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CRITICAL STATE BEHAVIOR OF AN UNSATURATED SILTY SAND

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ABSTRACT

Critical state models for unsaturated soils have been proposed in recent years; however, the proposed models have been based on limited experimental data. In this paper a laboratory study for verifying influence of matric suction on the shear strength and mechanical behavior of a silty sand is presented. For this purpose, a set of triaxial tests in saturated and unsaturated conditions have been carried out, with the aim of considering the suction effect and its changes during fast rate of loading (such as earthquake). Axis translation technique and double-walled triaxial cell have been used to measure the soil matric suction and variation of pore air volume respectively. Then according to the obtained results, the effect of matric suction on critical state parameter was studied. The data for critical state conditions from these tests are presented with respect to matric suction, based on the critical-state parameters, which is commonly proposed. The results indicated that the mechanical behavior of silty sand depend non-linearly on the matric suction. On the other hand, it is necessary to present a new definition of critical state line equations in different coordinates.

Keywords: Unsaturated Soil, Critical State, Suction, Volume Change, Constant Water content Triaxial Test

1. INTRODUCTION

The shear strength and critical state theories for unsaturated soils have received increased attention during the past three decades. The critical-state concept has been well established as a useful framework within which saturated soil behavior can be interpreted [1]. The critical-state behavior is described as a state of soil, in which its volume is constant under large shear strains. The behavior of saturated soils is controlled by effective stresses, and water content and volume are interrelated. Therefore the saturated critical state can be expressed through the deviator stress, q, the mean effective stress, p', the mean net stress p'', and the specific volume, ν :

$$q = Mp'' \tag{1}$$

$$\nu = \Gamma - \lambda \ln p' \tag{2}$$

Where M is the slope of the critical-state line in $(q : p'')$ space, Γ is the intercept at $p'=1$ kPa, and λ is the slope of the critical-state line in $(v : lnp')$ space. So far many studies have been conducted on critical state of saturated soils [1-4]. While the critical-state behavior of unsaturated soils is well known, there are still gaps in the knowledge of the critical-state concept and its application for unsaturated soils.

The stress states in soil consist of certain combinations of stress variables that can be referred to as stress state variables. Stress state is independent of the physical properties of a soil. The number of stress state variables required for the description of the stress state of a soil depends primarily upon the number of phases involved. Saturated soils are characterized by water phase in voids in classical soil mechanics. The effective stress, $(\sigma - u_w)$, for saturated soil has often been regarded as a physical low. Unsaturated soils are characterized by the presence of air phase, water phase and air–water interface in voids. It is thus difficult to define convenient stress state variables for unsaturated soils. During the past three decades there has been an increasing use of two independent stress variables to describe the behavior of unsaturated soils [5-14]. Because, unsaturated soils have an additional phase (the air phase), and it is therefore no longer possible to interpret their behavior through effective stresses, nor to assume that water content and volume are linked. For unsaturated soils, the stress state can be represented by two stress state variables, the net stress $(\sigma - u_a)$ and the matric suction $(u_a - u_w)$ [14], where u_a is the pore air pressure and u_w is the pore-water pressure. In addition to specific volume (or void ratio), the phase state of the soil has to be represented by an additional variable: this can be either gravimetric water content (w), volumetric water content (θ) or degree of saturation (S_r). As regards, proposed models for unsaturated soil have been based on limited experimental data, especially for silty sand, since the measurement of unsaturated critical-state parameters involves time-consuming, expensive and sophisticated testing procedure [6 and 15-18].

In the present experimental work, there is focused on critical state behavior of unsaturated silty sand. For this aim, a set of consolidated and constant water content undrained tests carried out in saturated and unsaturated states respectively. It is used a special triaxial apparatus for unsaturated tests in soil laboratory of Bu-Ali Sina university.

2. MATERIALS AND METHODS

2.1. Sampling procedure and soil properties

The engineering index properties of the tested soil are presented in Table-1 and the grain size-distribution is shown in Figure 1. The soil consists of 51% sand and 20% silt and 29% clay. The soil is classified as *SM* according to the Unified Classification System. For specimens preparation, dry sand (from Shooshab river), kaolinit and silt have been mixed with respect to the considered different weight ratios and the required amount of dry soil mass and water for each layer of specimens have been determined exactly. Triaxial soil specimens were formed by dynamic compacting soil at a nominal water content of 7% in seven uniform layers, using a specially fabricated unit. This procedure produced uniform 38 mm diameter by 70 mm long cylindrical specimens all having the same structure. This length to diameter ratio of 2 selected in order to minimize the effects due to end platens of the apparatus and to reduce the likelihood of buckling during testing. The physical properties of specimens are given in Table-2.

Table-1. *Classification properties of soil.*

Liquid limit %	**16**
Plastic limit %	**-**
Plasticity index %	**NPI**
Specified gravity (G_s)	**2.68**
Clay percent %	**29**
Silt percent %	**20**
Soil type: (unified system)	**SM**

Table-2. *Physical properties of compacted soil specimen.*

Wet unit weight (kN/m^3)	**19.23**
Dry unit weight (kN/m^3)	**17.97**
Initial S_r %	**40.8**
Moisture content %	**7**
Void ratio	**0.46**
Porosity	**0.32**

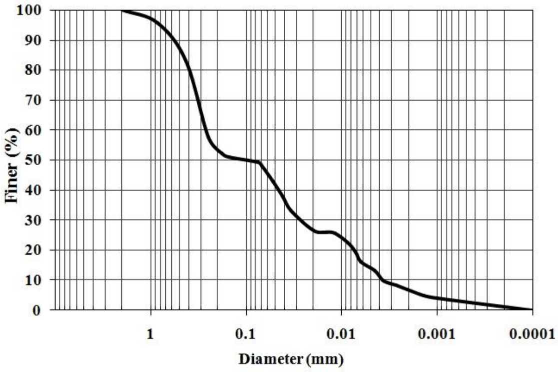

Figure 1. *Grain size distribution of the soil.*

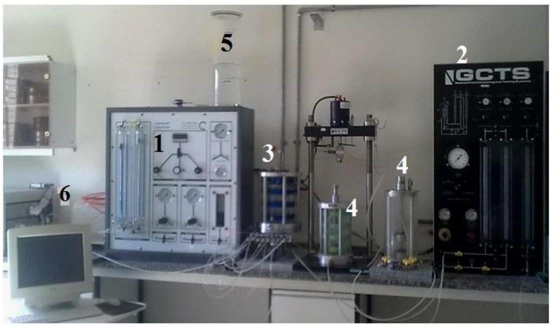

Figure 2. *Triaxial compression test equipments: (1)unsaturated control panel; (2) saturated control panel; (3) unsaturated triaxial cell; (4) saturated triaxial cell; (5) water de-airing tank system; (6) data logger.*

2.2. Testing equipment

In order to evaluate the stress–strain behavior of unsaturated soils, the triaxial compression and direct shear tests are performed on unsaturated specimens with various degree of saturation or matric suction. In the current study, the triaxial compression test apparatus used to determine the critical-state parameters of unsaturated soils (Figure 2).The matric suction is generally controlled by using the axis translation technique [15 and 19-21]. Triaxial tests were conducted using two conventional triaxial cells made for unsaturated soil testing [15]. the apparatus has ability to control and measure the pore air and pore-water pressure in the soil specimen independently by using axis translation technique. the pore-water pressures (u_w) was controlled through a saturated ceramic discs with a high air entry value. For that purpose, two ceramic discs with air entry value of 500 kPa were sealed onto the base and upper pedestals of the triaxial cell. The constant pore air pressure was applied to the base and upper pedestals by coarse corundum that were sealed in the middle of ceramic discs (Figure 3). This two-way flows of water and air causes an acceptable decrease in test time and also production of homogeneous specimen.

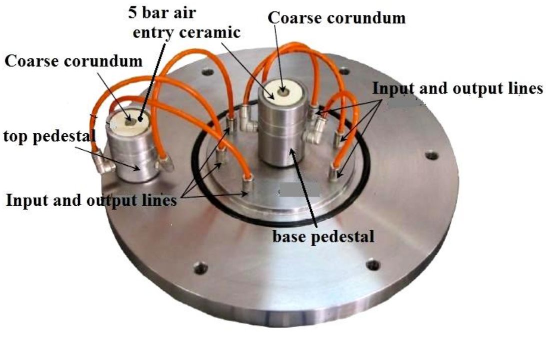

Figure 3. *Base plate Components of unsaturated triaxial cell.*

Figure 4. *Prepared soil specimen.*

For unsaturated soils, the total volume change equals to the sum of the water and air volume changes. The overall volume and pore-water-volume changes of the soil specimen are generally measured owing to the difficulties in measuring air-volume changes. The difference between the overall and pore-water volume change gives the air volume change [15]. In this work, the water content variation was measured by means of the volume change line connected to water compartment below the high-air entry ceramic disc. A flushing system was used to remove air bubbles that can accumulate at the base of the porous ceramic disks during unsaturated soil testing. The air bubbles originated from the diffusion of dissolved air into water through the ceramic disks. If not removed, these air bubbles could impede the flow of water from and into the specimen and cause incorrect measurements of pore-water pressures and volume changes

[22]. Another volume change transducer was connected between the constant pressure device and triaxial cell to measure overall volume change of soil specimens.

2.3. Testing program

The triaxial test programme comprised both CU and CW tests.

The unsaturated stress–strain behaviors of soil specimens were determined by constant water content triaxial (CW) test. In this work, for unsaturated triaxial test, soil specimens were enclosed in two rubber membranes with two slotted aluminium sheets separated by layer silicon grease between the membranes. In this way, air which is diffused into cell water through the rubber membrane was eliminated [19]. After placing and sealing the specimen inside the triaxial chamber (Figure 4), the wetting process was then started by decreasing the value of matric suction until the specimen was achieves to initial matric suction of 50 or 100 kPa. In this stage, water was absorbed by the soil specimen so that volume of the water in the soil specimen remained constant (i.e. after 4-5 days). The wetting curves during the equalization stage are plotted in Figure 5. After the equalization stage the soil specimen is first consolidated by applying matric suction and net confining. During this consolidation stage, the moisture content of the specimen is reached the equilibrium state. In constant water content tests, unsaturated specimens were sheared to failure under constant gravimetric water content conditions. In these tests the pore-water valve was shut off while pore-air was allowed to drain freely from the specimen. During CW tests, matric suction will vary if significant changes in pore-water pressure occur during shearing.

The saturated stress–strain behaviors of soil specimens were determined by means of conventional triaxial compression test apparatus. The saturated triaxial compression tests were carried out under consolidated and undrained condition. Prior the tests, the soil specimens were saturated until a value of pore pressure coefficient (B_w) exceeding 0.95 [23-24]. For this purpose, after taking necessary measurements, the specimens have been first subjected by CO_2 at least for 3 hr and then saturated by de-aired water. Specimens have been considered to be fully saturated if B is at least equal or greater than 0.95. In this study, backpressure of 300 kPa has been applied during the tests to achieving the saturation state.

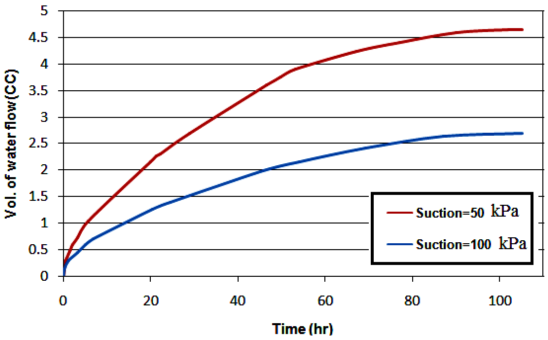

Figure 5. *Wetting curves during equalization stage.*

Table-3. *Initial stress values for the CW tests.*

Test	σ_3	u_a	u_w	σ_3-u_a	u_a-u_w
CW- S 50-25	275	250	200	25	50
CW-S 50-50	300	250	200	50	50
CW-S 50-100	350	250	200	100	50
CW-S 100-25	275	250	150	25	100
CW-S 100-50	300	250	150	50	100
CW-S 100-100	350	250	150	100	100

3. TEST RESULTS

Figures 6(a) and 6(b) show the stress-strain and pore-water volume change versus axial strain for the saturated soil specimens. In most cases, the pore-water volume changes tend to stabilize close to the end of loading (i.e. 20-30% strain). Figure 6(c) shows the stress paths of the saturated series on the (q : p') plane. Despite having different initial mean stresses, the specimens approached a unique critical-state line with a slope M equal to 1.5.

The results of the CW triaxial tests carried out under constant net confining stresses of 25, 50, and 100 kPa are presented Figures7 and 8. The specimens were designated using a convention similar to that used in the CU tests. For example, CW-S 50-100 represents a specimen that was tested under the constant water content condition at a net confining stress of 50kPa and an initial matric suction of 100kPa, as shown in Table-3. Figures 7(a) and 8(a) are the stress–strain curves for the unsaturated specimens (test series CW-S 50 and CW- S100). It was observed that matric suction has considerable effects on stress-strain curves and for the same net confining pressure, the strength of the unsaturated specimens is significantly greater than the strength of a saturated specimen. However, the general shape of the stress-strain curves is similar to those of saturated specimens. For example, the shear strength for specimen number (CW-S 100–50) is roughly twice that for specimen number (CU-S 0–50). This shows that the matric suction has significant influence on the shear strength of unsaturated soils.

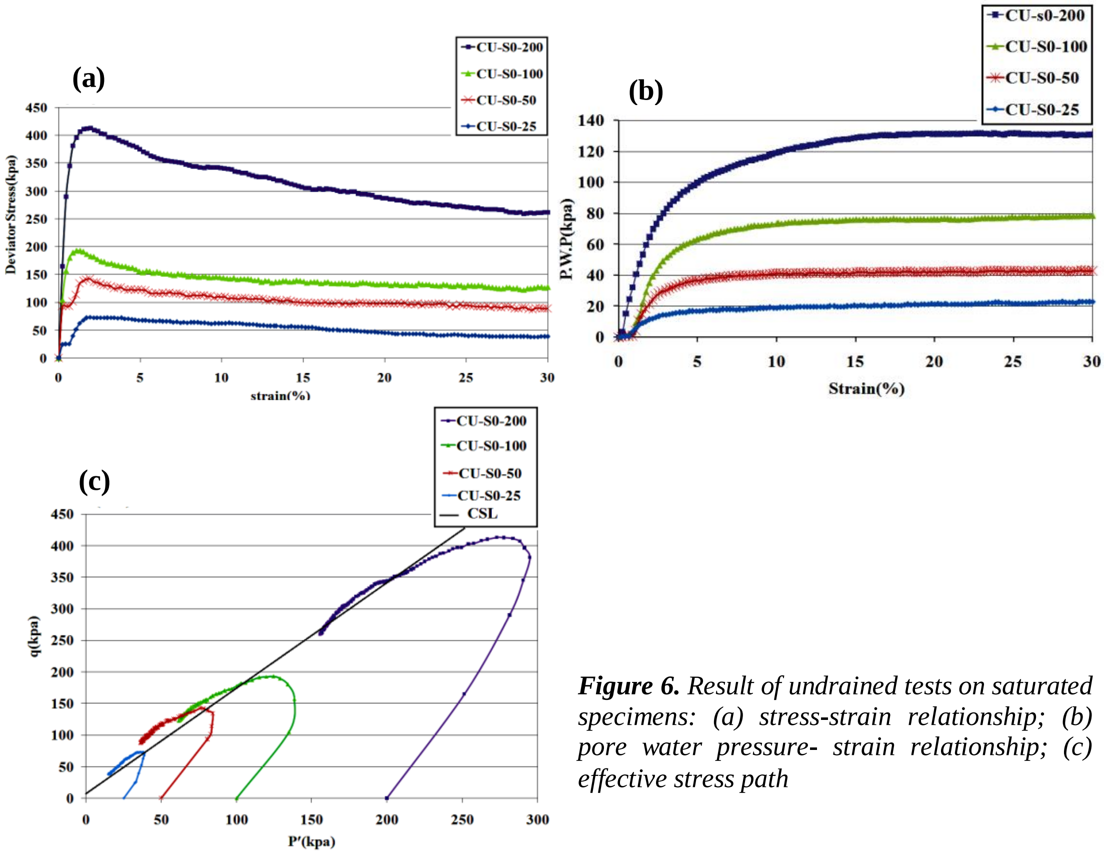

Figure 6. *Result of undrained tests on saturated specimens: (a) stress-strain relationship; (b) pore water pressure- strain relationship; (c) effective stress path*

The changes of suction versus strain curves for the unsaturated soil specimens are presented Figures 7(b) and 8(b). Also Figures 7(c) and 8(3) presents the total volume changes during loading for unsaturated specimens. According to Figures 7(b) and 8(b), the matric suctions of the specimens were observed to increase from the initial value to a maximum value of matric suction. The trends of the total volume changes are similar to suction changes. Both relationships become stable close to the termination of loading. As shown in these Figures, volume change decreases with increasing initial suction and also change in suction (or pore-water pressure) increases with increasing initial suction in specimens, at the same net confining pressure. The magnitude of dilation was largest in specimens tested at the lowest net confining stress of 25 kPa. The matric suction at critical state for the CW tests ranged from 73 to 126 kPa.

4. CRITICAL STATE LINE FOR UNSATURATED SPECIMENS

The definition of critical state used herein is related to state, in which volume, suction (or pore-water pressure) and shear strength are constant when that soil is subjected to large strain under different stress paths. The critical states for the unsaturated soils are shown on stress–strain curves (Figures 7(a) and 8(a)), pore-water volume change versus strain curves (Figures 7(b) and 8(b)), and total volume change versus strain curves (Figures 7(c) and 8(c)). These test results seem to confirm that the net mean stress, p'', deviator stress, q, and specific volume, v, can be used as critical state variables for unsaturated soils (Table-4). The variables p'', q, and v have been suggested as the critical state variables for unsaturated soils by several researchers [3 and 25-26].

$$q = Mp'' + q_0(s) \tag{3}$$

$$v = v_0(s) - \lambda \ln p'' \tag{4}$$

Where M and λ are the slopes of the CSL for unsaturated soils on the (q : p''), and (v : p''), planes, respectively. $q_0(s)$ is the final intercept of the CSL with the q axis; and $v_0(s)$ is the specific volume of the soil at critical state with p''= 1 kPa (a reference pressure). The critical state lines for unsaturated tests presents in Figures 8 and 9. The slopes of the critical-state line were identical for the unsaturated series of tests as shown in these Figures and CSLs of are parallel to each other in the unsaturated state.

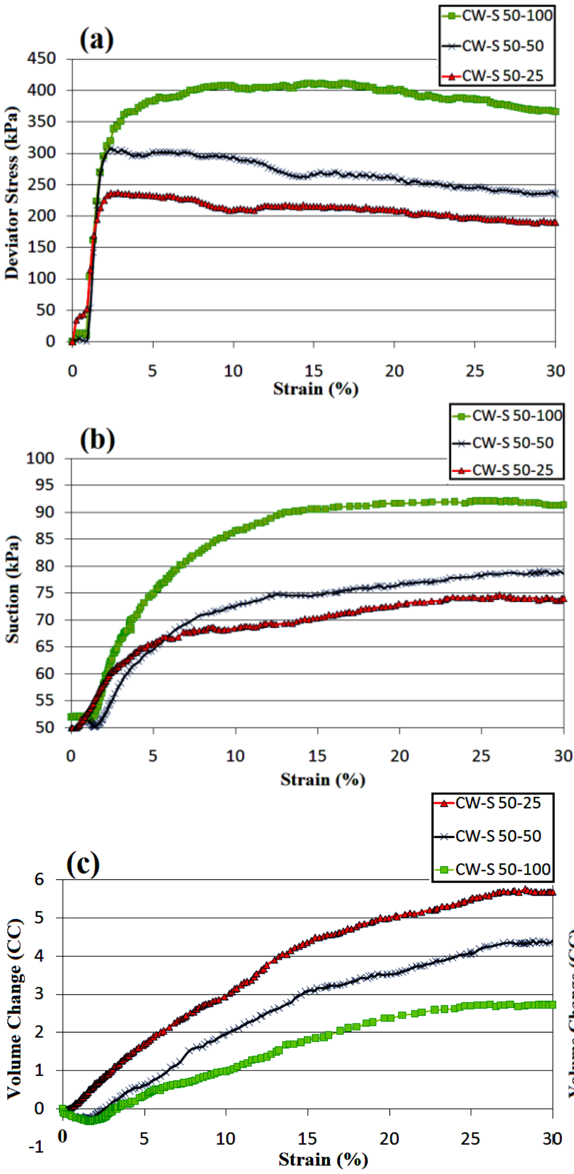

Figure 7. Results of constant water test at matric suction of 50 kPa and various confining stresses, plotted against axial strain: (a) deviator stress, q; (b) suction, s; (c) volume change

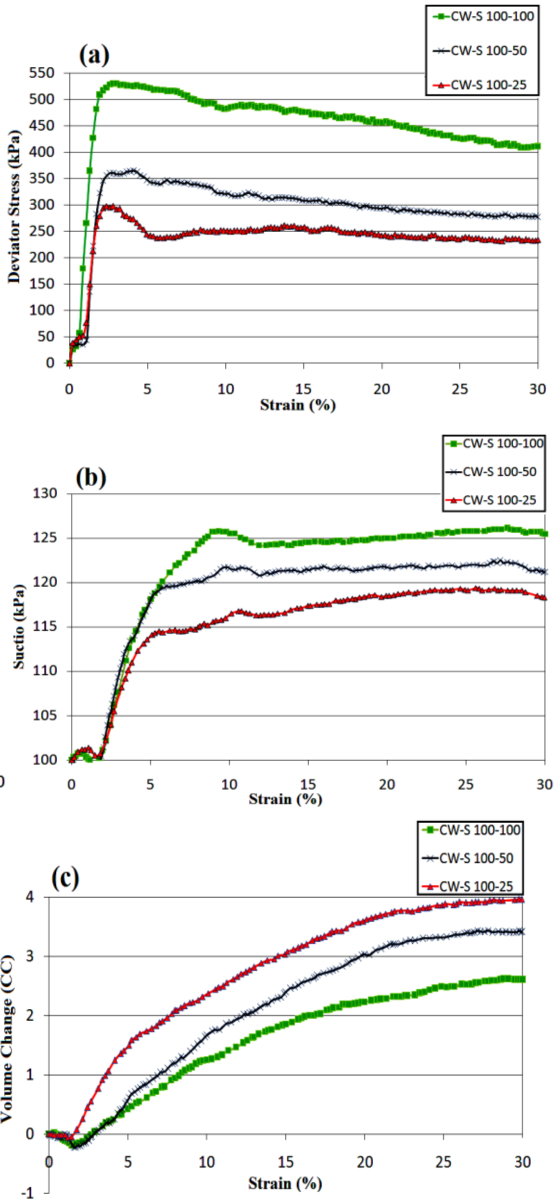

Figure 8. Results of constant water test at matric suction of 100 kPa and various confining stresses, plotted against axial strain: (a) deviator stress, q; (b) suction, s; (c) volume change

The slopes of CSLs are M equal to 1.33 and λ equal to $5.5*10^{-4}$. As shown in these Figures, intercepts q_0 and v_0 are function of matric suction. According to the tests results, it is clear that shear strength unsaturated soil not only at peak state but also at induced large strain state depended on matric suction. A conclusion can be deduced that, the unsaturated sandy soil mixed with fine present more undrained resistant under different stress paths with respect to saturated state. On the other hand for a fast rate of loading such as earthquake, the matric suction lead to more values of cyclic resistances of soil.

***Table-4**. Critical state values for the stress and phase variables.*

Test	$(q)_{cr}$	$(u_a\text{-}u_w)_{cr}$	$v=(1+e)_{cr}$	$(S_r)_{cr}$	$(p'')_{cr}$
CW-S 50-25	190	74	1.55	43.5	88.5
CW-S 50-50	234	78.5	1.51	47.2	128
CW-S 50-100	366	91.4	1.48	50.3	222
CW-S100-25	234	118.3	1.52	45.8	103
CW-S 100-50	276	121.2	1.48	50	143
CW-S 100-100	410.5	125.2	1.44	54.4	237

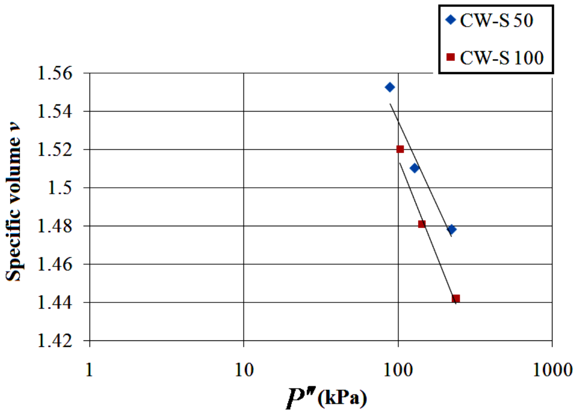

***Figure 8.** Critical state lines on the (v : p″) plane for the unsaturated specimens.*

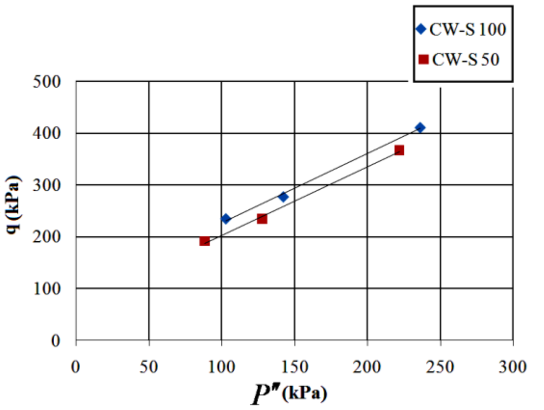

***Figure 9.** Critical state lines on the (q : p″) plane for the unsaturated specimens.*

5. CONCLUSIONS

This paper presents the shear strength and dilative characteristics of an unsaturated silty sand. For this a new unsaturated triaxial apparatus was used. In this triaxial apparatus, the matric suction was controlled by axis translation technique and volumetric behavior of specimens controlled by double-walled triaxial cell. The tests include consolidated undrained tests on saturated specimens and constant water content tests on unsaturated specimens. Unsaturated tests were carried out under two different matric suctions of 50 and 100 kPa. Critical behavior of soil was studied in terms of net stress and matric suction. The tests results show that matric suction plays two important roles on the silty sand behavior. Firstly, increase in matric suction will improve the shear strength, resulting in a different behavior in comparison to the fully saturated specimens and secondly, soil dilation behavior is influenced by matric suction. For a given cofining pressure, greater soil-dilation is observed at lower matric suction. Based on the test results, the critical state lines for the unsaturated soil specimens with respect to different matric suction or degree of saturation are parallel to each other. It has been observed that the intercepts q_0 and v_0 are function of matric suction (u_a-u_w).

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