Heat Transfer Research 45(3), 279–292 (2014)

NUMERICAL SIMULATION OF NATURAL CONVECTION AROUND AN OBSTACLE PLACED IN AN ENCLOSURE FILLED WITH DIFFERENT TYPES OF NANOFLUIDS

Mohammad Hemmat Esfe,[1] Ali Akbar Abbasian Arani,[1] Arash Karimipour,[2,*] & Seyed Sadegh Mirtalebi Esforjani[2]

[1]*Faculty of Mechanical Engineering, Kashan University, Kashan, Iran*

[2]*Department of Mechanical Engineering, Najafabad Branch, Islamic Azad University, Isfahan, Iran*

[*]Address all correspondence to Arsah Karimipour
E-mail: arashkarimipour@gmail.com

The present numerical study deals with natural convection in an enclosure with a heated cylindrical block filled with nanofluid. The governing equations have been discretized using the finite volume method, and the SIMPLE algorithm has been used to couple velocity and pressure fields. The effect of the Rayleigh number, type, solid volume fraction of nanoparticles, radius, and position of the hot body is studied, and obtained results are presented in the form of streamline and isotherm plots, a Nusselt diagram, and comparative tables. Water, Cu–water, and Al_2O_3–water have been utilized as working fluids. In this study, the particle diameters of Cu and Al_2O_3 nanoparticles are 90 and 47 nm, respectively. On the basis of the results, increasing solid volume fraction and Ra number causes a noticeable enhancement in the rate of heat transfer. Also, increasing the radius of the hot circular cylinder, differences in values of Nusselt number between base fluid and nanofluid significantly increase.

KEY WORDS: *natural convection, nanofluid, finite volume method, square cavity, Rayleigh*

1. INTRODUCTION

Free convection has many applications in industry, such as ventilation systems, fire control systems, heat transfer in buildings, solar thermal collector systems, electronic device cooling, radioactive waste material disposal (Bilgen, 2005), double-glazed windows, nuclear reactor cooling (Bilgen, 2002), aerospace science, and chemical apparatuses. Therefore study and research about new aspects of this issue seem essential.

Previous studies in the field of square cavities can be classified into three different categories according to types and conditions of lids: (1) studies that focus on cavities with fixed lids, in which there exists no movement on the surface of the lids, and

NOMENCLATURE

h	heat transfer coefficient, W/m^2·K	X, Y	dimensionless Cartesian coordinates
K	thermal conductivity, W/m·K	g	gravity, m/s^2
R	radius of circular cylinder	**Greek Symbols**	
L	length of the cavity, m	α	thermal diffusivity, m^2/s
Nu	Nusselt number	β	thermal expansion coefficient, 1/K
Nu_y	local Nusselt number	μ	dynamic viscosity, kg/ms
p	pressure, Pa	ρ	density, kg/m^3
P	dimensionless pressure	Θ	dimensionless temperature
Pr	Prandtl number, $\nu f/\alpha f$	φ	volume fraction of the nanoparticles
Ra	Rayleigh number, $(g\beta f\Delta TL^3)/(\nu f\alpha f)$	Ψ	stream function
Re	Reynolds number, $(\rho f u_0 L)/\mu \mathrm{f}$	**Subscripts**	
T	temperature, K	f	pure fluid
u,v	dimensional velocity components in x and y directions, m/s	nf	nanofluid
		s	solid
U, V	dimensionless velocity components in X and Y directions	p	particle
		eff	effective
x, y	dimensional Cartesian coordinates, m	max	maximum

also, inside the cavity, there are no fins, heaters, and heating and cooling devices; (2) studies on cavities in which one or more lids are moving with constant or variable velocity; and (3) research on cavities with fixed lids and the presence of fins, baffles, and insulation cylinders for heating or cooling inside them. It should be noted that the third case is among the most practical studies, and this article is also about the same area. Of course, previous studies on cavities can also be classified according to the fluid in the cavity.

Frederick (1989) has executed a numerical study on an air-containing cavity in which thin fins are located in the middle of the cold lids of the cavity for different ranges of Rayleigh number. The results of his study demonstrated 47% reduction in heat transfer compared to a finless cavity. Scozia and Frederick (2005) studied free heat transfer convection in a long rectangular cavity having fins on its cold lids. They showed that Nusselt number has maximum and minimum values, and these values are greatly dependent on fin length and Rayleigh number.

All of the earlier-mentioned studies were conducted on the effects of applying concentrated smooth fins; however, Tasnim and Collins (2005) investigated heat transfer control in a square cavity having curved insulated lids. They observed that stream and temperature fields vary owing to obstructive effects of the curved lids, and also, by increasing the length and angle of the curved insulation sheets, Nusselt number decreases. Regardless of studies on fins, various studies have also been done on cylinders with different geometries.

Numerical study of heat transfer in a cavity having square cylinders at its center was the topic of research by Mezrhab et al. (2006). The fluid in their study was air (Pr = 0.71), and variations in temperature and velocity fields for different values of Rayleigh number and thermal conduction ratio have been investigated.

Numerical study on natural convection in a cavity having an inclined hot square cylinder was done by Kumar De and Dalal (2006). In their study, by varying Rayleigh number from 103 to 106, they analyzed the cavity aspect ratio and displacement of the square cylinder on temperature and stream fields. There is another study that was conducted by Oztop et al. (2009) on investigation of mixed convection heat transfer in a cavity with moving lids having circular cylinders. They found out that heat transfer and fluid flow significantly vary with location of the cylinders.

In recent years, considering the importance of using nanofluids especially in the field of improving heat transfer, application of nanofluids inside the cavities has also became prevalent, and effects of nanofluid parameters such as type, volume fraction, and nanofluid temperature on the amount of heat transfer and stream characteristics have been studied as an important and practical issue.

The majority of previous studies on devising cylinders and fins to control heat transfer have been done assuming air and water as the working fluids; therefore studies on the same subject but using other types of fluids, especially nanofluids with different volume fractions, are highly beneficial. Research was conducted by Mahmoudi et al. (2010) focusing on natural convection around a heat source devised in a square cavity. The difference in their study was that they used nanofluid as the working fluid.

Owing to a lack of extensive research on nanofluids, on heat transfer inside a cavity with a hot square cylinder, and also considering application of this geometry in engineering, the purpose of this study is to investigate this geometry with respect to different effective factors such as cylinder diameter, strength and weakness of buoyancy force, type of suspended particles, and volume fraction of nanoparticles. The results of this study are presented in the form of temperature and stream contours and different figures.

2. BOUNDARY CONDITIONS AND GOVERNING EQUATIONS

A schematic of the problem is shown in Fig. 1. A hot cylinder with radius r and different positions at a high constant temperature (T_h) is considered to be located inside the cavity, and the side lids remain at a low constant temperature (T_c). The upper and lower lids are also insulated.

Fluid and nanofluids used in this problem are considered to be incompressible and Newtonian, and the stream is laminar and steady in all areas of the cavity. Except for the density, which varies according to the Boussinesq approximation, other thermophysical properties of water and nanofluid are assumed to be constant. The particle diameters of Cu and Al_2O_3 nanoparticles are 90 and 47 nm, respectively. Thermophysical properties of the base fluid and two types of nanoparticles are tabulated in Table 1.

Regarding the nanofluid, it should be noted that nanoparticles and base fluid are at thermal equilibrium and also that no slip occurs between them. In this study, nanoparticles are considered to be spherical with the same size, and radiation heat transfer between surfaces is neglected. Continuum, momentum, and energy equations for a laminar and steady free convective stream of nanofluid in a 2-D square cavity can be written as follows:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \ , \tag{1}$$

$$u \frac{\partial u}{\partial x} + v \frac{\partial v}{\partial y} = - \frac{1}{\rho_{if}} \frac{\partial p}{\partial x} + \upsilon_{nf} \nabla^2 u \ , \tag{2}$$

$$\begin{aligned} & u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} = -\frac{1}{\rho_{nf}}\frac{\partial p}{\partial y} + \upsilon_{nf}\nabla^2 v \\ & +\left(\rho\beta\right)_{nf} g(T - T_c) \end{aligned} \tag{3}$$

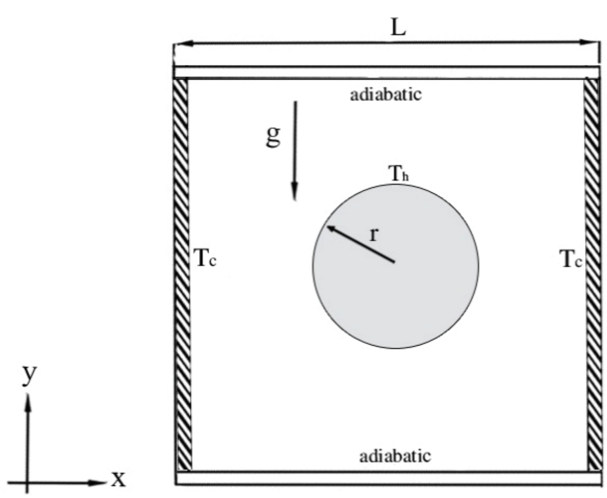


FIG. 1: Schematic view of the square cavity

TABLE 1: Thermophysical properties of base fluid and nanoparticles

Properties	Water	Cu	Al_2O_3
C_p, J/kg·k	4179	385	765
ρ, kg/m^3	997.1	8933	3970
K, W/mK	0.613	400	25
$\beta \times 10^{-5}$, 1/K	21	1.67	0.85
d_p, nm	–	90	47

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \alpha_{nf}\nabla^2 T . \tag{4}$$

Then, nondimensionalized forms of the governing equations are obtained using dimensionless parameters:

$$X = \frac{x}{L},\ Y = \frac{y}{L},\ R = \frac{r}{L},$$

$$U = \frac{uH}{\alpha_f},\ V = \frac{vH}{\alpha_f}, \tag{5}$$

$$\theta = \frac{T - T_c}{T_h - T_c},\quad P = \frac{pL^2}{\rho_{nf}\ \alpha_f^2},$$

$$\mathrm{Ra} = \frac{g\beta_f \Delta T L^3}{\upsilon_f \alpha_f},\quad \mathrm{Pr} = \frac{\upsilon_f}{\alpha_f}. \tag{6}$$

Nanofluid effective density, thermal diffusivity, specific heat capacity, and dynamic viscosity are calculated by the following equations:

$$\rho_{nf} = (1-\varphi)\,\rho_f + \varphi\rho_s, \tag{7}$$

$$\alpha_{nf} = k_{nf}/(\rho c_p)_{nf}, \tag{8}$$

$$(\rho c_p)_{nf} = (1-\varphi)\,(\rho c_p)_f + \varphi\,(\rho c_p)_s, \tag{9}$$

$$(\rho\beta)_{nf} = (1-\varphi)\,(\rho\beta)_f + \varphi\,(\rho\beta)_s, \tag{10}$$

$$\frac{\mu_{nf}}{\mu_f} = \frac{1}{(1-\varphi)^{2.5}} . \tag{11}$$

To calculate the nanofluid heat conduction coefficient, the Maxwell equation is used:

$$\frac{k_{nf}}{k_f} = \frac{\left(k_s + 2k_f\right) - 2\varphi\left(k_f - k_s\right)}{\left(k_f + 2k_s\right) + \varphi\left(k_f - k_s\right)} , \tag{12}$$

where k_p and k_f are nanoparticle thermal coefficient and base fluid thermal coefficient, respectively.

3. NUMERICAL METHOD AND VALIDATION

The previously mentioned equations are solved using the finite volume method, and the relation between pressure and velocity is obtained by the SIMPLE algorithm. The geometry of the problem, the presence of a curved sheet inside the cavity, leads us to use tetrahedron meshing with different densities, as shown in Fig. 2. Therefore all the computational area of the cavity is divided into four parts, and to have accuracy in the solution, inhomogeneous meshing, which is denser in the areas near the curves, is considered.

To validate the results obtained for this geometry and boundary condition, they are compared with results of Hadjisophocleous et al. (1998), Fusegi et al. (1991), Ha and Jung (2000), and Tiwari and Das (2007). The results of this study are in good agreement with the mentioned studies and are shown in Table 2.

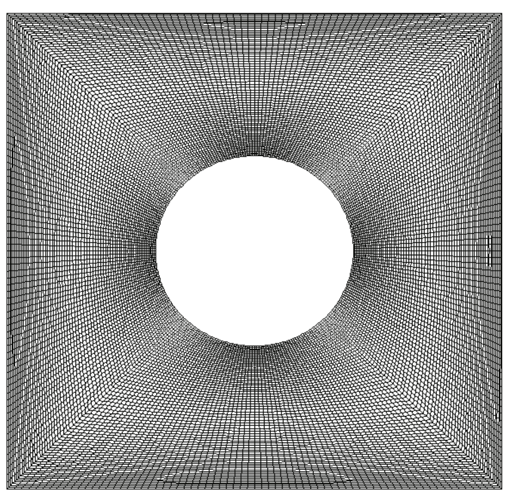


FIG. 2: Square cavity with tetrahedron mesh with different densities

TABLE 2: Comparison of present study with similar studies

		Present study	**Hadjisophocleous et al. (1998)**	**Fusegi et al. (1991)**	**Ha and Jung (2000)**	**Tiwari and Das (2007)**
$Ra = 10^3$	Nu	1.1165	1.141	1.0871	1.085	1.072
	Nu_{max}	1.5745	1.540	1.508		
	Nu_{min}	0.7084	0.727	0.6901		
$Ra = 10^4$	Nu	2.1742	2.29	2.195	2.1	2.070
	Nu_{max}	3.5055	3.84	3.5585		
	Nu_{min}	0.5535	0.670	0.5809		
$Ra = 10^5$	Nu	4.4792	4.964	4.450	4.361	4.464
	Nu_{max}	7.5897	8.93	7.9371		
	Nu_{min}	0.8487	1.01	0.7173		
$Ra = 10^6$	Nu	8.784	10.39	8.803		
	Nu_{max}	16.964	21.41	19.2675		
	Nu_{min}	1.295	1.58	0.9420		

4. RESULTS AND DISCUSSION

The results obtained from the numerical simulation of the free convective flow in a square cavity having hot circular cylinder in its middle, which are in the form of stream and temperature contours and various figures, are discussed in this section. The effects of some important parameters, such as Rayleigh number, radius, type of nanofluids, and nanoparticle volume fraction, on heat transfer and fluid flow inside the cavity are separately analyzed in the following sections.

4.1 Effect of Radius of Circular Cylinder on Streamlines and Nanofluid Isotherm Line

The radius of a hot circular cylinder is among the parameters that not only affect the value of the Rayleigh number but also the control of heat transfer in a cavity with a cylinder inside. To investigate the effects of these parameters on the base fluid and nanofluid with different values, volume fraction and stream and temperature contours for this case are selectively shown in this section.

As can be seen in the temperature contours of Fig. 3, by increasing the radius of the circular cylinder, which is located in the center of the cavity, the density of the temperature lines in the lower section of the cavity increases, and therefore heat exchange also increases in this area.

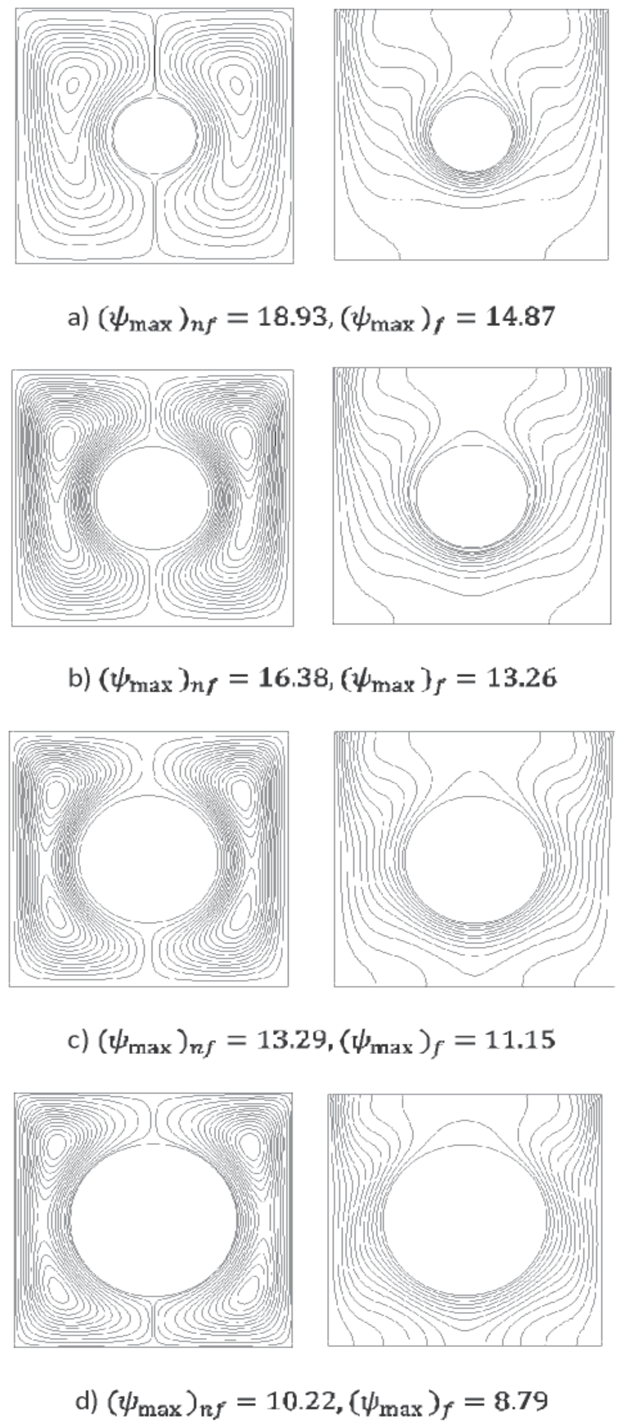


FIG. 3: Temperature (right) and streamline (left) contours for water–Cu nanofluid in volume fraction of 6% for (a) r = 0.15, (b) r = 0.2, (c) r = 0.25, and (d) r = 0.3

Density of the lines in the upper section of the vertical lid does not have a considerable change when increasing the radius of a hot circular cylinder. Therefore it is expected that temperature gradient and, consequently, Nusselt number in the previously mentioned cases are not much different from each other in the upper section of the cavity. In the mid-section and lower section of the cavity and near the lids, density of the lines gently increases by increasing the radius of the hot circular cylinder, which shows that there is more heat transfer in these areas.

Also streamlines at the smaller radius show that the flow intensity and strength increases in the upper section of the cavity, while, by increasing radius, flow in the mid-section also becomes stronger. Adding nanoparticles to the base fluid increases the value of ψ_{max}. This increase is different for different radii, and increasing the radius of the hot circular cylinder causes a decline in the rate of this increase. Value of ψ_{max} increases about 23.5%, 19.19%, and 16.2% for the concentration equal to 6% at $r = 0.15$, $r = 0.25$, and $r = 0.3$, respectively.

4.2 Investigating Heat Transfer with Variation of Rayleigh Number

In this section, to have a better understanding about the importance of Rayleigh number in heat transfer control, figures for average Nusselt number with respect to Rayleigh number are shown for base fluid and water–copper nanofluid with volume fraction of 8% at different radii of a hot circular cylinder when the cylinder is located at $c(x,y) = (0.5,0.5)$.

Figure 4 demonstrates average Nusselt number variation with respect to Rayleigh number for base fluid and nanofluid at different values of a hot cylinder radius. As can be noted from the figures, there is a similar trend for average Nusselt number variation with respect to Rayleigh number for base fluid and nanofluid. As the figures show, increasing Rayleigh number at every specific radius causes Nusselt number to significantly increase.

Conversely, the slope of the lines at higher values of Rayleigh number also considerably increases, and this confirms that there is a greater amount of convection heat transfer in this case. By increasing the radius of the hot circular cylinder, and owing to an increase in the contact surface of the fluid and the cylinder, buoyancy force increases and Nusselt number and, correspondingly, the total amount of heat transfer increases, which can be clearly observed in the preceding figures.

4.3 Investigating Nusselt Number Value in Two Types of Nanofluids with Different Values of Volume Fraction

Figure 5 demonstrates variation of Nusselt number in the base fluid, water–copper nanofluid, and water–aluminum oxide nanofluid with respect to Rayleigh number. All the calculations are done for a square cavity with a hot circular cylinder with $r = 0.3$ inside it at location of $c(x, y) = (0.5, 0.5)$, $r = 0.2$.

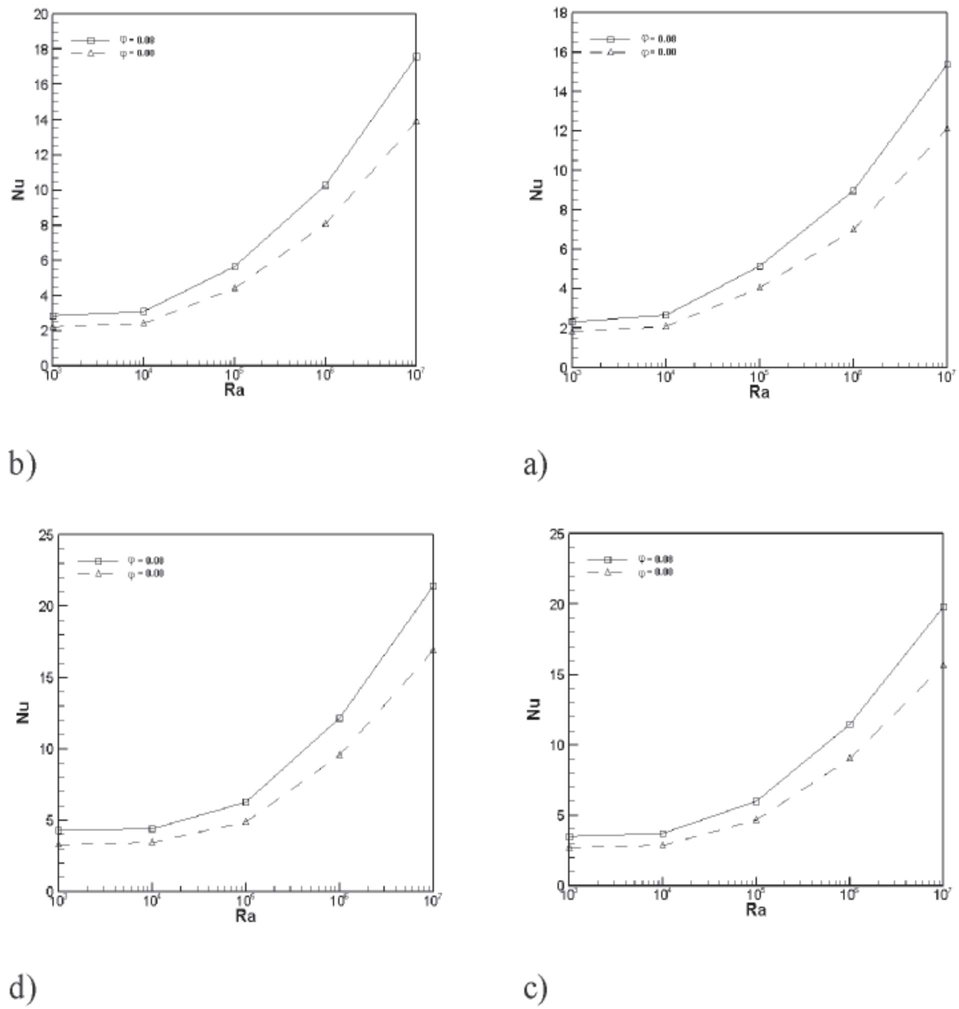
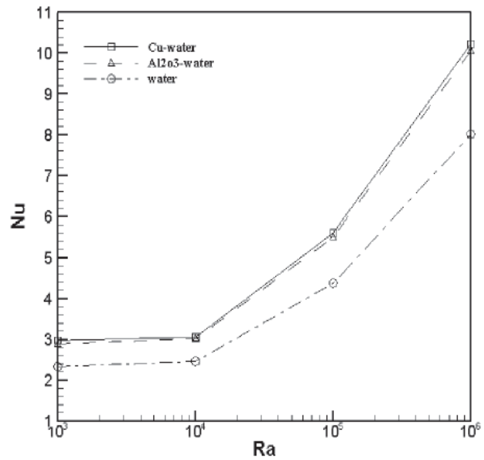


FIG. 4: Average Nusselt number variation with respect to Rayleigh number for the base fluid and nanofluid for (a) $r = 0.15$, (b) $r = 0.2$, (c) $r = 0.25$, and (d) $r = 0.3$

As can be seen in Fig. 5, when the volume fraction is 4%, the effect of adding nanoparticles to the base fluid is relatively stronger at high values of Rayleigh number compared to lower values of Rayleigh number.

Furthermore, it can be argued that the amount of heat transfer in nanoparticles having lower volume fraction are the same, and average Nusselt numbers of different nanofluids are also almost the same. Conversely, as can be understood from comparison of the preceding figures, by increasing the volume fraction of nanoparticles, the effect of different nanoparticles becomes more obvious in a way that water–copper



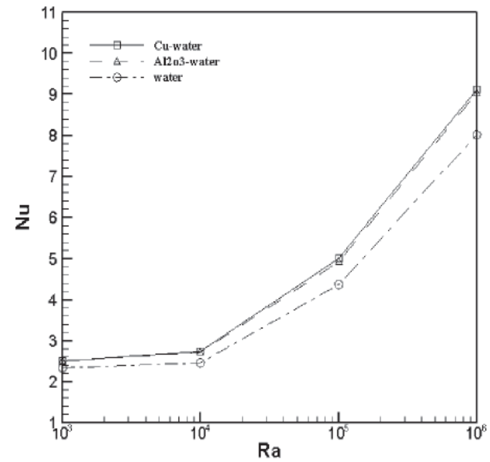


FIG. 5: Variation of Nusselt number in the base fluid, water–copper nanofluid, and water–aluminum oxide nanofluid with respect to Rayleigh number for (a) 4% of volume fraction and (b) 8% of volume fraction

nanofluid, which has a higher value of conduction heat transfer coefficient compared to water–allumina nanofluid, causes higher values of Nusselt number and, consequently, more heat transfer.

Generally, when determining Nusselt number for nanofluids, two factors, temperature gradient and ratio of conduction heat transfer coefficient of the nanofluid to base fluid, are very important. Increasing the volume fraction of nanoparticles, conduction heat transfer, and therefore thermal diffusivity, increases. Hence the temperature gradient decreases. However, as was argued before, for determining Nusselt number, in addition to temperature gradient, ratio of conduction heat transfer coefficients is also important, and because increasing the concentration of nanoparticles causes an increase in the ratio of the conduction heat transfer coefficients, and the amount of this increase is more than a reduction in temperature gradient; therefore it can be concluded that an increase in nanoparticles concentration causes Nusselt number to increase. Also, for higher values of nanoparticle volume fraction, an increase in average Nusselt number is higher for high Rayleigh numbers compared to low Rayleigh numbers.

4.4 Variations of Heat Transfer with Hot Circular Cylinder Radius

As is expected, Nusselt number and the amount of heat transfer increases by increasing the radius of the hot circular cylinder. Because one of the most important purposes of this study is to investigate effects of different parameters on stream field and heat transfer and their variations to control the amount of heat transfer in different

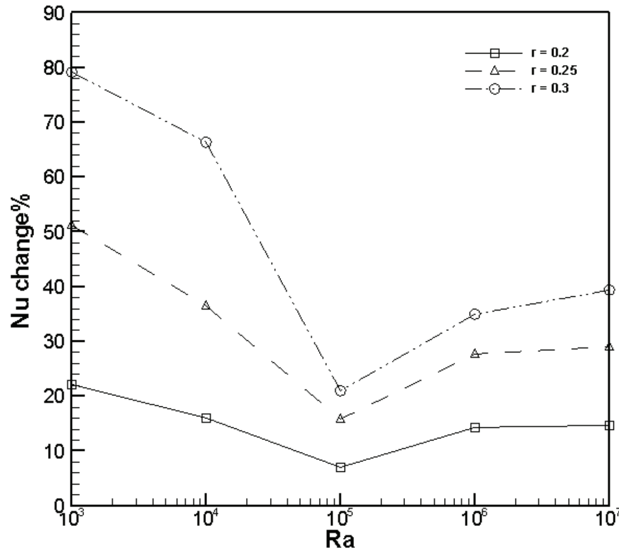


FIG. 6: Variations in percentage of Nusselt number with respect to Rayleigh number for water–copper nanofluid with volume fraction of 8% in different radii of the cylinder

cases and positions, finding the percentage of heat transfer variations with respect to variations of effective parameters, and finding the maximum and minimum of variations, are among the factors that can help to better understand and analyze this issue. Therefore, by using a common base, variations are calculated in percentages for different cases, and the corresponding results are presented.

If the radius of the circular cylinder is assumed to be equal to 0.15 times the length of the cavity as a reference case, the increase in percentage of Nusselt number is 22.5% for base fluid in Ra = 10^3 and for dimensionless radius equal to 0.2 with respect to the reference case. This value for R = 0.25 and R = 0.3 is 50.2 and 82.5, respectively.

To have a better understanding of Nusselt number variations with radius of the hot circular cylinder, the variations in percentage of Nusselt number for water–copper nanofluid with a volume fraction of 8% are presented in Fig. 5.

This figure shows that maximum variation of dimensionless Nusselt number occur at Ra = 10^3 and R = 0.3 is equal to 79.15%, which is slightly less than the similar case for base fluid. Also, minimum variation occurs at Ra = 10^5 and R = 0.2 and is equal to 7% and 8.5% for base fluid and nanofluid, respectively.

Conversely, as can be seen from the figure, variations are higher at lower values of Rayleigh number compared to higher values of Rayleigh number, and the slope of these variations also increases with an increase in hot cylinder radius.

5. CONCLUSIONS

In this article, free convection flow inside a square cavity and around a hot circular cylinder was investigated. Effects of different parameters, such as nanoparticle type, Rayleigh number, volume fraction of nanoparticles, radius of the hot cylinder, and position of the cylinder, were also investigated, and the following results were extracted:

1. Increasing Rayleigh number at all values of the cylinder radius and positions causes the Nusselt number in base fluid and nanofluid to increase; of course, the increase in nanofluid was slightly more than that in base fluid.

2. Increasing the volume fraction of nanoparticles, the effect of nanoparticle type becomes more important in a way that copper nanoparticles, which have a higher conduction coefficient than alumina nanoparticles, have more increase on average Nusselt number and consequently a greater amount of heat transfer.

3. Increasing the radius of the hot circular cylinder, differences in values of Nusselt number between base fluid and nanofluid significantly increase.

REFERENCES

Bilgen, E., Natural convection in cavities with a thin fin on the hot wall, *Int. J. Heat Mass Transfer*, vol. 48, pp. 3493–3505, 2005.

Bilgen, E., Natural convection in enclosures with partial partitions, *Renewable Energy*, vol. 26, pp. 257–70, 2002.

Frederick, R. L., Natural convection in an inclined square enclosure with a partition attached to its cold wall, *Int. J. Heat Mass Transfer*, vol. 32, pp. 87–94, 1989.

Scozia, R. and Frederick, R. L., Reformulation of the simplen discretization scheme to accommodate noncentralized interfaces, *Numer. Heat Transfer*, vol. 20, pp. 127–144, 1991.

Tasnim, S. H. and Collins, M. R., Suppressing natural convection in a differentially heated square cavity with an arc shaped baffle, *Int. Commun. Heat Mass Transfer,* vol. 32, pp. 94–106, 2005.

Mezrhab, A., Bouali, H., Amaoui, H., and Bouzidi, M., Computation of combined natural-convection and radiation heat-transfer in a cavity having a square body at its center, *Appl. Energy*, vol. 83, pp. 1004–1023, 2006.

De Kumar A. and Dalal, A., A numerical study of natural convection around a square, horizontal, heated cylinder placed in an enclosure, *Int. J. Heat Mass Transfer*, vol. 49, pp. 4608–4623, 2006.

Oztop, H. F., Zhao, Z., and Yub, B., Fluid flow due to combined convection in lid-driven enclosure having a circular body, *Int. J. Heat Fluid Flow*, vol. 30, pp. 886–901, 2009.

Mahmoudi, A. H., Shahi, M., Honarbakhsh Raouf, A., and Ghasemian, A., Numerical study of natural convection cooling of horizontal heat source mounted in a square cavity filled with nanofluid, *Int. Commun. Heat Mass Transfer,* vol. 37, pp. 1135–1141, 2010.

Hadjisophocleous, G. V., Sousa, A. C. M., and Venart, J. E. S., Prediction of transient natural convection in enclosures of arbitrary geometry using a nonorthogonal numerical model, *Numer. Heat Transfer, Part A,* vol. 13, pp. 373–392, 1998.

Fusegi, T., Kuwahara, K., and Farouk, B., A numerical study of three-dimensional natural convection in a differentially heated cubical enclosure, *Int. J. Heat Mass Transfer*, vol. 34, pp. 1543–1557, 1991.

Ha, M. Y. and Jung, M. J., A numerical study on three-dimensional conjugate heat transfer of natural convection and conduction in a differentially heated cubic enclosure with a heat-generating cubic conducting body, *Int. J. Heat Mass Transfer,* vol. 43, pp. 4229–4248, 2000.

Tiwari, R. K. and Das, M. K., Heat transfer augmentation in a two-sided lid-driven differentially heated square cavity utilizing nanofluids, *Int. J. Heat Mass Transfer*, vol. 50, pp. 2002–2018, 2007.