (SNRs). The noisy data are then used to calculate normal electric field which is needed as input measurement entries to the proposed model, producing equivalent magnetic current distribution on the PCB. The equivalent current distribution is then used to compute the field distribution on the far-field plane ( $h_2 = 50\text{mm}$ ,  $c=400\text{mm}$  and  $d=400\text{mm}$ ). A comparison of the actual and reconstructed results for SNR=30dB shown in Figs. 6 (a) and (b), respectively, demonstrates the validity of the proposed model. The results indicate that the model is capable of accurately reconstructing electric field distributions at far-field regions. A quantitative comparison of these results can also be found in Table. 1 where the mean-square deviation, MSD, between the actual magnitude of electric field  $E_n (n = 1, 2, \dots, N)$ , and its reconstructed counterpart,  $\hat{E}_n (n = 1, 2, \dots, N)$  in all cases are given

$$MSD = \frac{\sum_{n=1}^N (E_n - \hat{E}_n)^2}{\sum_{n=1}^N E_n^2} \tag{10}$$

**Table. 1** Values of MSD for various SNR

	SNR=30dB	SNR=20dB	SNR=10dB
MSD	0.031	0.172	0.457



**Fig. 6** Variations of the magnitude of electric field on the far-field plane (Fig. 1); (a) actual values and (b) predicted values.

**Conclusion**

A modeling technique, based on the electromagnetic equivalence principle, is proposed to determine radiated electric field from switch-mode power supplies on a typical printed circuit board (PCB). The proposed technique first utilizes the tangential magnetic field measured on a given rectangular plan in the vicinity of the board to identify the radiating current sources on the PCB by the method-of-moments. Having determined the radiating sources, the electric field distribution at a desired distance from the PCB is then computed, using the governing integral equations in free space. The main feature of the proposed technique is its direct approach for reconstruction of the radiating current sources which makes it much faster than the conventional techniques, involving heuristic optimization algorithms. It is shown that the proposed technique is readily converged for a wide range of SNR on the field measurement plane.

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