# Investigation of the Effect of an Inconsistent Blade on Natural Frequencies of a Rotating Multi Blade System

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*Abstract:* - In this study, dynamic characteristics of a rotating multi blade system which contains an inconsistent blade are determined. The main goal is to investigate the effect of an inconsistent blade on the system natural frequencies. For this purpose, natural frequencies of a perfect system and a system containing one inconsistent blade have been compared to show how this inconsistency affects the natural frequencies of the system. It has been assumed that the inconsistent blade differs from the other in term of material property. The vibration frequency characteristics have been analyzed using assumed mode method along with Hamilton's law. Based on the results, it is demonstrated that natural frequencies related to some modes of the system are affected in case of inconsistent blade while for the most of the modes, they are not affected.

*Keywords:* - Natural frequency, Multi blade system, inconsistent blade, Assumed mode method

**Nomenclature**

| Parameter | Definition |
|---|---|
| h | Cross section height |
| b | Cross section width |
| l | Beam length |
| $\alpha$ | Ratio of variation in Cross section height |
| $\beta$ | Ratio of variation in Cross section width |
| r | Disk radius |
| s | Stretch of the beam |
| u | Axial displacement the beam |
| v | Chord-wise displacement of the beam |
| $\Omega$ | Rotating velocity |
| T | Kinetic energy |
| $\rho$ | Density |
| A | Cross section area |
| U | Strain energy |
| E | Young's modulus |
| I | Second area moment of inertia |
| $U_D$ | The energy term of the system which are caused by the flexibility of the disk |
| $U_S$ | The energy term of the system which are caused by the flexibility of the shroud |
| $a_D$ | The positions where the beam is connected to the disk |
| $a_S$ | The positions where the beam is connected to the shroud |
| $k_D$ | Disk stiffness variable |
| $k_S$ | Shroud stiffness values |
| $\varphi$ | Bending mode function of the beam |
| q | The generalized coordinate |
| $\mu$ | The number of coordinates |
| $I_0$ | Second area moment of inertia of the cross section at the fixed end |

## 1. INTRODUCTION

Plenty of engineering items, such as turbines, compressor, helicopter rotors, propellers of ships or planes, robot handlers, and structures for space whirling contain rotating beams. To create well-designed structures, the analysis of their vibration characteristics, including natural frequencies and mode shapes is of great importance. A considerable variation can be noticed in vibration characteristics of rotating structures compared to those in non-rotating ones.

In order to evaluate the natural frequency of a single rotating cantilever beam, Southwell and Gough developed a model [1], in which, according to the Rayleigh energy theory, natural frequencies of a rotating cantilever beam could be measured via a widely-used simple equation called Southwell. Then, to increase the accuracy of natural frequencies, Schilhansl introduced a relatively different linear equation, in which only the rotating beam bending motions are considered [2]. The mentioned method of vibration analysis was developed in numbers of studies in order to determine natural frequency of rotating beams under various conditions [3-9]. In one of these researches, a dynamic analysis method, including a lot of hybrid deformation variables, was proposed to reach equations of motion [10]. Due to the linearity of the existent equations in this method, no replacement procedure is needed to analyze the vibration and the equations can be utilized directly. Compared to the previous ones, this method is less complicated, more compatible, and more precise [11, 12].

Beyond vibration of single rotating beam, various studies have been conducted to analyze vibration characteristics of rotating multi blade system [13-15], in order to develop application of modal analysis in vital industrial equipment such as turbines. The majority of the mentioned studies are limited to the systems which contain consistent blades. However, equipment with multi blade system usually suffers from the existence of one or more inconsistent blades in their systems. In this case, the vibration behavior of the system such as natural frequencies will be changed. In order to keep the working frequencies away from the system natural frequencies, it is essential to be aware of the way of variation in system natural frequencies. Hence, the concept of vibration characteristics of the multi blade system containing inconsistent blade can be important. In this study, comparing the natural frequencies of a multi blade system with and without inconsistency will demonstrate the way of variation of system natural frequencies.

## 2. EQUATION of MOTION

The model of blade is considered as a slender beam which consists of rectangular and tapered cross section beam. This model is shown in Fig. 1.

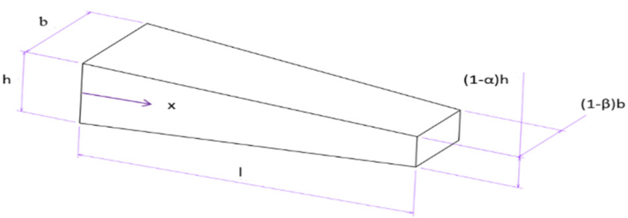


**Figure 1.** The schematic of the tapered blade model

The blade in this model contains homogeneous and isotropic material features. The shear and rotary inertia effects are neglected, since the beam has a slender shape. The rate of variation in the cross section dimensions is described by two dimensionless parameters α and β, as shown in Fig 1. Assuming a high value of out-of-plane rigidity for the blade, the out-of-plane motion is neglected and only the in-plane motion is considered. The schematic of deformed and un-deformed beams are presented in Fig. 2. The beam is considered as a cantilever of a length l which is fixed to a disk with a radius r. When the beam is deformed, the point $P_0$ placed at the position of point P. Hence, axial displacement u and the chord-wise displacement v can be used to define the deformation vector.

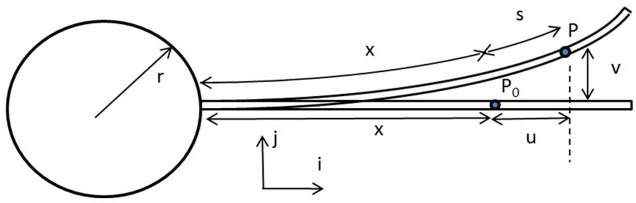


**Figure 2.** Configuration of rotating blade displacement

It was shown [5] that the relation between stretching of the beam (s), u and v is as below:

$$s = u + \frac{1}{2}\int_0^x \left(\frac{\partial v}{\partial \sigma}\right)^2 d\sigma \tag{1}$$

where σ is the dummy variable employed for integration. Using the defined deformation terms along with considering the beam rotating velocity (Ω) as a constant value (Ω=0), the kinetic and strain energies of the beam can be expressed as below:

$$T = \frac{1}{2}\int_0^l \rho A(x)\left\{\left(\dot{u} - \Omega v\right)^2 + \left(\dot{v} + \Omega(r + x + u)\right)^2\right\} dx \tag{2}$$

$$\begin{aligned} U &= \frac{1}{2}\int_0^l EA(x)\left(\frac{\partial s}{\partial x}\right)^2 dx + \\ &+ \frac{1}{2}\int_0^L EI(x)\left(\frac{\partial^2 v}{\partial x^2}\right)^2 dx \end{aligned} \tag{3}$$

where E is the Young's modulus. I(x) is the second area moment of inertia and A(x) is the cross-section area which are functions of x. In case of multi blade systems such as turbine, stiffness of the shroud and disc results in additional terms of energy. In this study, the disc and shrouds are considered as the discrete springs with corresponding stiffness values. Hence, due to flexibility of the disc and shrouds, the energy terms can be written as:

$$\begin{aligned} U_D &= \frac{1}{2}K_D\left(v^k(a_D) - v^{k-1}(a_D)\right)^2 + \\ &+ \frac{1}{2}K_D\left(v^{k+1}(a_D) - v^k(a_D)\right)^2 \end{aligned} \tag{4}$$

and

$$\begin{aligned} U_S &= \frac{1}{2}K_S\left(v^k\left(a_S\right) - v^{k-1}\left(a_S\right)\right)^2 + \\ &+ \frac{1}{2}K_S\left(v^{k+1}\left(a_S\right) - v^k\left(a_S\right)\right)^2 \end{aligned} \tag{5}$$

In the above expressions, $U_D$ and $U_S$ are the energy terms of the kth blade of the system which are caused by the flexibility of the disk and the shroud. $a_D$ and $a_S$ are the positions where the beam is connected to the disk and the shroud, respectively, and $K_D$ and $K_S$ are the corresponding stiffness values. The superscript k, k+1 and k-1 above the displacement term are the blade numbers. When the strain energy and kinematic expressions are placed in Eq. 6, the equation of motion can be derived:

$$\int \delta\left(T - U\right)dt = 0 \tag{6}$$

A group of hybrid deformation variables can be approached by an assumed-mode method called the Rayleigh-Ritz. The bending displacement is approximated by mode functions as follows:

$$v\left(x,t\right) = \sum_{i=1}^{\mu} \varphi_i\left(x\right)q_i\left(t\right) \tag{7}$$

where $\varphi_i$ is the bending mode function of the beam, $q_i$ is the corresponding generalized coordinate, and $\mu$ is the number of coordinate $q_i$. Previously, it was demonstrated that the coupling effect between extensional and bending motions has a weak influence on the vibrational characteristics of the rotating blades [13]. Hence, this the terms related to the coupling effects are ignored in this study. Using the expression of Eq. 7 for the bending displacement, the equations of the bending motion of the kth blade are derived as below (omitting the terms of coupling):

$$\begin{aligned}&\sum_{j=1}^{\mu}\left[m_{ij}\ddot{q}_j^{k} + \left(k_{ij}^{B} + \Omega^2\left(k_{ij}^{G} - m_{ij}\right)\right)q_j^{k}\right] - \\ &-k_{ij}^{D}\left(q_j^{k-1} - 2q_j^{k} + q_j^{k+1}\right) - \\ &-k_{ij}^{C}\left(q_j^{k-1} - 2q_j^{k} + q_j^{k+1}\right) = 0\end{aligned} \tag{8}$$

where i=1,2,3,…, etc. and

$$m_{ij} = \int_0^l \rho\left(x\right)\varphi_i\left(x\right)\varphi_j\left(x\right)dx \tag{9}$$

$$k_{ij}^{B} = \int_0^l EI\left(x\right)\varphi_{i,xx}\left(x\right)\varphi_{j,xx}\left(x\right)dx \tag{10}$$

$$k_{ij}^{G} = \int_0^l g\left(x\right)\varphi_{i,x}\left(x\right)\varphi_{j,x}\left(x\right)dx \tag{11}$$

$$k_{ij}^{D} = K_D\varphi_i\left(a_D\right)\varphi_j\left(a_D\right) \tag{12}$$

$$k_{ij}^{S} = K_S\varphi_i\left(a_S\right)\varphi_j\left(a_S\right) \tag{13}$$

$$\begin{aligned}g\left(x\right) &= \rho r\left(l - x\right) + \\ &+ \frac{\rho}{2l}\left(l - \left(\alpha + \beta\right)r\right)\left(l^2 - x^2\right) + \\ &+ \frac{\rho}{3l}\left(\alpha\beta r - \alpha l - \beta l\right)\left(l^3 - x^3\right) + \\ &+ \frac{\rho}{4l^2}\alpha\beta\left(l^4 - x^4\right)\end{aligned} \tag{14}$$

Since, parametric study is preferred for different purposes such as system design, it is beneficial to transform equations of motion into a dimensionless form. Hence, employing the variables of Eq. 15, the equation of motion is rewritten in a dimensionless form as shown in Eq. 16.

$$\tau = \frac{t}{T}, \xi = \frac{x}{l}, \theta = \frac{q}{l}, T = \sqrt{\frac{\rho l^4}{EI_0}} \tag{15}$$

$$\begin{aligned}&\sum_{j=1}^{\mu}[M_{ij}\ddot{\theta}_j^{k} + \\ &+\left(K_{ij}^{B} + \gamma^2\left(K_{ij}^{G} - M_{ij}\right)\right)\theta_j^{k}] - \\ &-K_{ij}^{D}\left(\theta_j^{k-1} - 2\theta_j^{k} + \theta_j^{k+1}\right) - \\ &-K_{ij}^{C}\left(\theta_j^{k-1} - 2\theta_j^{k} + \theta_j^{k+1}\right) = 0\end{aligned} \tag{16}$$

where i=1,2,3,…, etc. and

$$M_{ij} = \int_0^1 \left(1 - \alpha\xi\right)\left(1 - \beta\xi\right)\varphi_i\left(\xi\right)\varphi_j\left(\xi\right)d\xi \tag{17}$$

$$K_{ij}^{B} = \int_0^1 \left(1 - \alpha\xi\right)^3\left(1 - \beta\xi\right)\varphi_{i,\xi\xi}\left(\xi\right)\varphi_{j,\xi\xi}\left(\xi\right)d\xi \tag{18}$$

$$K_{ij}^{G} = \int_0^1 G\left(\xi\right)\varphi_{i,\xi}\left(\xi\right)\varphi_{j,\xi}\left(\xi\right)d\xi \tag{19}$$

$$K_{ij}^{D} = \beta_D\varphi_i\left(\xi_D\right)\varphi_j\left(\xi_D\right) \tag{20}$$

$$K_{ij}^{S} = \beta_S\varphi_i\left(\xi_S\right)\varphi_j\left(\xi_S\right) \tag{21}$$

$$\gamma = \Omega T\ ,\ \delta = \frac{r}{l},\ \beta_D = \frac{K_D l^3}{EI_0},$$
$$,\beta_s = \frac{K_s l^3}{EI_0},\ \xi_D = \frac{a_D}{l},\ \xi_S = \frac{a_S}{l} \qquad (22)$$

$$G(\xi) = \delta(1-\xi) + \\ + \frac{1}{2}\left(1-(\alpha+\beta)\delta\right)\left(1-\xi^2\right) + \\ + \frac{1}{3}\left(\alpha\beta\delta - \alpha - \beta\right)\left(1-\xi^3\right) + \\ + \frac{1}{4}\alpha\beta\left(1-\xi^4\right) \qquad (23)$$

Now by assembling n (total number of blades) sets of equation of motion (Eq. 16) corresponding to n blades, the total equation can be written as Eq. 24.

$$[M]\left\{\ddot{\theta}\right\} + \left([K] + \gamma^2\left(\left[K^G\right] - [M]\right)\right)\{\theta\} = 0 \quad (24)$$

For the modal analysis of the system, the matrix [θ] can be assumed as:

$$\{\theta\} = e^{-j\omega\tau}\{\eta\} \qquad (25)$$

Here, ω is the dimensionless natural frequency of the system. Employing Eq. 25, Eq. 24 can be written as Eq. 26 which is used for modal analysis of the multi blade system.

$$\omega^2[M]\{\eta\} = \left([K] + \gamma^2\left(\left[K^G\right] - [M]\right)\right)\{\theta\} \quad (26)$$

## 3. NUMERICAL RESULTS

A code was developed in MATLAB software to determine the natural frequencies of the systems. In order to evaluate the code, natural frequencies of a single rotating beam with uniform section (α=β=0) were determined and compared with those reported in Ref [10]. The natural frequency was determined in the case of δ=1 for different values of γ. The obtained natural frequencies and the corresponding values from the literature are in good agreement as presented in Table 1.

**Table 1.** Comparison of the first natural frequencies (results of MATLAB code and those of literature) in the bending vibration

| γ | Present | Cheng and Yan (2006) |
|---|---|---|
| 1 | 3.889 | 3.889 |
| 2 | 4.834 | 4.834 |
| 3 | 6.082 | 6.084 |
| 4 | 7.476 | 7.481 |
| 5 | 8.942 | 8.951 |

In this study, the natural frequencies of a multi blade system shown in Fig. 3 have been determined. This system contains 4 packets and each packet consists of 6 blades (total number of 24 blades). The first five bending mode functions of a cantilever beam are considered as the assumed modes of the blades. It is worth mentioning that, in the assembly step, the term related to the stiffness of the shroud should not be considered for the blades at the ends of the pockets, as there is not any shroud connection between packets. Fig. 4 shows the dimensionless natural frequencies which are plotted versus the dimensionless angular speed. The values of dimensionless parameters employed for the numerical analysis are given in Table 2.

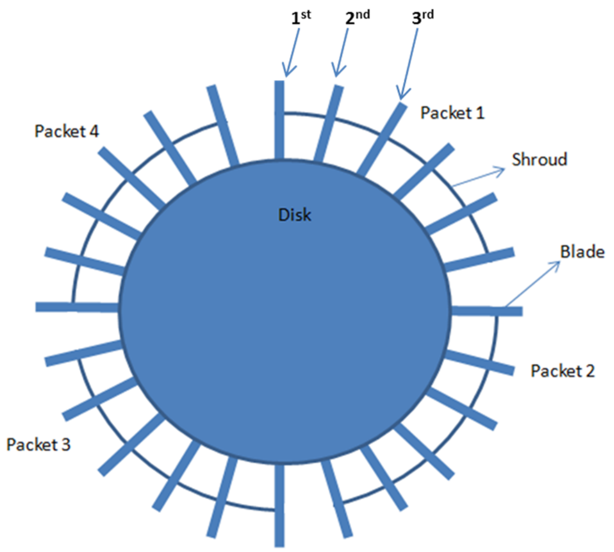


**Figure 3.** Configuration of the multi blade system

**Table 2.** The values of dimensionless parameters employed in the numerical analysis

| α | β | δ | $\beta_D$ | $\beta_S$ | $\xi_D$ | $\xi_S$ |
|---|---|---|---|---|---|---|
| 0.3 | 0.3 | 4 | 2e6 | 10 | 0.1 | 1 |

It is worth mentioning that $\xi_D$ and $\xi_S$ are the dimensionless terms related to the positions where the beam is connected to the disk and the shroud. $\beta_D$ and $\beta_S$ are the dimensionless stiffness values of disk and the shroud. δ is the ratio of the disk radius to the blade length (Eq. 22).

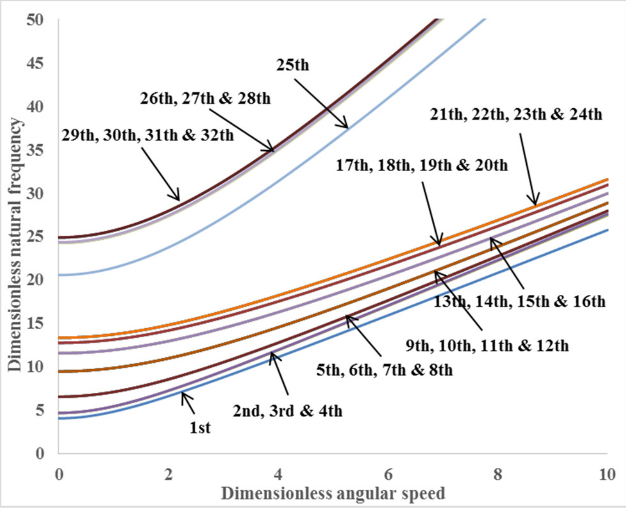


**Figure 4.** Dimensionless natural frequencies of the perfect multi blade system versus the dimensionless angular speed (γ)

In order to investigate the effect of an inconsistent blade on the natural frequency of the system, the dimensionless material properties of the one of the blades were changed. In case of defects such as crack, void and etc. in a blade, mechanical properties as well as Young's modulus would be significantly reduced. Addition to that, thermal shock would be other phenomena, which affect material properties of the blades. According to these kinds of material property variations, it is assumed that the Young's modulus of the inconsistent blade has been reduced to the 10% of the initiate value. Therefore, the values of $\beta_D$ and $\beta_S$ were considered to be 2e7 and 100, respectively (while these values were 2e6 and 10 for perfect blades). Different conditions are considered in terms of the position of inconsistent blade in the system (1$^{st}$, 2$^{nd}$ and 3$^{rd}$ blade as shown in Fig. 3) and rotating speed of the system (γ=0, 5 and 10). It is worth mentioning that other positions for the inconsistent blade are similar to the considered positions due to symmetrical geometry. In Table 3, natural frequencies of the system for the mentioned conditions are presented.

**Table 3.** Natural frequency of the system without inconsistency and the systems with one inconsistent blade in the position of 1$^{st}$, 2$^{nd}$ and the 3$^{rd}$ blade (see Fig. 3)

| γ=0 | | | | |
|---|---|---|---|---|
| Mode | Perfect | 1$^{st}$ | 2$^{nd}$ | 3$^{rd}$ |
| 1 | 4.067 | 4.067 | 4.067 | 4.067 |
| 2 | 4.685 | 4.685 | 4.685 | 4.685 |
| 3 | 4.685 | 4.686 | 4.687 | 4.687 |
| 4 | 4.696 | 4.696 | 4.696 | 4.697 |
| 5 | 6.553 | 6.553 | 6.553 | 6.553 |
| 6 | 6.553 | 6.553 | 6.553 | 6.553 |
| 7 | 6.553 | 6.556 | 6.556 | 6.555 |
| 8 | 6.556 | 7.087 | 6.836 | 6.585 |
| 9 | 9.464 | 9.464 | 9.464 | 9.464 |
| 10 | 9.464 | 9.464 | 9.464 | 9.464 |
| 11 | 9.464 | 9.464 | 9.464 | 9.464 |
| 12 | 9.464 | 10.339 | 9.464 | 10.048 |
| 13 | 11.561 | 11.561 | 11.561 | 11.561 |
| 14 | 11.561 | 11.561 | 11.561 | 11.561 |
| 15 | 11.561 | 11.561 | 11.561 | 11.561 |
| 16 | 11.561 | 12.322 | 11.841 | 11.993 |
| 17 | 12.757 | 12.757 | 12.757 | 12.757 |
| 18 | 12.757 | 12.757 | 12.757 | 12.757 |
| 19 | 12.757 | 12.757 | 12.757 | 12.757 |
| 20 | 12.757 | 13.249 | 13.146 | 12.890 |
| 21 | 13.341 | 13.341 | 13.341 | 13.341 |
| 22 | 13.341 | 13.341 | 13.341 | 13.341 |
| 23 | 13.341 | 13.341 | 13.341 | 13.341 |
| 24 | 13.341 | 15.324 | 15.874 | 15.875 |
| 25 | 20.556 | 20.556 | 20.556 | 20.556 |

| γ=5 | | | | |
|---|---|---|---|---|
| Mode | Perfect | 1$^{st}$ | 2$^{nd}$ | 3$^{rd}$ |
| 1 | 13.505 | 13.505 | 13.505 | 13.505 |
| 2 | 14.459 | 14.458 | 14.459 | 14.459 |
| 3 | 14.459 | 14.462 | 14.462 | 14.462 |
| 4 | 14.488 | 14.488 | 14.489 | 14.489 |
| 5 | 15.156 | 15.156 | 15.156 | 15.156 |
| 6 | 15.157 | 15.157 | 15.157 | 15.157 |
| 7 | 15.157 | 15.163 | 15.163 | 15.162 |
| 8 | 15.166 | 15.396 | 15.282 | 15.174 |
| 9 | 16.706 | 16.706 | 16.706 | 16.706 |
| 10 | 16.706 | 16.706 | 16.706 | 16.706 |
| 11 | 16.706 | 16.706 | 16.706 | 16.706 |
| 12 | 16.706 | 17.314 | 16.706 | 17.104 |
| 13 | 18.316 | 18.316 | 18.316 | 18.316 |
| 14 | 18.316 | 18.316 | 18.316 | 18.316 |
| 15 | 18.316 | 18.316 | 18.316 | 18.316 |
| 16 | 18.316 | 19.055 | 18.576 | 18.722 |
| 17 | 19.531 | 19.531 | 19.531 | 19.531 |
| 18 | 19.531 | 19.531 | 19.531 | 19.531 |
| 19 | 19.531 | 19.531 | 19.531 | 19.531 |
| 20 | 19.531 | 20.125 | 19.995 | 19.686 |
| 21 | 20.243 | 20.243 | 20.243 | 20.243 |
| 22 | 20.243 | 20.243 | 20.243 | 20.243 |
| 23 | 20.243 | 20.243 | 20.243 | 20.243 |
| 24 | 20.243 | 23.660 | 25.022 | 25.024 |
| 25 | 35.992 | 35.992 | 35.992 | 35.992 |

| γ=10 | | | | |
|---|---|---|---|---|
| Mode | Perfect | 1$^{st}$ | 2$^{nd}$ | 3$^{rd}$ |
| 1 | 25.748 | 25.748 | 25.748 | 25.748 |
| 2 | 27.469 | 27.466 | 27.468 | 27.469 |
| 3 | 27.469 | 27.481 | 27.481 | 27.482 |
| 4 | 27.579 | 27.578 | 27.580 | 27.582 |
| 5 | 27.922 | 27.925 | 27.925 | 27.923 |
| 6 | 27.932 | 27.931 | 27.931 | 27.931 |
| 7 | 27.932 | 27.950 | 27.948 | 27.936 |
| 8 | 27.958 | 28.071 | 28.010 | 27.960 |
| 9 | 28.852 | 28.852 | 28.852 | 28.852 |
| 10 | 28.853 | 28.853 | 28.853 | 28.853 |
| 11 | 28.853 | 28.853 | 28.853 | 28.853 |

| | | | | |
|---|---|---|---|---|
| 12 | 28.853 | 29.249 | 28.853 | 29.109 |
| 13 | 29.957 | 29.957 | 29.957 | 29.957 |
| 14 | 29.957 | 29.957 | 29.957 | 29.957 |
| 15 | 29.957 | 29.958 | 29.958 | 29.958 |
| 16 | 29.958 | 30.534 | 30.155 | 30.268 |
| 17 | 30.933 | 30.933 | 30.933 | 30.933 |
| 18 | 30.933 | 30.933 | 30.933 | 30.933 |
| 19 | 30.933 | 30.933 | 30.933 | 30.933 |
| 20 | 30.933 | 31.467 | 31.346 | 31.068 |
| 21 | 31.579 | 31.579 | 31.579 | 31.579 |
| 22 | 31.579 | 31.579 | 31.579 | 31.579 |
| 23 | 31.579 | 31.579 | 31.579 | 31.579 |
| 24 | 31.579 | 35.950 | 38.775 | 38.780 |
| 25 | 62.115 | 62.115 | 62.115 | 62.115 |

As observed in Table 3, natural frequencies of the first 7 mode are not changed, which indicates that any inconsistency in one of the blades of a multi blade system will not affect system dynamic characteristics. Variation of the natural frequencies of the system due to inconsistent blade only can be observed for the 8$^{th}$, 12$^{th}$, 16$^{th}$, 20$^{th}$ and 24$^{th}$ modes of the system. In order to have a better presentation of the variation in natural frequencies, the values corresponding to the 8$^{th}$, 12$^{th}$, 16$^{th}$ and 24$^{th}$ modes are presented in Table 4.

**Table 4.** Variation of the natural frequencies of the system due to inconsistent blades for the 8$^{th}$, 12$^{th}$, 16$^{th}$ and 24$^{th}$ mode (IBN: Inconsistent Blade Number, NF: Natural Frequency, NFP: Natural Frequency of Perfect system, PV: Percentage of Variation in natural frequency due to inconsistent blade)

| | γ=0 | | | |
|---|---|---|---|---|
| | IBN | 1$^{st}$ | 2$^{nd}$ | 3$^{rd}$ |
| 8$^{th}$ mode | NF | 7.087 | 6.836 | 6.585 |
| | NFP | 6.556 | 6.556 | 6.556 |
| | PV | 7.482 | 4.092 | 0.436 |
| 12$^{th}$ mode | NF | 10.339 | 9.464 | 10.048 |
| | NFP | 9.464 | 9.464 | 9.464 |
| | PV | 9.247 | 0 | 6.174 |
| 16$^{th}$ mode | NF | 12.322 | 11.841 | 11.993 |
| | NFP | 11.561 | 11.561 | 11.561 |
| | PV | 6.575 | 2.421 | 3.731 |
| 20$^{th}$ mode | NF | 13.249 | 13.146 | 12.89 |
| | NFP | 12.757 | 12.757 | 12.757 |
| | PV | 3.711 | 2.956 | 1.034 |
| 24$^{th}$ mode | NF | 15.324 | 15.874 | 15.875 |
| | NFP | 13.341 | 13.341 | 13.341 |
| | PV | 14.864 | 18.989 | 18.995 |

| | γ=5 | | | |
|---|---|---|---|---|
| | IBN | 1$^{st}$ | 2$^{nd}$ | 3$^{rd}$ |
| 8$^{th}$ mode | NF | 15.396 | 15.282 | 15.174 |
| | NFP | 15.166 | 15.166 | 15.166 |
| | PV | 1.492 | 0.762 | 0.052 |
| 12$^{th}$ mode | NF | 17.314 | 16.706 | 17.104 |
| | NFP | 16.706 | 16.706 | 16.706 |
| | PV | 3.64 | 0 | 2.377 |
| 16$^{th}$ mode | NF | 19.055 | 18.576 | 18.722 |
| | NFP | 18.316 | 18.316 | 18.316 |
| | PV | 4.036 | 1.42 | 2.219 |
| 20$^{th}$ mode | NF | 20.125 | 19.995 | 19.686 |
| | NFP | 19.531 | 19.531 | 19.531 |
| | PV | 2.949 | 2.319 | 0.785 |
| 24$^{th}$ mode | NF | 23.66 | 25.022 | 25.024 |
| | NFP | 20.243 | 20.243 | 20.243 |
| | PV | 16.876 | 23.606 | 23.616 |

| | γ=10 | | | |
|---|---|---|---|---|
| | IBN | 1$^{st}$ | 2$^{nd}$ | 3$^{rd}$ |
| 8$^{th}$ mode | NF | 28.071 | 28.01 | 27.96 |
| | NFP | 27.958 | 27.958 | 27.958 |
| | PV | 0.404 | 0.185 | 0.008 |
| 12$^{th}$ mode | NF | 29.249 | 28.853 | 29.109 |
| | NFP | 28.853 | 28.853 | 28.853 |
| | PV | 1.373 | 0 | 0.887 |
| 16$^{th}$ mode | NF | 30.534 | 30.155 | 30.268 |
| | NFP | 29.958 | 29.958 | 29.958 |
| | PV | 1.923 | 0.657 | 1.037 |
| 20$^{th}$ mode | NF | 31.467 | 31.346 | 31.068 |
| | NFP | 30.933 | 30.933 | 30.933 |
| | PV | 1.696 | 1.318 | 0.434 |
| 24$^{th}$ mode | NF | 35.95 | 38.775 | 38.78 |
| | NFP | 31.579 | 31.579 | 31.579 |
| | PV | 13.843 | 22.788 | 22.804 |

As illustrated in Table 4, the highest values of variation in natural frequencies are observed for 24$^{th}$ mode. Among the other four modes, the 12$^{th}$ and 16$^{th}$ modes demonstrate higher values of variation. The natural frequencies of the system mostly are increased due to the inconsistent blades. As observed in Table 4, in case of the 1$^{st}$ blade as the inconsistent one, the values of variations are more than other cases, except for the 24$^{th}$ mode. However, for the 24$^{th}$ mode, the values of variation become significant, in case of placing inconsistent blade in the position of 2$^{nd}$ or 3$^{rd}$ blade. Comparison of the values of variation for different angular speeds of the system, lead to the fact that higher variation values in the natural frequencies of the system are observed for the lower angular speed (γ=0), except for the 24th mode in which lower variation values are related to the lower angular speed. For better presentation of the way of variation, the natural frequencies of the systems with the inconsistency in the position of 1st, 2nd and 3rd blades are plotted versus system angular speeds in Fig. 5, Fig. 6 and Fig. 7.

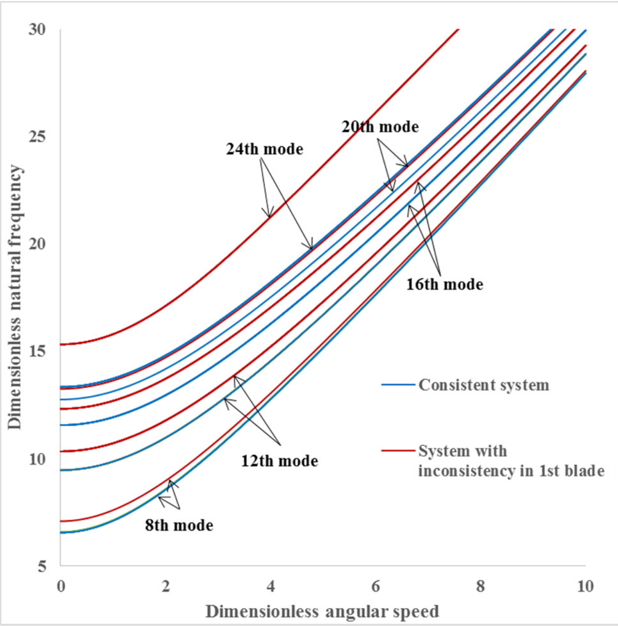


**Figure 5.** Dimensionless natural frequencies of a multi blade system without any inconsistency (perfect system) and a system containing one different blade in material parameters ($\beta_D$ =2e7 and $\beta_S$ = 100) in the position of the 1st blade (see Fig.3) versus the dimensionless angular speed (γ)

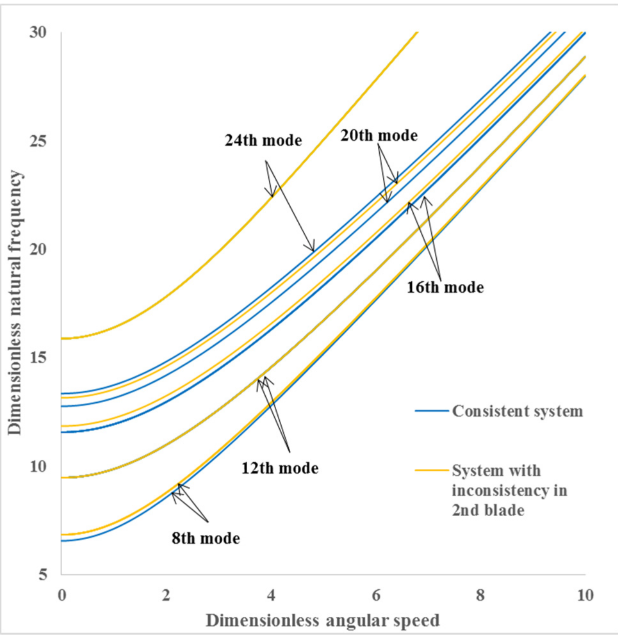


**Figure 6.** Dimensionless natural frequencies of a multi blade system without any inconsistency (perfect system) and a system containing one different blade in material parameters ($\beta_D$ =2e7 and $\beta_S$ = 100) in the position of the 2nd blade (see Fig.3) versus the dimensionless angular speed (γ)

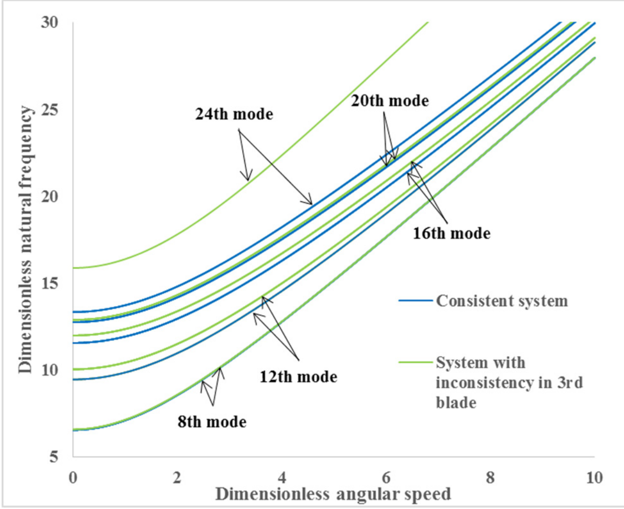


**Figure 7.** Dimensionless natural frequencies of a multi blade system without any inconsistency (perfect system) and a system containing one different blade in material parameters ($\beta_D$ =2e7 and $\beta_S$ = 100) in the position of the 3rd blade (see Fig.3) versus the dimensionless angular speed (γ)

As shown in Fig. 5, there is a significant jump in the natural frequency of the 24th mode. This jump in natural frequencies of the 24th mode seems to be more for the higher values of angular speed. On the other hand, for the other four modes, the rate of variation in natural frequencies due to the inconsistency is decreased while the angular speed is increased. As same as observed in Fig. 5, the highest values of natural frequency jump are observed for the 24th mode in Fig. 6 which relates to the case of 2nd blade as the inconsistent blade. In this case (Fig. 6), the natural frequencies of the 12th mode are not affected due to inconsistency which can be attributed to the mode shapes of the blades. Similar to those of Fig. 5 and Fig. 6, again 24th mode shows the highest jump in natural frequency curves of Fig. 7 which relates to the system with inconsistency in the 3rd blade. This time, the 8th mode is not affected due to the inconsistency which again can be attributed to the mode shapes. In order to investigate the effect of mode shape on the way of variation in natural frequencies, the 8th and 12th mode shapes for consistent multi blade system (considering the first assumed bending mode as the mode of deflection) are plotted in Fig. 8 and Fig. 9.

The 3rd blade of the system has very low displacement in the 8th mode shape plotted in Fig. 8. This is the reason why the 8th mode rarely affected by the inconsistency in the position of 3rd blade. Additionally, as shown in Fig. 9, since 2nd blade in 12th mode shape is not displaced, this mode is not affected by the inconsistency of the system in the position of 2nd blade. These information about the way of variation in system natural frequencies due to inconsistency can be employed to avoid the multi

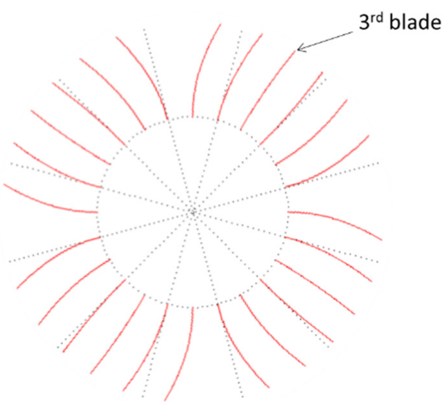


**Figure 8.** The 8th mode shape of the consistent multi blade system

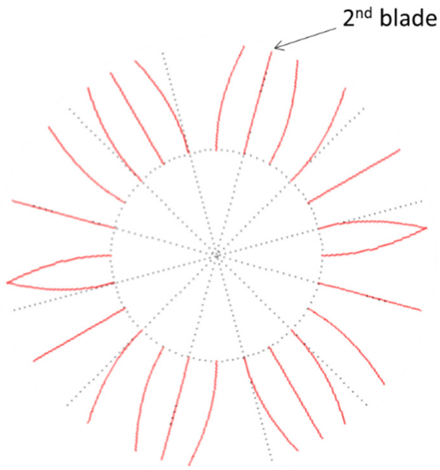


**Figure 9.** The 12th mode shape of the consistent multi blade system

blade systems to work in the frequencies close to their natural frequencies. This can be applicable, especially for the systems, in which, there is a high possibility to have an inconsistent blade. In the other word, natural frequencies of the multi blade systems, which have the possibility of system inconsistency, should be counted as an important issue in the dynamic behavior of the system.

## 4. CONCLUSIONS

In this study, the effect of the existence of an inconsistent blade on the natural frequencies of a rotating multi blade system was investigated. For this purpose, the natural frequencies of a perfect blade system and a system containing one different blade in material property have been compared for a domain of dimensionless angular speed. Based on the comparison results, although the majority of the natural frequencies (including first seven modes) did not show significant variation, however, some of the natural frequencies (here, 8th, 12th, 16th, 20th and 24th modes) were affected (natural frequency of the system is increased) in case of an inconsistent blade in different possible positions. Increasing in natural frequency was more significant for the 25th mode. The obtained information about the way of variation in system natural frequencies due to inconsistency can be employed to keep the multi blade systems working frequencies away from the system natural frequencies, especially for the systems, in which, there is a high possibility to have an inconsistent blade.

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