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Nonlinear Dynamic System Identification into Wiener Model using Subspace Identification and Support Vector Machine

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ABSTRACT

In this article, a new block- oriented nonlinear identification method is proposed. This modeling method uses the Wiener model comprised of a linear dynamic block that is followed by a nonlinear static block. The linear block is described by the subspace identification algorithm whereas the nonlinear one is represented via the Least Squares- Support Vector Machine. The proposed method is tested with a practical nonlinear chemical plant named as CSTR. A dataset of the input-output signals gathered from the system is applied to show the superiority of the method.

Keywords: Block-oriented identification, Wiener model, Subspace identification, Least Squares-Support Vector Machine, CSTR system.

1. INTRODUCTION

Throughout the literature, gray-box models of the nonlinear dynamical processes have been attracted a considerable attention due to their effective capabilities. These methods use input/output signals to form a description of the plant with some prior information about the system's dynamical equations.

In the recent decades, Block – Oriented Nonlinear methods are introduced to represent the nonlinear systems with lower parameters in comparison with the other methods [1]. The most common and simplest schemes are the Wiener and Hammerstein structures. Wiener model composed of a linear dynamic block followed by a static nonlinear block. Wiener structure has a good proficiency to describe almost all nonlinear plants with arbitrarily high accuracy [2]. Hammerstein subsystems are put into the model in the reverse order. these two models are applied to represent a huge number of nonlinear plants such as chemical plants [3, 4], high power amplifier [5], tubular reactors [6], control valve actuators [7], physiological systems [8], and solid oxide fuel cells [9] just to name a few (Fig.s 1, 2).

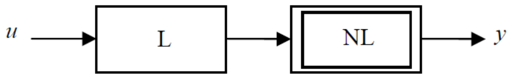

Fig. 1. Wiener Model

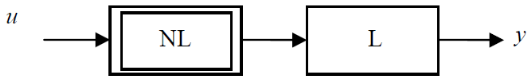

Fig. 2. Hammerstein Model

Subspace identification has a long history with many theoretical formulations to identify linear systems, successfully [10, 11]. N4SID, MOESP and CVA are some of the most convenient ones. Recently, some papers develop this idea for nonlinear dynamic systems. In this regard, kernel-based methods such as neural network and support vector machine can play an important role. Presented in this paper, Least Squares- Support Vector Machine (LS-SVM) is applied to extend the subspace identification method for nonlinear behavioral processes into Wiener structure.

This paper is organized as follows. The subspace identification and the LS-SVM regression are briefly presented in Section 2 and 3. In Section 4, the problem statement and the proposed method is described. Section 5 provides simulating example to show performance of the proposed method. Section 6 concludes the paper.

2. SUBSPACE IDENTIFICATION (N4SID METHOD)

In this paper, the linear dynamic part of the Wiener model is determined using Numerical Subspace State Space System IDentification (N4SID). Namely, given a data set of measured input-output signals, the linear approximation of the system can be described as follows:

$$\begin{cases} \mathbf{x}(k+1) = \mathbf{A}\mathbf{x}(k) + \mathbf{B}\mathbf{u}(k) \\ \mathbf{y}(k) = \mathbf{C}\mathbf{x}(k) + \mathbf{D}\mathbf{u}(k) \end{cases} \qquad k = 0, \ldots, N-1 \tag{1}$$

where $u(\mathrm{k}) \in \mathbb{R}^{n}$ and $y(\mathrm{k}) \in \mathbb{R}^{p}$ are the measured input and output signals at the time instant k, respectively and $x(k) \in \mathbb{R}^{n}$ contains the state vector of the model. $\mathbf{A} \in \mathbb{R}^{n\times n}$, $\mathbf{B} \in \mathbb{R}^{n\times m}$, $\mathbf{C} \in \mathbb{R}^{p\times m}$, and $\mathbf{D} \in \mathbb{R}^{p\times m}$ are called state, input, output, and direct feedthrough matrices, respectively . In this regard, the matrix pairs $\{\mathbf{A},\mathbf{C}\}$ and $\{\mathbf{A},\mathbf{B}\}$ are assumed to be observable and controllable, respectively. Subspace identification is composed of two basic stages: firstly, state space variables should be obtained from input-output data using the conventional linear algebra tools such as Linear-Quadratic decomposition or Singular Value Decomposition. In the second phase, the matrices of the state-space representation can be derived in a linear least squares problem. More details on the subspace identification procedure (N4SID method) can be found in [10].

3. SUBSPACE IDENTIFICATION (N4SID METHOD)

In this manuscript, the nonlinear static block is estimated using Least Squares-Support Vector Machine (LS-SVM). Based on the context of the convex optimization and statistical learning theories, the nonlinear regression problem is formulated in the dual space using Lagrange coefficients.

In the other words, a data set of the input-output signals $\{u(k), y(k)\}_{k=1}^{N}$ from a nonlinear model as

$$y(k) = w^{T}\varphi(u(k)) + b \tag{2}$$

where $u(k) \in \mathbb{R}^{p}$ determines the input data, $y(k) \in \mathbb{R}^{m}$ represents the output signal , and $\varphi(.)$ is used to denote the kernel function for the nonlinear mapping. The coefficients w and b can be found in the following optimization problem

$$\begin{aligned} &\min \qquad \frac{1}{2}w^{T}w + \frac{1}{2}\gamma\sum_{k=1}^{N} e(\mathrm{k})^{2} \\ &\textit{subject to} \qquad y(k) = w^{T} f(u(k)) + c \end{aligned} \tag{3}$$

where $\{e(k)\}_{k=1}^{N}$ show the output error terms and γ defines a regularization constant term [12].

4. PROBLEM DESCRIPTION

Consider the nonlinear state-space system

$$\begin{cases} \mathbf{x}(k+1) = \mathbf{f}\left(\mathbf{x}(k), \mathbf{u}(k)\right) \\ \mathbf{y}(k) = h\left(\mathbf{x}(k)\right) \end{cases} \tag{4}$$

where $\mathbf{x}(k) \in \mathbb{R}^{n}$ the state, $\mathbf{u}(k) \in \mathbb{R}^{m}$ the input, and $\mathbf{y}(k) \in \mathbb{R}^{p}$ are the output vectors and $\mathbf{f} \in \mathbb{R}^{n} \times \mathbb{R}^{m} \to \mathbb{R}^{n}$ and $h \in \mathbb{R}^{n} \to \mathbb{R}^{p}$ are assumed to be nonlinear smooth functions. Using a finite number of the measured input-output signals, the main goal of this article is to propose a Wiener model that has a same dynamic behavior as the system.

In the training phase, the linear dynamic block should be approximated using the N4SID method firstly, and then in the second phase the static nonlinear block is estimated in the LS-SVM procedure. These two steps are implemented in the offline mode according to the input-output data set $\{\mathbf{u}(k), \mathbf{y}(k)\}_{k=1}^{N\_offline}$. In the online test stage, the generated model outputs are compared to the system outputs.

In the LS-SVM regression, Gaussian kernel function is applied for the high dimensional feature spaces. Gaussian kernel function is also useful to give proper smoothness and good generality in the unknown input range condition.

In the LS-SVM regression approach, the nonlinear static block is represented as

$$\hat{y}(k) = f(L(k)) = w^{T}\varphi(L(k)) + b \tag{5}$$

where $W \in \mathbb{R}^{n_h}$ is the weight vector, $\varphi(.) : \mathbb{R}^{n} \to \mathbb{R}^{n_h}$ is the nonlinear map function, and b is the bias term. In this regard, the modeling error for sample k is defined as $e(k) = y(k) - \hat{y}(k)$ and the objective function in the LS-SVM regression is as bellows

$$\begin{aligned} &\min_{\{w,b,e\}} \ \mathrm{J(w, e)} = \frac{1}{2} w^{T} w + \frac{\gamma}{2} \sum_{k=1}^{M} e_k^{2} \\ &\textit{subject} \quad \textit{to} \qquad y_t(k) = w^{T}\varphi(L_t(k)) + b + e_k \end{aligned} \tag{6}$$

where the scalar y is a constant value to handling a governed trade-off between the smoothness and data fitting. Considering above formulation, the constrained optimization problem can be solved using Lagrangian as

$$\xi(w, b, e; \alpha) = \mathrm{J(w, e)} - \sum_{k=1}^{M} \alpha_k (w^{T}\varphi(L_t(k)) + b + e_k - y_t(k)) \tag{7}$$

where $\alpha_k (k = 1, \ldots, \mathrm{N})$ are the Lagrange coefficients. The optimal conditions for Lagrangian would be resulted to

$$\begin{aligned}
&\frac{\partial \zeta}{\partial w} = 0 \rightarrow w = \sum_{k=1}^{M} \alpha_k \varphi(L_t(k)) \\
&\frac{\partial \zeta}{\partial b} = 0 \rightarrow \sum_{k=1}^{M} \alpha_k = 0 \\
&\frac{\partial \zeta}{\partial e_k} = 0 \rightarrow \alpha_k = \gamma e_k \\
&\frac{\partial \zeta}{\partial \alpha_k} = 0 \rightarrow y_t(k) = w^T \varphi(L_t(k)) + b + e_k
\end{aligned} \tag{8}$$

Eliminating e_k and w in the above equations, the following linear equations can be extracted as

$$\begin{pmatrix} 0 & 1_M^T \\ 1_M & \Omega + \gamma^{-1} I_M \end{pmatrix} \begin{pmatrix} b \\ \alpha \end{pmatrix} = \begin{pmatrix} 0 \\ Y_t \end{pmatrix} \tag{9}$$

where $\Omega_{ij} = \varphi(L(\mathrm{i})^T L(\mathrm{j}))$, $Y = [y(1), y(2), \ldots, y(N\text{ - }offline)]$, $\alpha = [\alpha_1, \alpha_2, \ldots, \alpha_{N-offline}]^T$,and $\mathbf{1} = \mathrm{ones}(N\_offline, 1)$. The output of the LS-SVM regression, namely the Wiener model output, can be determined as

$$\hat{y}(k) = \sum_{i=1}^{M} \alpha_i K(L(i), L(k)) + b \tag{10}$$

where α_k and b are the solutions of Equation (9).

5. SIMULATION RESULTS

Performance of the suggested modeling procedure is illustrated using a practical chemical system. Continuous Stirred Tank Reactor (CSTR) is the selected benchmark. Chemical reactors often have significant heat effects, so it is important to be able to add or remove heat from them. In a CSTR the heat is added or removed by virtue of the temperature difference between a jacket fluid and the reactor fluid. Often, the heat transfer fluid is pumped through agitation nozzle that circulates the fluid through the jacket at a high velocity. The product concentration can be controlled by manipulating the feed flow rate [13].

The gathered input-output data from a practical plant [14] are applied to show the capability of the method. In this system, the flow of the coolant liquid is used to adjust the output concentration (Fig. 3).

In the offline training phase, the Wiener model subsystems are approximated using a dataset of 800 measured input-output samples. These signals are shown in Fig. 4, respectively. The reminder of the data is utilized to test the algorithm in the online state. Fig. 5 shows the input signal that is applied in the online stage. The measured output of the CSTR system in the online phase is shown in Fig. 6, too.

The Root Mean Squared Error (RMSE) criterion considered as

$$RMSE\% = \sqrt{\frac{1}{N}\sum_{i=1}^{N}\left(\frac{y_i - \hat{y}_i}{FS} * 100\right)^2} \tag{11}$$

which is computed at the end of the online identification task is equal to 1.1065

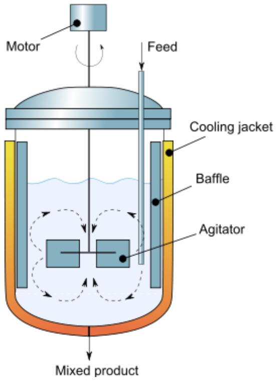

Fig 3. CSTR Layout

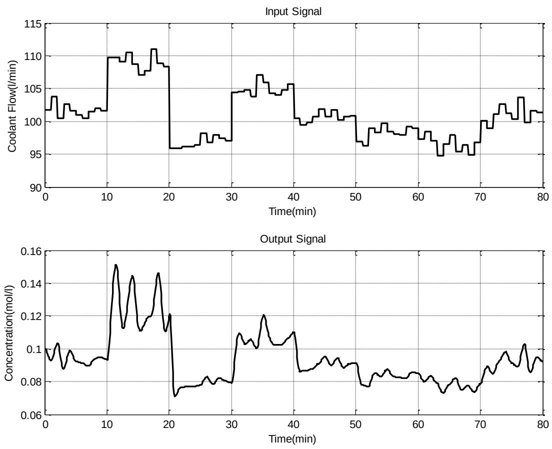

Fig. 4. Input-Output Signal in the offline mode

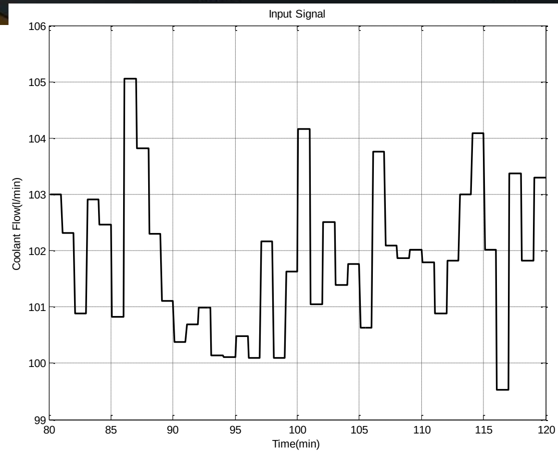

Fig. 5. Input Signal in the online mode

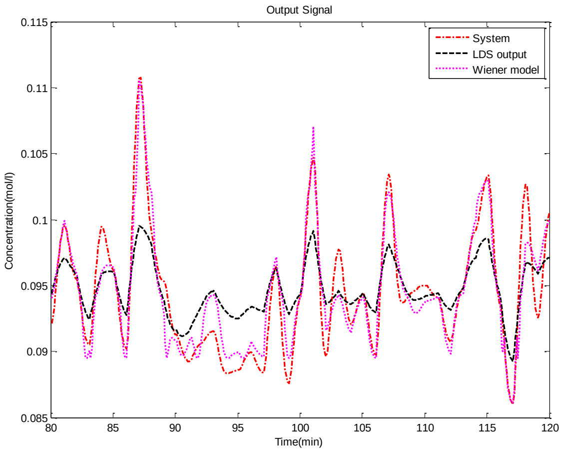

Fig. 6. Output Signal in the online mode

CONCLUSION

A novel block-oriented identification method is proposed for the nonlinear systems. The wiener model composed of a linear dynamic subsystem affected its output by a nonlinear static subsystem is used. A linear approximation computed by the N4SID subspace identification is considered as the linear subsystem. The nonlinear static subsystem which is estimated by the LS-SVM regression is applied to describe the output nonlinear effects. Simulation results that are conducted on the CSTR system approve the capability of the method.

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