

Coefficient estimates for a subclass of analytic functions by Srivastava-Attiya operator

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Abstract. In this paper, we investigate bounds of the coefficients for subclass of analytic and bi-univalent functions. The results presented in this paper would generalize and improve some recent works and other authors.

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1. Introduction

Let \mathcal{A} be a class of functions of the form

$$f(z) = z + \sum_{n=2}^{\infty} a_n z^n, \quad (1.1)$$

which are analytic in the open unit disk $\mathbb{U} = \{z \in \mathbb{C} : |z| < 1\}$. Further, let \mathcal{S} denote the class of functions $f \in \mathcal{A}$ which are univalent in \mathbb{U} .

For $f(z)$ defined by (1.1) and $h(z)$ defined by

$$h(z) = z + \sum_{n=2}^{\infty} b_n z^n,$$

the Hadamard product $(f * h)(z)$ of the functions $f(z)$ and $h(z)$ defined by

$$(f * h)(z) = z + \sum_{n=2}^{\infty} a_n b_n z^n,$$

In 2007, Srivastava and Attiya [21] (see also Răducanu and Srivastava [18] and Prajapat and Goyal [17]) for the class \mathcal{A} introduced and investigated linear operator

$\mathcal{J}_\mu^b : \mathcal{A} \rightarrow \mathcal{A}$ that defined in terms of the Hadamard product by

$$\mathcal{J}_\mu^b f(z) = z + \sum_{k=2}^{\infty} \Theta_k a_k z^k,$$

where

$$\Theta_k = \left| \left(\frac{1+b}{k+b} \right)^\mu \right|,$$

and (throughout this paper unless otherwise mentioned) the parameters μ , b are considered as $\mu \in \mathbb{C}$ and $b \in \mathbb{C} \setminus \{0, -1, -2, \dots\}$, (see for more details [20]).

Remark 1.1. (1) For $\mu = 1$ and $b = v$ ($v > -1$), we get generalized Libera-Bernardi integral operator [19];

(2) For $\mu = \sigma$ ($\sigma > 0$) and $b = 1$, we get Jung-Kim-Srivastava integral operator [12].

For each $f \in \mathcal{S}$, the Koebe one-quarter theorem [9] ensures that the image of \mathbb{U} under f contains a disk of radius $\frac{1}{4}$. Hence every function $f \in \mathcal{S}$ has an inverse f^{-1} , which is defined by

$$f^{-1}(f(z)) = z \quad (z \in \mathbb{U}),$$

and

$$f(f^{-1}(w)) = w \quad \left(|w| < r_0(f); r_0(f) \geq \frac{1}{4} \right),$$

where

$$g(w) = f^{-1}(w) = w - a_2 w^2 + (2a_2^2 - a_3) w^3 + \dots \quad (1.2)$$

A function $f \in \mathcal{A}$ is said to be bi-univalent in \mathbb{U} if both f and f^{-1} are univalent in \mathbb{U} . Let Σ denote the class of bi-univalent functions in \mathbb{U} given by (1.1).

Recently many researchers have introduced and investigated several interesting subclasses of the bi-univalent function class Σ and they have found non-sharp estimates on the first two Taylor-Maclaurin coefficients $|a_2|$ and $|a_3|$ and other problems, see for example, [3, 2, 4, 5, 6, 7, 8, 10, 11, 13, 14, 15, 22, 23, 24].

For two functions f and g that are analytic in \mathbb{U} , we say that the function f is *subordinate* to g and write $f(z) \prec g(z)$, if there exists a Schwarz function ω , that is analytic in \mathbb{U} with $\omega(0) = 0$ and $|\omega(z)| < 1$ such that $f(z) = g(\omega(z))$ for all $z \in \mathbb{U}$.

In particular, if the function g is univalent in \mathbb{U} , then $f(z) \prec g(z)$ if and only if $f(0) = g(0)$ and $f(\mathbb{U}) \subseteq g(\mathbb{U})$.

In this work, we obtain estimates of coefficients for a subclass of bi-univalent functions considered by Selvaraj et al. [20]. The results presented in this paper would generalize and improve some recent works and other authors.

2. The subclass $\mathcal{S}_{\Sigma,t}^{\mu,b}(\gamma, \lambda, \phi)$

Throughout this paper, we assume that ϕ is an analytic function with positive real part in the unit disk \mathbb{U} , satisfying $\phi(0) = 1$, $\phi'(0) > 0$ and symmetric with respect to the real axis. Such a function has series expansion of the form

$$\phi(z) = 1 + B_1z + B_2z^2 + B_3z^3 + \cdots \quad (B_1 > 0), \quad (2.1)$$

Let that $u(z)$ and $v(z)$ are Schwarz function in \mathbb{U} with

$$u(0) = v(0) = 0, \quad |u(z)| < 1, \quad |v(z)| < 1$$

and suppose that

$$u(z) = \sum_{n=1}^{\infty} p_n z^n \quad \text{and} \quad v(z) = \sum_{n=1}^{\infty} q_n z^n \quad (z \in \mathbb{U}). \quad (2.2)$$

Then [16, p. 172]

$$|p_1| \leq 1, \quad |p_2| \leq 1 - |p_1|^2, \quad |q_1| \leq 1, \quad |q_2| \leq 1 - |q_1|^2. \quad (2.3)$$

By (2.1), we get

$$\phi(u(z)) = 1 + B_1 p_1 z + (B_1 p_2 + B_2 p_1^2) z^2 + \cdots \quad (z \in \mathbb{U}) \quad (2.4)$$

and

$$\phi(v(w)) = 1 + B_1 q_1 w + (B_1 q_2 + B_2 q_1^2) w^2 + \cdots \quad (w \in \mathbb{U}). \quad (2.5)$$

In 2014, Selvaraj et al. [20] introduced subclass of Σ and obtained estimates on the coefficients $|a_2|$ and $|a_3|$ for functions in this subclass as follows:

Definition 2.1. [20] A function $f \in \Sigma$ given by (1.1) is said to be in the class $\mathcal{S}_{\Sigma,t}^{\mu,b}(\gamma, \lambda, \phi)$ if the following conditions are satisfied:

$$1 + \frac{1}{\gamma} \left(\frac{[(1-t)z]^{1-\lambda} (\mathcal{J}_\mu^b f(z))'}{[\mathcal{J}_\mu^b f(z) - \mathcal{J}_\mu^b f(tz)]^{1-\lambda}} - 1 \right) \prec \phi(z),$$

and

$$1 + \frac{1}{\gamma} \left(\frac{[(1-t)w]^{1-\lambda} (\mathcal{J}_\mu^b g(w))'}{[\mathcal{J}_\mu^b g(w) - \mathcal{J}_\mu^b g(tw)]^{1-\lambda}} - 1 \right) \prec \phi(w),$$

where $|t| \leq 1$ ($t \neq 1$); $\gamma \in \mathbb{C} \setminus \{0\}$; $\lambda \geq 0$; $z, w \in \mathbb{U}$ and g is given by (1.2).

Theorem 2.2. [20] Let the function $f(z)$ given by (1.1) be in the class $\mathcal{S}_{\Sigma,t}^{\mu,b}(\gamma, \lambda, \phi)$. Then

$$|a_2| \leq \frac{|\gamma| B_1 \sqrt{2B_1}}{\sqrt{|\gamma B_1^2 \Lambda(\lambda, t) \Xi(\lambda, t) - 2(B_2 - B_1)[\Lambda(\lambda, t) + 2]^2 \Theta_2^2 + 2\gamma B_1^2 \Upsilon(\lambda, t) \Theta_3|}} \quad (2.6)$$

and

$$|a_3| \leq \frac{B_1 |\gamma|}{\Upsilon(\lambda, t) \Theta_3} + \left(\frac{B_1 |\tau|}{[\Lambda(\lambda, t) + 2] \Theta_2} \right)^2, \quad (2.7)$$

where

$$\Lambda(\lambda, t) = (\lambda - 1)(1 + t), \quad \Upsilon(\lambda, t) = [(\lambda - 1)(1 + t + t^2) + 3]$$

and

$$\Xi(\lambda, t) = [(\lambda - 2)(1 + t) + 4].$$

3. Coefficient estimates

In the section, we get that the following theorem which is an refinement of inequalities (2.6) and (2.7).

Theorem 3.1. *Let the function $f(z)$ given by (1.1) be in the class $\mathcal{S}_{\Sigma, t}^{\mu, b}(\gamma, \lambda, \phi)$, $|t| \leq 1$ ($t \neq 1$), $\gamma \in \mathbb{C} \setminus \{0\}$ and $\lambda \geq 0$. Then*

$$|a_2| \leq \frac{|\gamma|B_1\sqrt{2B_1}}{\sqrt{2B_1[\Lambda(\lambda, t) + 2]^2\Theta_2^2 + |\gamma B_1^2\Lambda(\lambda, t)\Xi(\lambda, t) - 2B_2[\Lambda(\lambda, t) + 2]^2\Theta_2^2 + 2\gamma B_1^2\Upsilon(\lambda, t)\Theta_3|}}$$

and

$$|a_3| \leq \begin{cases} \frac{|\tau|B_1}{\Upsilon(\lambda, t)\Theta_3} & B_1 \leq \frac{[(\lambda - 1)(1 + t) + 2]^2\Theta_2^2}{|\gamma|\Theta_3[(\lambda - 1)(1 + t + t^2) + 3]} \\ \frac{\Phi(\Theta_1, \Theta_2, \lambda, t)}{\Psi(\Theta_1, \Theta_2, \lambda, t)\Upsilon(\lambda, t)\Theta_3} & B_1 > \frac{[(\lambda - 1)(1 + t) + 2]^2\Theta_2^2}{|\gamma|\Theta_3[(\lambda - 1)(1 + t + t^2) + 3]}. \end{cases}$$

where

$$\Phi(\Theta_1, \Theta_2, \lambda, t) = |\tau|B_1 |\gamma B_1^2\Lambda(\lambda, t)\Xi(\lambda, t) - 2B_2[\Lambda(\lambda, t) + 2]^2\Theta_2^2 + 2\gamma B_1^2\Upsilon(\lambda, t)\Theta_3| + 2|\gamma|^2\Theta_3\Upsilon(\lambda, t)B_1^3,$$

and

$$\Psi(\Theta_1, \Theta_2, \lambda, t) = 2B_1[\Lambda(\lambda, t) + 2]^2\Theta_2^2 + |\gamma B_1^2\Lambda(\lambda, t)\Xi(\lambda, t) - 2B_2[\Lambda(\lambda, t) + 2]^2\Theta_2^2 + 2\gamma B_1^2\Upsilon(\lambda, t)\Theta_3|.$$

Proof. Let $f \in \mathcal{S}_{\Sigma, t}^{\mu, b}(\gamma, \lambda, \phi)$ and $g = f^{-1}$. Then there are analytic functions $u, v : \mathbb{U} \rightarrow \mathbb{U}$, with $u(0) = v(0) = 0$, given by (2.2) such that

$$1 + \frac{1}{\gamma} \left(\frac{[(1-t)z]^{1-\lambda}(\mathcal{J}_\mu^b f(z))'}{[\mathcal{J}_\mu^b f(z) - \mathcal{J}_\mu^b f(tz)]^{1-\lambda}} - 1 \right) = \phi(u(z)), \quad (3.1)$$

and

$$1 + \frac{1}{\gamma} \left(\frac{[(1-t)w]^{1-\lambda}(\mathcal{J}_\mu^b g(w))'}{[\mathcal{J}_\mu^b g(w) - \mathcal{J}_\mu^b g(tw)]^{1-\lambda}} - 1 \right) = \phi(v(w)). \quad (3.2)$$

From (2.4), (2.5), (3.1) and (3.2), we obtain

$$[(\lambda - 1)(1 + t) + 2]\Theta_2 a_2 = \gamma B_1 p_1, \quad (3.3)$$

$$\begin{aligned} & [(\lambda - 1)(1 + t + t^2) + 3]\Theta_3 a_3 + \frac{1}{2}(\lambda - 1)(1 + t)[(\lambda - 2)(1 + t) + 4]\Theta_2^2 a_2^2 \\ & = \gamma[B_1 p_2 + B_2 p_1^2], \end{aligned} \quad (3.4)$$

$$- [(\lambda - 1)(1 + t) + 2]\Theta_2 a_2 = \gamma B_1 q_1, \quad (3.5)$$

and

$$\begin{aligned} & [(\lambda - 1)(1 + t + t^2) + 3]\Theta_3(2a_2^2 - a_3) \\ & + \frac{1}{2}(\lambda - 1)(1 + t)[(\lambda - 2)(1 + t) + 4]\Theta_2^2 a_2^2 = \gamma[B_1 q_2 + B_2 q_1^2]. \end{aligned} \quad (3.6)$$

From (3.3) and (3.5), we get

$$p_1 = -q_1. \quad (3.7)$$

Adding (3.4) and (3.6), and using (3.7), we have

$$\begin{aligned} & ((\lambda - 1)(1 + t)[(\lambda - 2)(1 + t) + 4]\Theta_2^2 + 2\Theta_3[(\lambda - 1)(1 + t + t^2) + 3])a_2^2 \\ & - 2\gamma B_2 p_1^2 = \gamma B_1(p_2 + q_2). \end{aligned} \quad (3.8)$$

From (3.3), we have

$$\begin{aligned} & (\gamma B_1^2\{(\lambda - 1)(1 + t)[(\lambda - 2)(1 + t) + 4]\Theta_2^2 + 2\Theta_3[(\lambda - 1)(1 + t + t^2) + 3]\} \\ & - 2B_2[(\lambda - 1)(1 + t) + 2]^2\Theta_2^2)a_2^2 = \gamma^2 B_1^3(p_2 + q_2). \end{aligned}$$

By (2.3) and (3.3), we get

$$\begin{aligned} & |(\gamma B_1^2\{(\lambda - 1)(1 + t)[(\lambda - 2)(1 + t) + 4]\Theta_2^2 + 2\Theta_3[(\lambda - 1)(1 + t + t^2) + 3]\} \\ & - 2B_2[(\lambda - 1)(1 + t) + 2]^2\Theta_2^2)a_2^2| \leq |\tau|^2 B_1^3(|p_2| + |q_2|) \\ & \leq 2|\gamma|^2 B_1^3(1 - |p_1|^2) \\ & = 2|\gamma|^2 B_1^3 - 2B_1[(\lambda - 1)(1 + t) + 2]^2\Theta_2^2|a_2|^2. \end{aligned}$$

Therefore,

$$\begin{aligned} & |a_2| \leq \quad (3.9) \\ & \leq \frac{|\gamma|B_1\sqrt{2B_1}}{\sqrt{2B_1[\Lambda(\lambda, t) + 2]^2\Theta_2^2 + |\gamma B_1^2\Lambda(\lambda, t)\Xi(\lambda, t) - 2B_2[\Lambda(\lambda, t) + 2]^2\Theta_2^2 + 2\gamma B_1^2\Upsilon(\lambda, t)\Theta_3|}}, \end{aligned}$$

where

$$\Lambda(\lambda, t) = (\lambda - 1)(1 + t), \quad \Upsilon(\lambda, t) = [(\lambda - 1)(1 + t + t^2) + 3]$$

and

$$\Xi(\lambda, t) = [(\lambda - 2)(1 + t) + 4].$$

Next, in order to find the bound on the coefficient $|a_3|$, by subtracting (3.6) from (3.4), and using (3.7), we get

$$\begin{aligned} & 2[(\lambda - 1)(1 + t + t^2) + 3]\Theta_3 a_3 = 2\Theta_3[(\lambda - 1)(1 + t + t^2) + 3]a_2^2 \\ & + \tau B_1(p_2 - q_2). \end{aligned} \quad (3.10)$$

Using (2.3) and (3.7), we have

$$\begin{aligned} & 2[(\lambda - 1)(1 + t + t^2) + 3]\Theta_3|a_3| \\ & \leq |\gamma|B_1(|p_2| + |q_2|) + 2\Theta_3[(\lambda - 1)(1 + t + t^2) + 3]|a_2|^2 \\ & \leq 2|\gamma|B_1(1 - |p_1|^2) + 2\Theta_3[(\lambda - 1)(1 + t + t^2) + 3]|a_2|^2. \end{aligned}$$

From (3.3), we get

$$\begin{aligned} & |\gamma|B_1[(\lambda-1)(1+t+t^2)+3]\Theta_3|a_3| \\ & \leq [|\gamma|\Theta_3[(\lambda-1)(1+t+t^2)+3]B_1 - [(\lambda-1)(1+t)+2]^2\Theta_2^2] |a_2|^2 + |\gamma|^2B_1^2. \end{aligned}$$

From (3.9), for $[|\gamma|\Theta_3[(\lambda-1)(1+t+t^2)+3]B_1 - [(\lambda-1)(1+t)+2]^2\Theta_2^2] > 0$ we have

$$\begin{aligned} & |\gamma|B_1[(\lambda-1)(1+t+t^2)+3]\Theta_3|a_3| \\ & \leq [|\gamma|\Theta_3[(\lambda-1)(1+t+t^2)+3]B_1 - [(\lambda-1)(1+t)+2]^2\Theta_2^2] \\ & \quad \times \frac{2|\gamma|^2B_1^3}{2B_1[\Lambda(\lambda,t)+2]^2\Theta_2^2 + |\gamma B_1^2\Lambda(\lambda,t)\Xi(\lambda,t) - 2B_2[\Lambda(\lambda,t)+2]^2\Theta_2^2 + 2\gamma B_1^2\Upsilon(\lambda,t)\Theta_3|} \\ & \quad + |\gamma|^2B_1^2. \end{aligned}$$

Therefore,

$$\begin{aligned} |a_3| & \leq [|\gamma|\Theta_3[(\lambda-1)(1+t+t^2)+3]B_1 - [(\lambda-1)(1+t)+2]^2\Theta_2^2] \\ & \quad \times \frac{2|\gamma|B_1^2}{\Psi(\Theta_1, \Theta_2, \lambda, t)[(\lambda-1)(1+t+t^2)+3]\Theta_3} + \frac{|\gamma|B_1}{[(\lambda-1)(1+t+t^2)+3]\Theta_3}, \end{aligned}$$

where

$$\begin{aligned} \Psi(\Theta_1, \Theta_2, \lambda, t) & = 2B_1[\Lambda(\lambda,t)+2]^2\Theta_2^2 \\ & \quad + |\gamma B_1^2\Lambda(\lambda,t)\Xi(\lambda,t) - 2B_2[\Lambda(\lambda,t)+2]^2\Theta_2^2 + 2\gamma B_1^2\Upsilon(\lambda,t)\Theta_3|. \end{aligned}$$

Consequently,

$$|a_3| \leq \begin{cases} \frac{|\gamma|B_1}{[(\lambda-1)(1+t+t^2)+3]\Theta_3} & B_1 \leq \frac{[(\lambda-1)(1+t)+2]^2\Theta_2^2}{|\gamma|\Theta_3[(\lambda-1)(1+t+t^2)+3]} \\ \frac{\Phi(\Theta_1, \Theta_2, \lambda, t)}{\Psi(\Theta_1, \Theta_2, \lambda, t)[(\lambda-1)(1+t+t^2)+3]\Theta_3} & B_1 > \frac{[(\lambda-1)(1+t)+2]^2\Theta_2^2}{|\gamma|\Theta_3[(\lambda-1)(1+t+t^2)+3]}, \end{cases}$$

where

$$\begin{aligned} \Phi(\Theta_1, \Theta_2, \lambda, t) & = |\tau|B_1 |\gamma B_1^2\Lambda(\lambda,t)\Xi(\lambda,t) - 2B_2[\Lambda(\lambda,t)+2]^2\Theta_2^2 + 2\gamma B_1^2\Upsilon(\lambda,t)\Theta_3| \\ & \quad + 2|\gamma|^2\Theta_3[(\lambda-1)(1+t+t^2)+3]B_1^3. \end{aligned}$$

This completes the proof. \square

Remark 3.2. Theorem 3.1 is an improvement of the estimates obtained by Selvaraj et al. [20] in Theorem 2.2. For the coefficient $|a_2|$, it is clear that

$$\begin{aligned} & \frac{|\gamma|B_1\sqrt{2B_1}}{\sqrt{2B_1[\Lambda(\lambda,t)+2]^2\Theta_2^2 + |\gamma B_1^2\Lambda(\lambda,t)\Xi(\lambda,t) - 2B_2[\Lambda(\lambda,t)+2]^2\Theta_2^2 + 2\gamma B_1^2\Upsilon(\lambda,t)\Theta_3|}} \\ & \leq \frac{|\gamma|B_1\sqrt{2B_1}}{\sqrt{|\gamma B_1^2\Lambda(\lambda,t)\Xi(\lambda,t) - 2(B_2 - B_1)[\Lambda(\lambda,t)+2]^2\Theta_2^2 + 2\gamma B_1^2\Upsilon(\lambda,t)\Theta_3|}}. \end{aligned}$$

On the other hand, for the coefficient $|a_3|$, we make the following cases:

(i) For $B_1 \leq \frac{[(\lambda - 1)(1 + t) + 2]^2 \Theta_2^2}{|\gamma| \Theta_3 [(\lambda - 1)(1 + t + t^2) + 3]}$, it is clear that

$$\frac{|\gamma| B_1}{\Upsilon(\lambda, t) \Theta_3} \leq \frac{B_1 |\gamma|}{\Upsilon(\lambda, t) \Theta_3} + \left(\frac{B_1 |\tau|}{[\Lambda(\lambda, t) + 2] \Theta_2} \right)^2.$$

(ii) For $B_1 > \frac{[(\lambda - 1)(1 + t) + 2]^2 \Theta_2^2}{|\gamma| \Theta_3 [(\lambda - 1)(1 + t + t^2) + 3]}$, it is clear that

$$\frac{\Phi(\Theta_1, \Theta_2, \lambda, t)}{\Psi(\Theta_1, \Theta_2, \lambda, t) \Upsilon(\lambda, t) \Theta_3} \leq \frac{B_1 |\gamma|}{\Upsilon(\lambda, t) \Theta_3} + \left(\frac{B_1 |\tau|}{[\Lambda(\lambda, t) + 2] \Theta_2} \right)^2.$$

Remark 3.3. If we set $\lambda = 0$ in Theorem 3.1, then we get an improvement of the estimates obtained by Selvaraj et al. [20, Corollary 2.1].

Remark 3.4. If we set $\lambda = 1$ in Theorem 3.1, then we get an improvement of the estimates obtained by Selvaraj et al. [20, Corollary 2.2].

Remark 3.5. If $\mathcal{J}_\mu^b f(z)$ be the identity map and $\lambda = 0$ in Theorem 3.1, then we get an improvement of the estimates obtained by Selvaraj et al. [20, Corollary 2.3].

Remark 3.6. If $\mathcal{J}_\mu^b f(z)$ be the identity map and $\lambda = 1$ in Theorem 3.1, then we get an improvement of the estimates obtained by Selvaraj et al. [20, Corollary 2.4].

Remark 3.7. If $\mathcal{J}_\mu^b f(z)$ be the identity map and $\gamma = 1$, $t = 0$ in Theorem 3.1, then we get an improvement of the estimates obtained by Deniz [8, Theorem 2.8].

Remark 3.8. If $\mathcal{J}_\mu^b f(z)$ be the identity map and $\gamma = 1$, $\lambda = 1$ in Theorem 3.1 is an improvement of the estimates obtained by Ali et al. in [3, Theorem 2.1].

Remark 3.9. If we take

$$\phi(z) = \frac{1 + Az}{1 + Bz} = 1 + (A - B)z + (B - A)Bz^2 + \cdots \quad (-1 \leq B < A \leq 1, z \in \mathbb{U})$$

and

$$\varphi(z) = \left(\frac{1 + z}{1 - z} \right)^\alpha = 1 + 2\alpha z + 2\alpha^2 z^2 + \cdots \quad (0 < \alpha \leq 1, z \in \mathbb{U}),$$

which gives $B_1 = A - B$, $B_2 = (B - A)B$ and $B_1 = 2\alpha$, $B_2 = 2\alpha^2$, in Theorem 3.1, then we can deduce interesting results analogous, respectively. Also, for suitable choices the parameter μ and b in Theorems 3.1 and some Remarks above we have an improvement of results involving Libera-Bernardi integral operator [19] and Jung-Kim-Srivastava integral operator [12].

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