Example 3.4 Consider the flow of air through a pipe of inside diameter = 0.15 m and length = 30 m. The inlet flow conditions are $M_1 = 0.3$, $p_1 = 1$ atm, and $T_1 = 273$ K. Assuming $f = \text{const} = 0.005$, calculate the flow conditions at the exit, M_2, p_2, T_2, and p_{o_2}.

SOLUTION From Table A.1: For $M_1 = 0.3$, $p_{o_1}/p_1 = 1.064$. Thus

$$p_{o_1} = 1.064(1\ \text{atm}) = 1.064\ \text{atm}$$

From Table A.4: For $M_1 = 0.3$, $4\bar{f}L_1^*/D = 5.299$, $p_1/p^* = 3.619$, $T_1/T^* = 1.179$, and $p_{o_1}/p^* = 2.035$. Since $L = 30\ \text{m} = L_1^* - L_2^*$, then $L_2^* = L_1^* - L$ and

$$\frac{4\bar{f}L_2^*}{D} = \frac{4\bar{f}L_1^*}{D} - \frac{4\bar{f}L}{D} = 5.2993 - \frac{(4)(0.005)(30)}{0.15} = 1.2993$$

From Table A.4: For $4\bar{f}L^*/D = 1.2993$, $\boxed{M_2 = 0.475}$, $T_2/T^* = 1.148$, $p_2/p^* = 2.258$, and $p_{o_2}/p_o^* = 1.392$. Hence

$$p_2 = \frac{p_2}{p^*}\frac{p^*}{p_1}p_1 = 2.258\frac{1}{3.169}(1\ \text{atm}) = \boxed{0.624\ \text{atm}}$$

$$T_2 = \frac{T_2}{T^*}\frac{T^*}{T_1}T_1 = 1.148\frac{1}{1.179}273 = \boxed{265.8\ \text{K}}$$

$$p_{o_2} = \frac{p_{o_2}}{p^*}\frac{p^*}{p_{o_1}}p_{o_1} = 1.392\frac{1}{2.035}1.064 = \boxed{0.728\ \text{atm}}$$

Certain physical trends reflected by the numbers obtained from such solutions are summarized here:

1. For *supersonic* inlet flow, i.e., $M_1 > 1$, the effect of friction on the downstream flow is such that
   a. Mach number decreases, $M_2 < M_1$
   b. Pressure increases, $p_2 > p_1$
   c. Temperature increases, $T_2 > T_1$
   d. Total pressure decreases, $p_{o_2} < p_{o_1}$
   e. Velocity decreases, $u_2 < u_1$
2. For *subsonic* inlet flow, i.e., $M_1 < 1$, the effect of friction on the downstream flow is such that
   a. Mach number increases, $M_2 > M_1$
   b. Pressure decreases, $p_2 < p_1$
   c. Temperature decreases, $T_2 < T_1$
   d. Total pressure decreases, $p_{o_2} < p_{o_1}$
   e. Velocity increases, $u_2 > u_1$

From the above, note that friction always drives the Mach number towards 1, decelerating a supersonic flow and accelerating a subsonic flow. This is em-

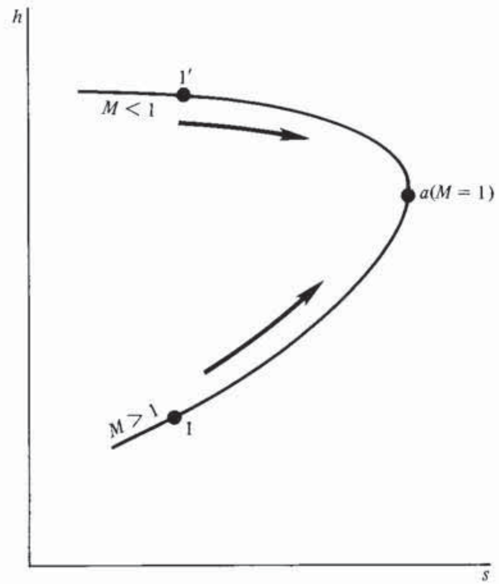

Figure 3.13 The Fanno curve.

phasized in Fig. 3.13, which is a Mollier diagram of one-dimensional flow with friction. The curve in Fig. 3.13 is called the *Fanno curve*, and is drawn for a set of given initial conditions. Point *a* corresponds to maximum entropy, where the flow is sonic. This point splits the Fanno curve into subsonic (upper) and supersonic (lower) portions. If the inlet flow is supersonic and corresponds to point 1 in Fig. 3.13, then friction causes the downstream flow to move closer to point *a*, with a consequent decrease of Mach number towards unity. Each point on the curve between points 1 and *a* corresponds to a certain duct length *L*. As *L* is made larger, the conditions at the exit move closer to point *a*. Finally, for a certain value of *L*, the flow becomes sonic. For this condition, the flow is *choked*, because any further increase in *L* is not possible without a drastic revision of the inlet conditions. For example, if the inlet conditions at point 1 were obtained by expansion through a supersonic nozzle, and if *L* were larger than that allowed for attaining Mach 1 at the exit, then a normal shock would form inside the nozzle, and the duct inlet conditions would suddenly become subsonic.

Consider the alternative case where the inlet flow is subsonic, say given by point 1′ in Fig. 3.13. As *L* increases, the exit conditions move closer to point *a*. If *L* is increased to a sufficiently large value, then point *a* is reached and the flow at the exit becomes sonic. The flow is again choked, and any further increase in *L* is impossible without an adjustment of the inlet conditions to a lower inlet Mach number, i.e., without moving the inlet conditions to the left of point 1′ in Fig. 3.13.

Finally, note that friction always causes the total pressure to decrease whether the inlet flow is subsonic or supersonic. Also, unlike the Rayleigh curve