A Digital Time-Domain Method for Instantaneous Harmonics Extraction in Active Power Filters

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Abstract: **In this paper, a digital time domain method for instantaneous harmonics extraction in active power filters is presented. The proposed recursive method is based on creating an orthogonal space to extract the reactive current. This method reduces the order of computation considerably in comparison with the similar methods in frequency domain.**

1. Introduction

Instantaneous reactive power compensation methods are generally considered in low and medium active power filters (APF) [1]. Such strategies have intrinsically smoother transient response behaviour in comparison with other methods. Conventional (non-hybrid) methods for instantaneous reactive power compensation are generally based on extracting and tracking the instantaneous reference current [1-3]. Also hybrid topologies are presented in which a proportion of the source harmonics current is used to control APF system [4-6]. A typical structure of an APF system is shown in Fig.1.

Extracting the reference current instantaneously can be accomplished in both frequency and time domain. The frequency domain methods are based on applying FFT, which requires processors with this capability and considerably higher order of computation. Furthermore, to obtain proper results, bit allocation should be increased or floating-point systems ought to be used, which increases the cost and complexity. Modified-FFT based algorithms are also developed which relatively reduces the order of computation in comparison to conventional FFT algorithms [7].

In this paper, extracting the instantaneous reference current is developed in time domain by using a recursion algorithm. In this method the order of computation is reduced in comparison to other similar algorithms in frequency domain and the error due to truncation is almost instantaneous and non-accumulative. The implementation is simple and the required processor memory/speed is reduced considerably. This method can be extended to control hybrid topologies including the one proposed by Fujita and Akagi [4]. The presented method derives source harmonic current 'i_{sh}' in three-phase systems without applying PQ theorem. This is an advantage considering that using PQ theorem (dq-frames) in unbalanced / unsymmetrical systems has drawbacks [6]. In the proposed method an orthogonal space is created to extract the reactive current term by considering the source voltage waveform.

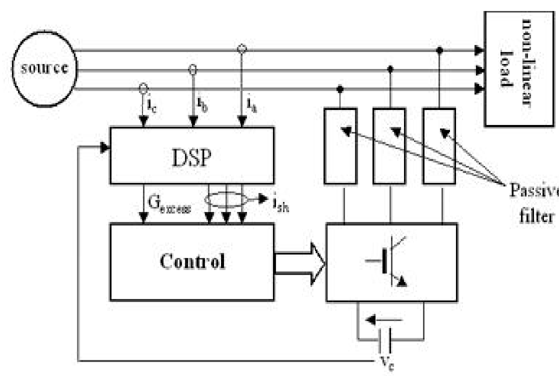

Fig. 1: A general structure of an APF system

2. Reactive Current

Assume that a load voltage and current are v(t) and i(t) respectively at any instant of time and are periodic with same period 'T'. Thus the active power and apparent power are defined as following:

$$P \overset{\Delta}{=} \frac{1}{T}\int_T v(t)i(t)dt \quad (1)$$

$$S \overset{\Delta}{=} V_{rms}.I_{rms} \quad (2)$$

And power factor is defined as:

$$PF = \frac{P}{S} \quad (3)$$

Desired compensation would happen when 'PF = 1'. If the load is voltage dependent, its current can be divided into active '$i_a(t)$' and reactive '$i_r(t)$' terms as defined below:

$$i(t) = i_a(t) + i_r(t) \quad (4)$$

Such that

$$\text{I)}\quad \frac{1}{T}\int_T i_r(t)v(t)dt = 0 \quad (5)$$

II) $I_{a,rms}$ = Minimum (6)

In other words, the reactive current not only does not transfer any active power (5), but also the reactive current is as such to minimize the effective active current (6). Using Cauchy-Schwartz inequality shown in (7), it can be proven that in order to minimize $I_{a,rms}$, the active current term must be proportional to voltage.

Proof:

$$\left[\int_a^b fh\right]^2 \le \int_a^b f^2 . \int_a^b h^2 \quad (7)$$

The equality in the above equation is true if and only if 'f = G h', where G is a constant. By substituting 'f = v(t)', 'h = $i_a(t)$' and '[a, b]=T' the following relation is established.

$$\frac{1}{T}\int_T v(t) i_a(t) \le I_{a,rms} . V_{rms} \quad (8)$$

Therefore $I_{a,rms}$ is minimum if and only if

$$i_a(t) = G \cdot v(t) \quad (9)$$

Where 'G' is the non-linear load conductance and can be obtained as follows:

$$\int_T v(t) i(t) dt = \int_T v(t) i_a(t) dt = G \int_T v^2(t) dt$$

Thus

$$G = \frac{\frac{1}{T}\int_T v(t) i(t) dt}{\frac{1}{T}\int_T v^2(t) dt} = \frac{P}{V_{rms}^2} \quad (10)$$

$$i_r(t) = i(t) - G \cdot v(t) \quad (11)$$

In all the above equations the integral limits are shown as $\int_T$, which means only the integral duration is important and the starting time can be neglected. In other words:

$$\int_T \equiv \int_{t-T}^{t} \quad (12)$$

The instantaneous active and reactive power can be obtained from the above derivation. Thus the instantaneous active and apparent power are:

$$P(t) = \frac{1}{T}\int_{t-T}^{t} v(\tau) i(\tau) d\tau \quad (13)$$

$$S(t) = V_{rms}(t) . I_{rms}(t) \quad (14)$$

And the instantaneous power factor and conductance are:

$$PF(t) = \frac{P(t)}{S(t)} \quad (15)$$

$$G(t) = \frac{P(t)}{V_{rms(t)}^2} \quad (16)$$

Therefore the instantaneous reactive current at steady-state is:

$$i_r(t) = i(t) - G(t) v(t) \quad (17)$$

3. Instantaneous Reactive Current Compensation and Inverter Active Flow

A general single-phase shunt non-hybrid reactive power compensation system based on reactive current injection is shown in Fig.2. The mentioned circuit can be viewed as a compensator for one-phase of a three-phase system.

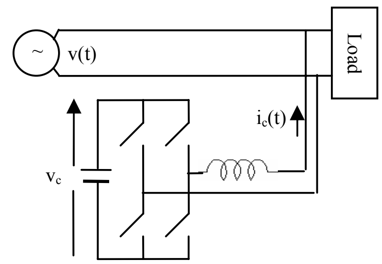

Fig. 2: A single-phase non-hybrid APF

In Fig.2, if at every instant of time, $i_c(t)$ is equal to $i_r(t)$, then the voltage supply is providing only the active current term which produces useful energy and the compensator is providing the reactive current term. Let's assume, it is desired to increase the capacitor voltage at instant t = nT to be more than the cpacitor voltage at t = (n-1)T. This means in this period the capacitor should receive energy or in other words an active power flow is created from source toward inverter. The required active power for this voltage increase is called P_{excess} and is defined as following:

$$P_{excess} = -\frac{1}{T}\int_{(n-1)T}^{nT} v(t) i_c(t) dt \quad (18)$$

Following the same formulation used in section 2, the optimized current (for not creating any secoundary reactive power) is:

$$i_c(t) = -G_{excess} . v(t) \quad (19)$$

and

$$G_{excess} = \frac{P_{excess}}{V_{rms}^2} \quad (20)$$

To control the active power flow various methods are presented [1-3,8]. However, in most cases the designers have used a PI-controller [1,3,8]. The general block diagrm of such a controller is shown in Fig.3.

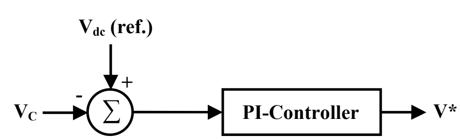

Fig. 3: DC-Link voltage control

Since in Fig. 3 the reference voltage (V_{dc}) is constant this method is called constant-voltage control. If the Controller has a proper DC-Gain, the output signal of this controller (V*) can be used as a measure to obtain G_{excess} according to relation (20).

4. Control Strategy and Implementation Method

From the above discution and using equations (17) and (19), the following relation, which establishes the control strategy, is obtained:

$$i_C(t) = i(t) - [G(t) + G_{excess}]v(t) \qquad (21)$$

In the above relation G(t) Controls the injected reactive power and G_{excess} controls the DC-link voltage. To digitally implement this control strategy a proper algorithm is required to calculate G(t) instantaneously. Therefore, the required discrete relations must be obtained using the same idea as the continuous equation (21).

Let's assume that N samples from voltage and current are taken in one period. Thus:

$$P(n) = \frac{1}{N}\sum_{m=n-N+1}^{n} v(m)\cdot i(m) \qquad (22)$$

$$V_{rms}^2(n) = \frac{1}{N}\sum_{m=n-N+1}^{n} v^2(m) \qquad (23)$$

Therefore, for any sample 'n', G(n) is obtained as following:

$$G(n) = \frac{P(n)}{V_{rms}^2(n)} = \frac{N\cdot P(n)}{N\cdot V_{rms}^2(n)} \qquad (24)$$

The same relations can be deduced for the (n+1)th sample as below:

$$N\cdot P(n+1) = N\cdot P(n) + \Delta P \qquad (25)$$

$$N\cdot V_{rms}^2(n+1) = N\cdot V_{rms}^2(n) + \Delta V^2 \qquad (26)$$

where

$$\Delta P = v(n+1)\cdot i(n+1) - v(n-N+1)\cdot i(n-N+1) \qquad (27)$$

$$\Delta V^2 = v^2(n+1) - v^2(n-N+1) \qquad (28)$$

$$G(n+1) = \frac{N\cdot P(n+1)}{N\cdot V_{rms}^2(n+1)} \qquad (29)$$

Calculation of $N\cdot P(n)$ and $N\cdot V_{rms}^2(n)$ can be performed recursively at any instant 'n', to obtain G(n).

5. Reference Current Extraction Algorithm

It is assumed that the required G_{excess} to control the inverter DC-link voltage is already calculated. To obtain G(n) both of n'th and (n-N+1)'th samples of the load voltage and current are required. Therefore at any instant, 'N' previous samples are needed. To accomplish this task shif registers can be used, but circular arrays are faster and simpler [7]. Configuration of a circular array is shown in Fig.4. In this structure, at any instant 'n', a pointer has the address of an element of the circular array. This element contain v(n-N+1) and i(n-N+1). These values are used to calculate ΔP and ΔV^2 according to equations (27) and (28). After that, these values are updated and replaced by v(n) and i(n) to be used in the next cycle and the pointer is incremented.

The proposed recursion algorithm based on equations (22) – (29) is shown if Fig.5. This algorithm requires two circular arrays to save the samples of 'v(n)' and 'i(n)' in one period. Two pointers 'pv' and 'pi' point to the 'v(n-N+1)' and 'i(n-N+1)' respectively. These values with 'v(n)' and 'i(n)' are used to calculate ΔP and ΔV^2 for instant 'n'. After calculation, the circular arrays will be updated by replacing the older values by 'v(n)' and 'i(n)' to use in the next cycle. Then G(n) will be calculated to obtain '$i_r(n)$'.

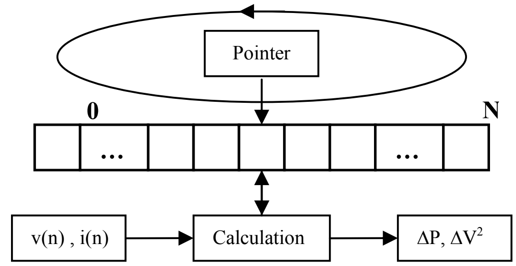

Fig. 4: Circular array

The simulation results are shown in Fig.6. A single-phase full-bridge rectifier is used as the non-linear load. The variations of G(t) when G_{excess}=0 and harmonics current are shown respectively according to start time and when the load is changed.

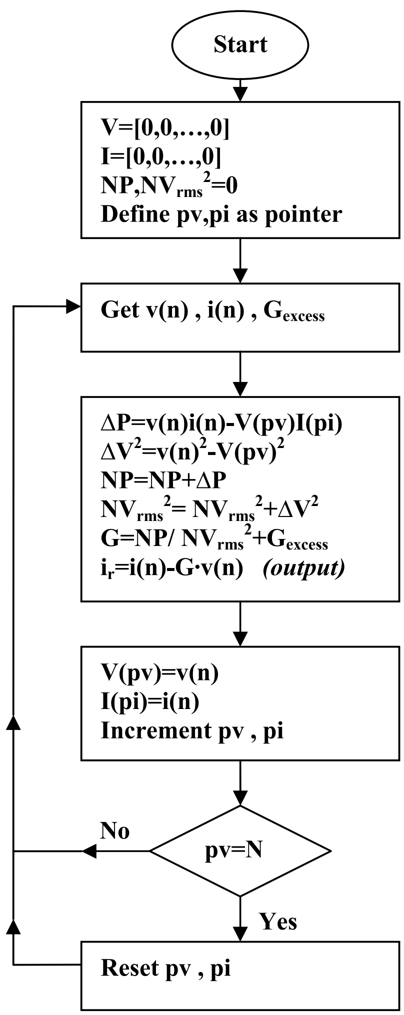

Fig. 5: Proposed algorithm

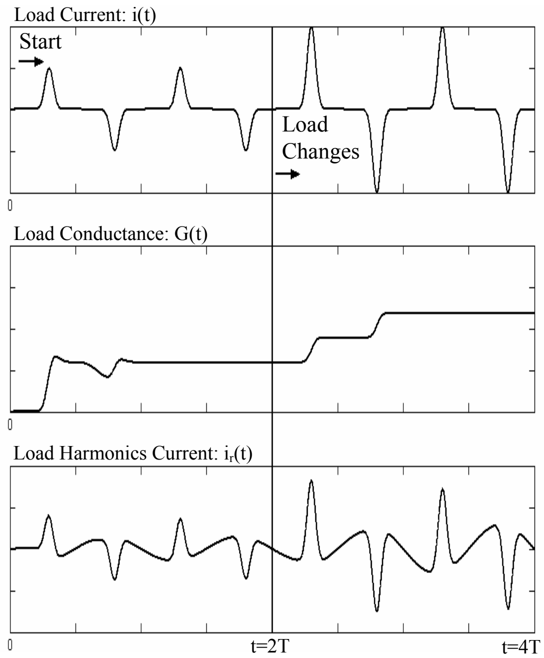

Fig. 6: Simulation results of a non-linear load

6. Displacement / Harmonics Extraction

Generally the load reactive current includes displacement and harmonics terms. The displacement term '$i_d(t)$' has same frequency as the source voltage but is not inphase; and harmonics term '$i_h(t)$' does not have same frequency as the source voltage.

$$i_r(t) = i_d(t) + i_h(t) \quad (30)$$

Compensating the displacement term is not economical using APF. It can be easily compensated by capacitor/inductor banks or using STATCON. Also in hybrid topologies it is desired to extract only the harmonics term 'i_{sh}'. The previous formulation can not extract only the harmonics term when the load current contains displacement. In the following section, the previous formulation will be modified to able to extract only the harmonics term.

Consider a case when the load voltage has harmonics because of source impedance or its nature. By using Fourier series:

$$v(t) = \sum_{n=-\infty}^{+\infty} V_n e^{jn\omega_0 t} \quad (31)$$

Where

$$V_n = \frac{1}{T}\int_T v(t)e^{-jn\omega_0 t}dt \qquad \omega_0 = \frac{2\pi}{T} \quad (32)$$

Now, w(t) is defined as below:

$$w(t) \overset{\Delta}{=} v(t - \frac{3T}{4}) \quad (33)$$

(if '$v(t) = \sin(\omega_0 t)$' then '$w(t) = \cos(\omega_0 t)$')

And, v(t) and w(t) are orthogonal if:

$$\frac{1}{T}\int_T v(t)w(t)dt = 0 \quad (34)$$

By substituting equations (31) and (33) in equation (34) and simplifying, the following relation is obtained:

$$\frac{1}{T}\int_T v(t)w(t)dt = \sum_{n=-\infty}^{+\infty} (j)^n |V_n|^2$$

$$= |V_0|^2 + 2\sum_{n=-\infty}^{+\infty} (-1)^n |V_{2n}|^2$$

For v(t) and w(t) to be orthogonal, it is only sufficient that v(t) would have no DC term and even harmonics. It is the performed scenario in three-phase systems because the source voltage is almost odd-symmetrical. By using the same idea as discussed in section 4, the following relations are obtained:

$$i(n) = G(n) \cdot v(n) + D(n) \cdot w(n) + i_h(n) \quad (35)$$

$$D(n) = \frac{Q(n)}{V_{rms}^2(n)} = \frac{N \cdot Q(n)}{N \cdot V_{rms}^2(n)} \quad (36)$$

$$Q(n) = \frac{1}{N}\sum_{m=n-N+1}^{n} w(m) \cdot i(m) \quad (37)$$

By using equations (35) and (36) the proposed algorithm can be modified to extract only the harmonics term of the load current. Consider that it is not necessary to create another circular array to save w(n). It can be achieved using only the circular array that saves v(n) and calculating D(n). Fig.7 shows the simulation results of a three-phase full-bridge rectifier.

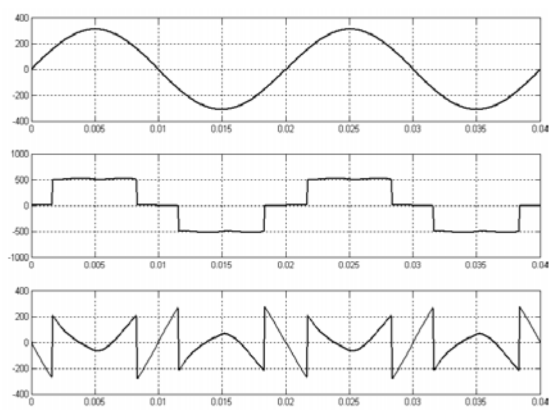

Fig. 7: Simulation results: source voltage (top), load current (middle) and extracted load current harmonics (bottom)

7. Some Considerations

- Both of FFT and PQ theorem (dq-frames) ignore the actual wave form of the source voltage and assume that it is a pure sine wave. The proposed method can extract reactive current term based on orthogonal bases by considering the source voltage waveform.

- In a three-phase balanced source voltage system, the following relation is performed:

$$v_a(t)^2 + v_b(t)^2 + v_c(t)^2 = 3 \cdot V_{rms}^2 \quad (38)$$

Then, in such systems there is no need to calculate V_{rms}^2 using circular arrays.

- For balancing unbalanced systems using non-hybrid topologies, the following procedure can be used:

1) Calculate G(t) for each phase of the system separately and define them as $G_a(t)$, $G_b(t)$ and $G_c(t)$.

2) If the compensation performed, then:

$i_k(t) = G_k(t) \cdot v_k(t) \quad$ for $k = a,b,c$

Since it is desired to have a same current magnitude for each phase to have a balanced system, define $G_s(t) = \text{average}\{G_k(t)\}$ and use it instead of $G_k(t)$ for each phase. Thus, system is made balanced as $i_k(t) = G_s(t) \cdot v_k(t)$. This method can be easily extended when it is not desired to compensate displacement current by using the same idea for D(t).

- In hybrid topologies, for unbalanced systems the proposed method can be used to extract 'i_{sh}' separately for each phase of the system without the PQ theorem drawbacks [9].

8. Conclusion

The proposed method performs extracting harmonics in time domain and reduces the order of computation considerably. This method can be used in hybrid / non-hybrid topologies to control an APF system. The proposed algorithm can be implemented digitally using DSP or FPGA.

9. References

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