

## Determination of aggregated load power consumption, under non-sinusoidal supply using an improved load model

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### ABSTRACT

The harmonic content of supply voltage results in additional power losses and hence increases the load power consumption. The role of the power quality equipments on the power consumption without using an accurate model cannot be determined, too.

In this paper, an improved model for aggregated loads proposed, which estimates the effects of voltage harmonics on the power consumption. The distinguished aspect of the proposed model is its parameters identification method which is based on the practical techniques, such as employing a capacitor bank or varying dummy loads in steps.

The proposed model has been verified by the comparison of measured and simulated results.

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### 1. Introduction

The most of electrical loads are designed to have their best efficiency in the nominal sinusoidal supply voltages. Distorted and deviated voltages lead to changes in the power and energy consumption of loads. The estimation of voltage harmonics effects on the power consumption, illustrates the cost aspects of the consumer. It can also be an evaluation of the economical benefit for utilizing the custom power equipments and passive filters.

For example, electrical motors are typical loads which their efficiency are extremely affected by voltage harmonics. In 1998, Lee [1] has studied the effect of fundamental component and various harmonics on temperature increase and efficiency decrease of electrical motors. In 2000, Lee [2] has studied the effect of harmonic voltage phase angle on motors. The test results show that in addition to magnitude and harmonic order, the harmonics phase angle affects the current, power factor and efficiency of the motor, too. It has been shown that for some special harmonics with specified phases, the input current of motor is less than the nominal and the efficiency of motor is more than the nominal efficiency. Also it has been shown that some special harmonics with specified phases have less effect on the efficiency, the power factor and the temperature increase of the motor.

According to NEMA, the Harmonic Voltage Factor (HVF) is defined as follows:

$$HVF = \sqrt{\sum_{n=5}^{n=\infty} \frac{V_n^2}{n}} \quad (1)$$

where  $V_n$  is the amplitude of  $n$ th voltage harmonic and  $n$  is the order of harmonic. In ref. [3], it is shown that by increasing HVF, motor capacity decreases.

In 2000, Jalilian has analysed the induction motor efficiency under voltage harmonics [4]. In this paper, it is shown that the THD parameter is insufficient for the determination of losses. A new parameter which is called Weighted THD is defined in ref. [4] as follows:

$$WTHD = \sqrt{\sum_{n=2}^{n=\infty} V_n^2 n^{-0.8}} \quad (2)$$

In this parameter, the order of each harmonic has been considered and the effect of the lower order harmonics on increasing the power losses has been shown. Furthermore, it has been shown that the capacity of motors decreases proportionally with WTHD.

Electrical transformers are designed to provide the nominal power in fundamental frequency. Harmonic current distortion increases the losses in transformers [5,6]. Furthermore, voltage harmonics decrease the efficiency of single phase equipments, power electronic equipments and power distribution networks [7–10].

In most cases, loads consist of electrical motors, power electronic equipments, linear and non-linear loads which have unknown sensitivity to the voltage harmonics.

One of the common models which can estimate the relation between the load power consumption and the supply voltage is ZIP model [11]. This model assumes that the load has three parts; constant power, constant current and constant impedance parts. But this model is unable to take the harmonics effect into account.

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The conventional method for harmonic loads modelling is the Norton method which shows the relation between loads consumed power and voltage harmonics [12]. In this model, the load has been considered as a constant current source and parallel impedance. In this method, voltage harmonics are considered in the load consumed power but load harmonic currents are related only to the supply harmonic voltages of the same order.

This method has an error which has been described by Zhao et al. [12]. In the method of Zhao et al. [12], the load has been considered as a combination of linear and non-linear loads. For a linear load, harmonic currents of load are related only to the supply voltage harmonics of the same order but for a non-linear load, harmonic currents of load are the function of supply voltage harmonics of all orders. In this method with the assumption of small variations in the amplitude of the fundamental voltage and voltage harmonics, non-linear equations become linear around the operating point. Based on this model, the load consumed power is determined as a function of voltage harmonics. The model proposed by Zhao et al. [12], has a practical demerit, in this method the identification of many parameters in real circumstances is very difficult. This problem is overcome by a new concept presented in the present paper. In this concept, priority levels are given to the model parameters, and hence, considerable numbers of these parameters can be cancelled. By this method, the model parameters with high levels of priorities can be easily determined in practical circumstances.

## 2. Load model determination

In the model presented in ref. [12], voltage and current harmonics in each phase has been presented by the following equation:

$$\begin{aligned} v(t) &= \sum_{h=1}^{\infty} \sqrt{2} V_h \sin(2\pi h t / T + \theta_h) \\ i(t) &= \sum_{h=1}^{\infty} \sqrt{2} I_h \sin(2\pi h t / T + \phi_h) \quad h = 1, 2, 3, \dots, n \end{aligned} \quad (3)$$

So real and imaginary parts of voltage and current harmonics are as follows:

$$\begin{aligned} V_{hr} + jV_{hi} &= V_h \angle \theta_h \\ I_{hr} + jI_{hi} &= I_h \angle \phi_h, \quad h = 1, 2, 3, \dots, n \end{aligned} \quad (4)$$

Based on this consideration, the model of the load for each harmonic order consists of three components; a constant impedance, a constant current source and a harmonic voltage dependent source. On the other hand, we have

$$\begin{aligned} I_{Lhr} &= I_{0hr} + F_{hr}(V_1, V_2, V_3, \dots) + (Y_{hr} V_{hr} - Y_{hi} V_{hi}) \\ I_{Lhi} &= I_{0hi} + F_{hi}(V_1, V_2, V_3, \dots) + (Y_{hi} V_{hr} + Y_{hr} V_{hi}) \end{aligned} \quad (5)$$

where  $I_{0hr}$  is the  $h$ th order real constant current,  $I_{0hi}$  is the  $h$ th order imaginary constant current,  $F_{hr}(V_1, V_2, V_3, \dots)$ ,  $F_{hi}(V_1, V_2, V_3, \dots)$  is the real and imaginary non-linear functions of voltages,  $Y_{hr}$  is the real part of impedance in  $h$ th harmonic and  $Y_{hi}$  is the imaginary part of impedance in  $h$ th harmonic.

The variation of voltage harmonics is often less than 5%; as a result the non-linear function can become linear. Therefore, harmonic functions ( $F_{hr}$  and  $F_{hi}$ ) can be simplified as follows:

$$\begin{aligned} F_{hr}(V_1, V_2, V_3, \dots) &= a_1 V_{1r} + a_2 V_{2r} + a_3 V_{2i} + \dots + a_{2k-2} V_{kr} + a_{2k-1} V_{ki} \\ F_{hi}(V_1, V_2, V_3, \dots) &= b_1 V_{1r} + b_2 V_{2r} + b_3 V_{2i} + \dots + b_{2k-2} V_{kr} + b_{2k-1} V_{ki} \end{aligned} \quad (6)$$

Substituting these equations in Eq. (5) result in the following equations:

$$\begin{aligned} I_{Lhr} &= I_{0hr} + a_1 V_{1r} + \dots + a_{2k-2} V_{kr} + a_{2k-1} V_{ki} + a'_{2h-2} V_{hr} + a'_{2h-1} V_{hi} \\ I_{Lhi} &= I_{0hi} + b_1 V_{1r} + \dots + b_{2k-2} V_{kr} + b_{2k-1} V_{ki} + b'_{2h-2} V_{hr} + b'_{2h-1} V_{hi} \end{aligned} \quad (7)$$

where

$$\begin{aligned} a'_{2h-2} &= Y_{hr} + a_{2h-2} \\ a'_{2h-1} &= a_{2h-1} - Y_{hi} \\ b'_{2h-2} &= Y_{hr} + b_{2h-2} \\ b'_{2h-1} &= b_{2h-1} + Y_{hi} \end{aligned}$$

In order to omit the imaginary part of the fundamental voltage, the fundamental voltage phase angle is assumed to be the reference angle.

Zhao et al. [12] assume that Eq. (7) coefficients can be determined by the uniform variation of harmonic voltage parameters between  $-5\%$  and  $+5\%$ . Then, the value of the current for all combination of voltage harmonic parameters must be calculated and the coefficient can be obtained by the mean square method.

The main disadvantage of this method is the technique which has been used for calculating the coefficient of the model and numerous parameters which has been used. This model needs several simulations to calculate the model parameters.

It is obvious that in practical conditions, it is not possible to change supply voltage in a wide range. In present paper, a new method is proposed for the determination of the load power consumption. The proposed model can be used in practical applications and needs limited numbers of data. Based on this improved model, the effect of harmonic voltages on the load power consumption can be estimated.

Using Eq. (4), the load consumed power can be calculated as follows:

$$\begin{aligned} P &= \sum_{h=1}^n P_h = \sum_{h=1}^n (V_{hr} \cdot I_{hr} + V_{hi} \cdot I_{hi}) \\ &= \sum_{h=1}^n ((V_{hr0} + v_{hr}) \cdot (I_{hr0} + a_{hr1} v_{1r} + \dots + a_{hrn} v_{nr} + a_{hin} v_{ni}) \\ &\quad + (V_{hi0} + v_{hi}) \cdot (I_{hi0} + b_{hr1} v_{1r} + \dots + b_{hrn} v_{nr} + b_{hin} v_{ni})) \\ &= P_0 + \sum_{h=1}^n (c_{hr} \cdot v_{hr} + c_{hi} \cdot v_{hi}) \\ &\quad + \sum_{h=1}^n \sum_{t=1}^n (d_{ht0} v_{hr} v_{tr} + d_{ht1} v_{hr} v_{ti} + d_{ht2} v_{hi} v_{tr} + d_{ht3} v_{hi} v_{ti}) \\ &\quad + \sum_{h=1}^n (g_{hr} \cdot v_{hr}^2 + g_{hi} \cdot v_{hi}^2) \end{aligned} \quad (8)$$

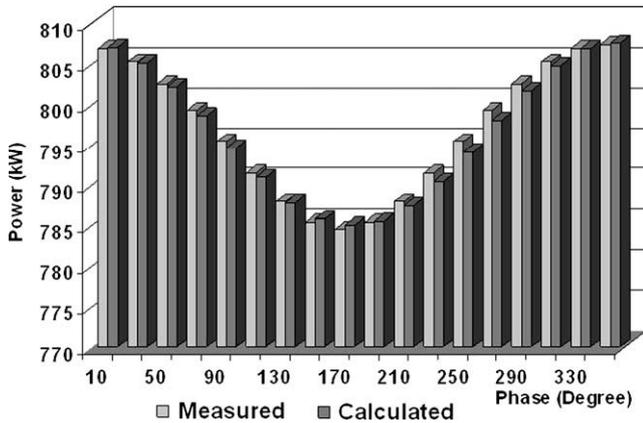
As a case study, an aggregated load which includes a linear load and a three phase uncontrolled rectifier with resistive load has been considered. In this paper, the coefficient of the model can be determined by changing supply voltage harmonics in several steps and using mean square method. Table 1 shows the coefficients of Eq. (8) for the case study. It is assumed that there are no third and even harmonics in supply voltage and the coefficients of these harmonics are assumed to be zero and other parameters which are less than 0.001 have been omitted.

This model shows the relation between loads consumed power and supply harmonic voltages. Also it shows the harmonic condition and voltage level which has minimum power consumption. Fig. 1 shows the variations of actual measured power and estimated power by the model. In this figure, the phase angle of 5th harmonic voltage has been changed from  $0^\circ$  to  $360^\circ$  and its amplitude is set to the 5% of the amplitude.

Fig. 1 shows that by changing the phase angle of supply harmonic voltage, the consumed power can change between 784 kW and 807 kW.

**Table 1**  
Coefficient of power consumption model for uncontrolled rectifier with resistive load and linear load.

Coefficients W & W/V	Value	Coefficient W/V <sup>2</sup>	Value	Coefficient W/V <sup>2</sup>	Value
$P_0$	794.6	$V_{1r}^2$	0.003	$V_{1r} V_{5r}$	0.001
$V_{1r}$	2.960	$V_{5r}^2$	0.002	$V_{5r} V_{7r}$	-0.001
$V_{5r}$	0.423	$V_{5i}^2$	0.004	$V_{5i} V_{7i}$	0.002
$V_{5i}$	0.000	$V_{7r}^2$	0.002	$V_{5i} V_{11i}$	0.003
$V_{7r}$	0.210	$V_{7i}^2$	0.003	$V_{5i} V_{13i}$	0.002
$V_{7i}$	0.000	$V_{11r}^2$	0.002	$V_{7r} V_{11i}$	0.001
$V_{11r}$	0.163	$V_{11i}^2$	0.004	$V_{7i} V_{11i}$	0.004
$V_{11i}$	0.000	$V_{13r}^2$	0.002	$V_{7i} V_{13i}$	0.003
$V_{13r}$	0.114	$V_{13i}^2$	0.003	$V_{11r} V_{13r}$	-0.001
$V_{13i}$	0.001			$V_{11i} V_{13i}$	0.001



**Fig. 1.** Variation of power consumption for uncontrolled rectifier and linear load vs. phase angle of fifth harmonic.

One of the useful applications of this model is the estimation of effects of power quality compensators on the energy consumption.

**3. Proposed model**

It is obvious that in the practical conditions, the variation of supply voltage has a limited range. In the power systems, the voltage supply can be changed by the switching of capacitive bank in steps or by changing a dummy load which has been installed in the system. In practical conditions, the number of capacitive bank steps is not more than 20 steps. Therefore the number of param-

eters which should describe the model must be less than the number of equations. In ref. [12], there is no limitation on the number of harmonics. For example, if we consider 32 harmonics in the model, the number of parameters, which describe the model, will be  $(64 + 32^2)$  and it is clear that when the number of equations is 16, it is not possible to determine all parameters and the number of parameters should be reduced.

To analyse the proposed model and the parameter identification in practical conditions (by means of switching capacitive bank in steps) a speed control drive of DC motor with a linear load has been simulated.

Fig. 2 shows this simulated system by Simulink toolbox. The capacitive bank has 15 steps and each step is equal to 50 kVAR. By entrance of each capacitive bank step, because of absorption of harmonic current and reactive power, the voltage of the load is changed. This change leads to the change of the load current.

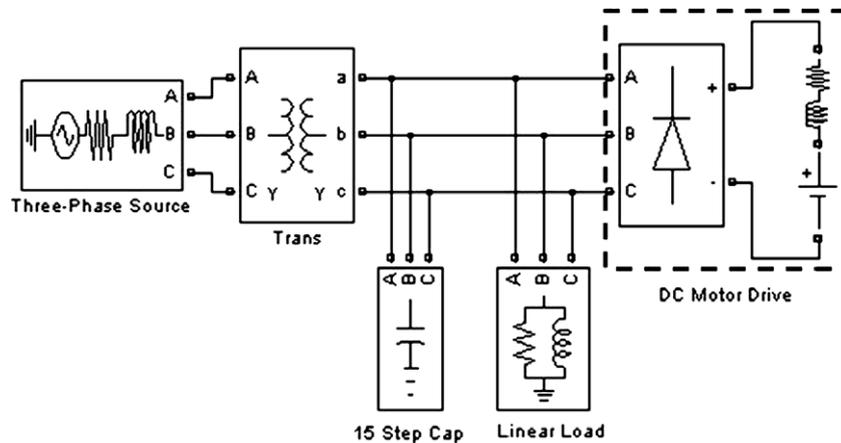
In practical conditions, the variation of the real part and the imaginary part of different harmonics should not be the same. For example, the real part of the 5th voltage harmonic may have a wide range of variation than the imaginary part of the 13th voltage harmonic. Table 2 shows the variance of different harmonic voltage parameters when all 15 capacitive bank steps has been switched. It is obvious that the variation ranges of some parameters are less than the others.

The same condition can be seen for second order parameters. In these conditions, parameters with wider variation range have more effects on the model than other parameters. For example, even harmonics and third harmonic have fewer effects on the model than 5th harmonic.

The variation of different parameters for this model shows that even harmonics, high order odd harmonics and odd harmonics with the order higher than the seven are negligible in comparison with the other harmonics.

**Table 2**  
Variance of different parameters.

$V_{1r}$	$V_{2r}$	$V_{2i}$	$V_{3r}$	$V_{3i}$	$V_{4r}$	$V_{4i}$
42.0	0.1	0.1	0.3	0.3	0.1	0.1
$V_{5r}$	$V_{5i}$	$V_{6r}$	$V_{6i}$	$V_{7r}$	$V_{7i}$	$V_{8r}$
7.5	71.8	0.1	0.1	44.	28.9	0.1
$V_{8i}$	$V_{9r}$	$V_{9i}$	$V_{10r}$	$V_{10i}$	$V_{11r}$	$V_{11i}$
0.2	0.4	0.4	0.1	0.1	1.8	1.8
$V_{12r}$	$V_{12i}$	$V_{13r}$	$V_{13i}$	$V_{14r}$	$V_{14i}$	$V_{15r}$
0.1	0.1	1	1	0	0.1	0.5
$V_{15i}$	$V_{16r}$	$V_{16i}$	$V_{17r}$	$V_{17i}$	$V_{18r}$	$V_{18i}$
0.5	0.4	0.4	0.3	4.4	0.1	0.1



**Fig. 2.** Simulated speed control drive of DC motor in parallel with a linear load.

**Table 3**  
Coefficient of proposed models for power consumption of DC motor speed control unit and linear load.

$P_0$ (W)	$V_1$ (W/V)	$V_{5i}$ (W/V)	$V_{11r}$ (W/V)	$V_{5r}$ (W/V)	$V_{13r}$ (W/V)	$V_{13i}$ (W/V)	$V_{11i}$ (W/V)	$V_{7i}$ (W/V)	$V_{7r}$ (W/V)
948.88	2.79	0.16	0.04	-0.03	0.03	-0.01	-0.01	0.00	0.01
948.88	2.79	0.16	0.04	-0.03	0.03	-0.01	-0.01	0.01	0
948.88	2.79	0.16	0.04	-0.03	0.03	-0.01	-0.01	0	0
948.88	2.79	0.16	0.04	-0.03	0.03	-0.01	0	0	0
948.89	2.79	0.16	0.04	-0.03	0.03	0	0	0	0
948.89	2.79	0.16	0.04	-0.03	0	0	0	0	0
948.89	2.79	0.16	0.04	0	0	0	0	0	0
948.90	2.79	0.16	0	0	0	0	0	0	0
948.99	2.79	0	0	0	0	0	0	0	0
950.91	0	0	0	0	0	0	0	0	0

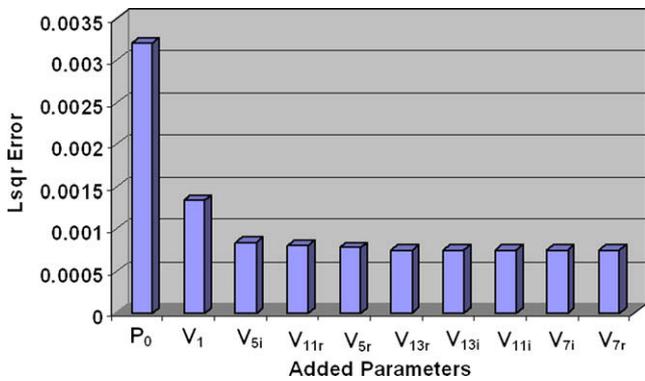


Fig. 3. Error reduction pattern in proposed models.

In the next step, if the approximated model of the load is known, then the priority of parameters can be determined by the simulation.

The coefficients of the model which describe the power consumption of Fig. 2 can be determined by this method. By changing the different harmonic voltages uniformly between -5% and +5%, the consumed power can be determined based on the simulations. The most important parameters can be determined by choosing one parameter and obtaining the model parameters with the least square method. The model which has the minimum least square error between all of one parameter models is the best one.

Also, based on this method, the best models with two, three and more parameters can be determined.

Table 3 shows the prioritization of parameters and calculated coefficients for the best one, two, three and more parameter models. Fig. 3 shows the error of the proposed models. It is obvious that

**Table 4**  
Correlation between different data, obtained by changing normal data.

	$V_{1r}$	$V_{5r}$	$V_{5i}$	$V_{7r}$	$V_{7i}$	$V_{11r}$	$V_{11i}$	$V_{13r}$	$V_{13i}$	$V_{17r}$	$V_{17i}$	$V_{19r}$	$V_{19i}$
$V_{1r}$	1	0.0	0.0	-0.1	-0.1	-0.1	0.1	0.0	-0.1	0.1	0.0	-0.1	0.1
$V_{5r}$	0.0	1	-0.1	0.0	0.0	0.1	-0.1	0.0	-0.1	-0.1	0.0	-0.1	-0.1
$V_{5i}$	0.0	-0.1	1	-0.1	0.1	0.1	0.0	0.0	-0.1	0.0	0.0	-0.1	0.0
$V_{7r}$	-0.1	0.0	-0.1	1	-0.1	0.0	0.0	0.0	-0.1	0.0	0.0	-0.1	0.0
$V_{7i}$	-0.1	0.0	0.1	-0.1	1	0.1	0.0	-0.2	-0.1	0.1	0.0	-0.1	0.1
$V_{11r}$	-0.1	0.1	0.1	0.0	0.1	1	0.0	0.0	0.1	-0.1	0.0	-0.1	-0.1
$V_{11i}$	0.1	-0.1	0.0	0.0	0.0	0.0	1	0.0	0.1	0.0	0.0	-0.1	-0.1
$V_{13r}$	0.0	0.0	0.0	0.0	-0.2	0.0	0.0	1	0.0	0.0	0.0	-0.1	-0.1
$V_{13i}$	-0.1	-0.1	-0.1	-0.1	-0.1	0.1	0.1	0.0	1	-0.5	-0.9	-0.1	-0.1
$V_{17r}$	-0.1	0.0	-0.1	0.0	-0.1	0.0	-0.1	0.0	-0.1	1	0.2	-0.1	-0.1
$V_{17i}$	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	-0.1	1	-0.1	-0.1
$V_{19r}$	-0.1	0.0	-0.1	0.0	-0.1	0.0	-0.1	0.0	-0.1	0.0	-0.1	1	-0.1
$V_{19i}$	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	-0.1	0.0	0.0	1

**Table 5**  
Correlation between different, which is obtained by switching capacitive bank.

	$V_{1r}$	$V_{5r}$	$V_{5i}$	$V_{7r}$	$V_{7i}$	$V_{11r}$	$V_{11i}$	$V_{13r}$	$V_{13i}$	$V_{17r}$	$V_{17i}$	$V_{19r}$	$V_{19i}$
$V_{1r}$	1	-0.2	1.0	0.9	0.6	-0.6	0.0	-0.5	-0.5	0.6	0.5	0.6	0.4
$V_{5r}$	-0.2	1	0.0	0.1	-0.9	0.6	0.4	0.3	0.5	-0.3	-0.5	-0.3	-0.5
$V_{5i}$	1.0	0.0	1	0.9	0.5	-0.5	0.1	-0.5	-0.4	0.6	0.3	0.6	0.3
$V_{7r}$	0.9	0.1	0.9	1	0.3	-0.3	0.4	-0.4	-0.1	0.5	0.1	0.5	0.0
$V_{7i}$	0.6	-0.9	0.5	0.3	1	-0.8	-0.2	-0.6	-0.6	0.6	0.5	0.6	0.5
$V_{11r}$	-0.6	0.6	-0.5	-0.3	-0.8	1	0.2	0.9	0.4	-0.7	-0.2	-0.6	-0.1
$V_{11i}$	0.0	0.4	0.1	0.4	-0.2	0.2	1	0.0	0.8	-0.2	-0.7	-0.3	-0.6
$V_{13r}$	-0.5	0.3	-0.5	-0.4	-0.6	0.9	0.0	1	0.2	-0.8	0.1	-0.7	0.3
$V_{13i}$	-0.5	0.5	-0.4	-0.1	-0.6	0.4	0.8	0.2	1	-0.5	-0.9	-0.6	-0.8
$V_{17r}$	0.6	-0.3	0.6	0.5	0.6	-0.7	-0.2	-0.8	-0.5	1	0.2	0.9	0.0
$V_{17i}$	0.5	-0.5	0.3	0.1	0.5	-0.2	-0.7	0.1	-0.9	0.2	1	0.4	1.0
$V_{19r}$	0.6	-0.3	0.6	0.5	0.6	-0.6	-0.3	-0.7	-0.6	0.9	0.4	1	0.2
$V_{19i}$	0.4	-0.5	0.3	0.0	0.5	-0.1	-0.6	0.3	-0.8	0.0	1.0	0.2	1

**Table 6**  
Obtained parameters for models.

	$P_0$	$V_{1r}$	$V_{5r}$
Model 1	931.9	2.3	-0.86
Model 2	938.7	2.2	0

**Table 7**  
Comparison of actual and estimated constants.

	$P_0$	Error (%)
Model 1	931.9	1.7
Model 2	938.7	1
Real	948.88	0

after 4th parameter, the error reduction is not considerable. As a result these parameters are less important parameters than the others and can be neglected.

Between remained parameters, the real and imaginary part of 7th voltage harmonic can be neglected because of their small effects on the model.

Based on the proposed method, the number of simulation parameters lower than specified values can be reduced. However, there is another difference between simulation and practical con-

ditions. In simulations, used data for model identification, has uniform distribution and there is no statistical correlation between them. However, in practical conditions, data which generated by switching the capacitive bank, is not independent and has a particular pattern. Table 4 shows the correlation coefficients between voltage parameters, used for model identification, and have uniform distribution. Table 5 shows the correlation coefficient between data which is generated by switching the capacitive bank 15 steps. It is obvious that there is considerable correlation between some of this data. For example, the variation of the imaginary part of 5th harmonic is the same as the variation of the real part of the fundamental voltage.

If the used parameters data are dependent, the obtained coefficients are completely different from the results obtained based on the independent parameters data. For example, to estimate the equation  $Z = 4X^3 + 3X + Y^3 + Y + 1$  around  $X = -0.1$  and  $Y = -0.1$ , if we consider  $X$  and  $Y$  with normal distribution and data which is used for identification is independent and has no correlation, the coefficient of the identified model by the least square method is given by Eq. (9). However, if the used data has high correlation (e.g.,  $Y = X$ ) then the identified model by the least square method is given by Eq. (10). As a result, it is important to notice that the coefficient obtained based on dependent data is completely different from coefficients obtained based on independent data.

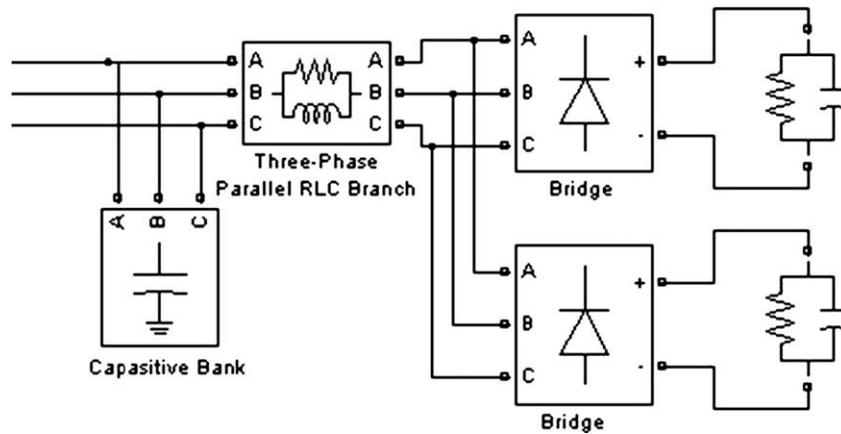


Fig. 4. Circuit diagram of electrolyse system.

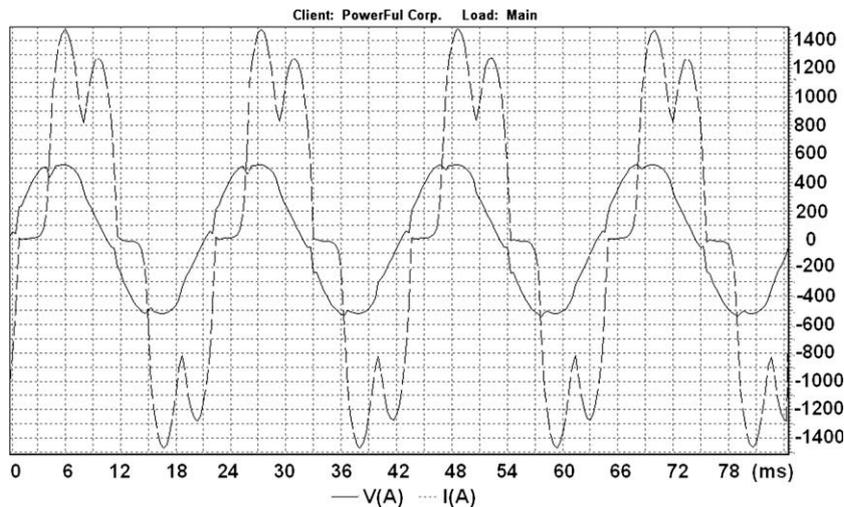
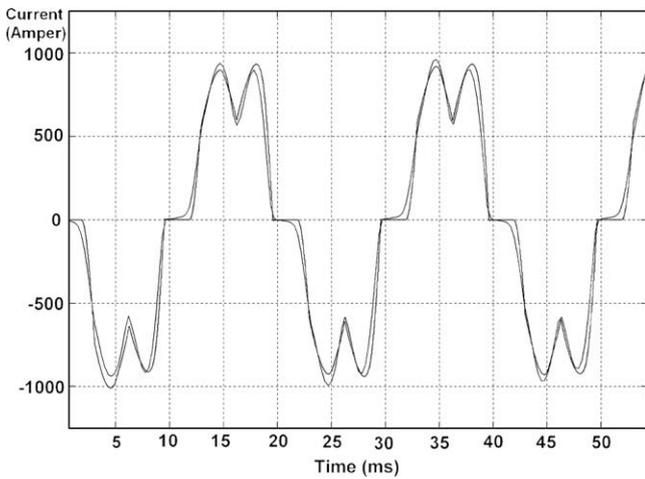


Fig. 5. Voltage and current waveforms, no capacitive bank.

**Table 8**  
Load voltage and power for different steps of capacitor bank.

Capacitor bank step	Voltage contents													Power P (kW)
	V <sub>1r</sub>	V <sub>5r</sub>	V <sub>5i</sub>	V <sub>7r</sub>	V <sub>7i</sub>	V <sub>11r</sub>	V <sub>11i</sub>	V <sub>13r</sub>	V <sub>13i</sub>	V <sub>17r</sub>	V <sub>17i</sub>	V <sub>19r</sub>	V <sub>19i</sub>	
0	519.8	-11.0	8.1	-5.8	3.5	4.0	0.3	3.9	0.4	-2.9	0.0	-2.3	-0.2	339.5
1	521.6	-11.5	8.7	-7.0	3.8	7.6	-0.5	9.0	-2.0	-10.6	11.9	21.8	2.6	343.8
2	523.2	-13.4	10.0	-9.9	6.8	9.5	-12.7	-25.2	-35.7	0.0	0.6	0.6	1.2	346.0
3	524.9	-13.5	14.3	-7.8	11.8	-21.6	-10.2	-3.7	-0.3	1.4	0.3	0.9	0.2	342.2
4	526.7	-14.1	15.9	-9.3	15.2	-6.6	1.4	-2.7	0.1	1.1	-0.1	0.7	-0.1	346.2
5	528.4	-15.1	19.1	-9.3	25.2	-3.1	1.8	-1.6	0.3	0.7	-0.1	0.5	-0.1	348.7
6	530.4	-15.2	23.0	1.7	43.9	-1.3	1.3	-0.8	0.1	0.5	0.0	0.4	0.0	353.8
7	532.1	-13.0	23.6	35.6	28.6	-0.6	-0.8	-0.6	-0.8	0.4	0.3	0.2	0.2	362.5
8	533.7	-13.5	26.2	27.9	-0.9	-1.3	-0.8	-1.0	-0.6	0.4	0.1	0.2	0.1	364.9
9	535.4	-13.5	33.0	17.4	-6.8	-1.2	-0.4	-0.9	-0.3	0.4	0.0	0.2	0.0	366.8
10	537.1	-10.8	42.4	11.3	-7.4	-1.0	-0.2	-0.8	-0.2	0.3	0.0	0.2	0.0	370.1
11	538.8	-2.7	53.1	7.3	-6.5	-0.7	-0.3	-0.6	-0.2	0.3	0.0	0.2	0.0	375.0



**Fig. 6.** Comparison of simulated results and actual values.

Considering the accurate values which are shown in Table 3, it can be seen that these models have acceptable accuracy. The Comparison of constant values shows that the error of these two models is acceptable.

For more accurate results in this model identification, the following points should be considered:

- (a) The variation of parameters should be considerable.
- (b) Parameters variation and their distribution for different considering harmonics should be similar.
- (c) Less correlated data is needed.
- (d) Considering above three points, the more data samples would result in more accurate model.

In practical circumstance, the supply voltage variation could be obtained by

- Using a capacitor banks switching.
- The variation of a dummy load.
- Tap changing in a supply transformer.
- Current injection using custom power devices.

Therefore, current injection method is the most satisfactory method.

**Table 11**  
Variance of different parameters.

V <sub>1r</sub>	V <sub>5r</sub>	V <sub>5i</sub>	V <sub>7r</sub>	V <sub>7i</sub>	V <sub>11r</sub>	V <sub>11i</sub>
42.9	10.3	194.8	234.6	238.3	57.9	20.1
V <sub>13r</sub>	V <sub>13i</sub>	V <sub>17r</sub>	V <sub>17i</sub>	V <sub>19r</sub>	V <sub>19i</sub>	
61.6	97.0	10.4	10.8	37.8	0.6	

$$Z = 0.59 + 3.14(X + 0.1) + 1.04(Y + 0.1) \tag{9}$$

$$Z = 0.59 + 2.09(X + 0.1) + 2.09(Y + 0.1) \tag{10}$$

However, the second model which is identified by the dependent data has acceptable accuracy around used data but the generalization of this model out of used data is impossible.

As a result, if there is considerable correlation between two model parameters data, the obtained coefficients is completely different from main model. To overcome this problem, we should omit one of these parameters, considering their importance.

Table 5 shows that there is considerable correlation between V<sub>5i</sub> and V<sub>1r</sub>. Therefore V<sub>5i</sub> can be neglected in comparison with V<sub>1r</sub>. Table 6 shows the parameters of these models (see Table 7).

**Table 9**  
The mean values of variations.

V <sub>1r</sub>	V <sub>5r</sub>	V <sub>5i</sub>	V <sub>7r</sub>	V <sub>7i</sub>	V <sub>11r</sub>	V <sub>11i</sub>	V <sub>13r</sub>	V <sub>13i</sub>	V <sub>17r</sub>	V <sub>17i</sub>	V <sub>19r</sub>	V <sub>19i</sub>
528.6	-12.18	22	3.56	9.28	-0.95	-1.60	-1.62	-2.98	-0.84	1.00	1.64	0.28

**Table 10**  
Estimated model based on simulation.

P <sub>0</sub>	V <sub>1r</sub>	V <sub>5r</sub>	V <sub>5i</sub>	V <sub>7r</sub>	V <sub>7i</sub>	V <sub>11r</sub>	V <sub>11i</sub>	V <sub>13r</sub>	V <sub>13i</sub>	V <sub>17r</sub>	V <sub>17i</sub>	V <sub>19r</sub>	V <sub>19i</sub>
366.3	1.37	0.18	0.16	0.18	-0.05	0.05	0.08	0.07	0.01	0.01	0.05	0.04	0.01

**Table 12**  
Estimated model based on measurement.

$P_0$	$V_{1r}$	$V_{5r}$	$V_{5i}$	$V_{7r}$	$V_{7i}$	$V_{11r}$	$V_{11i}$	$V_{13r}$	$V_{13i}$	$V_{17r}$	$V_{17i}$	$V_{19r}$	$V_{19i}$
365.89	1.44	0.21	0.12	0.15	-0.04	0.12	-0.03	0.20	-0.21	0	0	0	0

**Table 13**  
Error comparison and its variance for different variations around the mean values.

Average error (%)	Error variance (%)	Variation around mean value (%)
0.52	0.02	1
0.52	0.07	2
0.55	0.12	3
0.61	0.19	4
0.69	0.25	5
0.80	0.32	6
0.87	0.41	7
0.98	0.50	8
1.07	0.61	9
1.17	0.75	10

#### 4. Experimental results

To verify the proposed modelling method, a 420 kW water electrolysis system has been tested.

This electrolysis system is supplied via a 1250 kVA transformer. A 12-step capacitive bank is connected in parallel to this system in order to compensate the total reactive power of loads.

The electrolysis system includes two rectifier units with 2200 mF capacitance, as shown in Fig. 4. To decrease the harmonic content, a 3% series reactor (i.e. 3% voltage drop in nominal current) is used, too.

The measured voltage and current of the electrolysis system are shown in Fig. 5.

Table 8 shows the variation of the measured load voltage and power when the capacitive bank is connected.

Fig. 6 shows the simulated and the measured currents when there is no capacitive bank in the circuit.

It is clear that the model is has a very good agreement with measurement results. In order to estimate the coefficients from the model, the mean values of variations are taken into consideration and are given in Table 9.

The relation between consumed power and voltage parameters for variations around average values can be achieved by a number of simulations for various supply voltage harmonics. The determined model is shown in Table 10.

The first step in the estimation of the model parameters is the determination of the variance of the different parameters. These data sets are shown in Table 11. The even and third harmonic parameters are omitted because of their small values. As shown in Table 11, the variance of 17th and 18th harmonics is negligible, too. Table 12 shows the determined model based on these assumptions.

To verify the accuracy of this model with the model presented in Table 10, the voltage parameters are changed uniformly around their mean values and the model error is calculated. Table 13 shows the mean error and its variance.

It is clear that the model is suitable for values which are near the mean values, but when the deviation increases the accuracy decreases.

#### 5. Conclusion

The power consumption of most power consumer equipments changes in the presence of the voltage harmonics.

In this paper, an improved modelling and estimation method have been presented based on practical condition. This model can consider the effects of voltage harmonics on the load power consumption. Comparing the simulation and measurement results verify the accuracy of this estimation method. The proposed model has a good agreement with measurement results, too.

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