

## تکلیف در منزل (۲)

موضوع: تحلیل حساسیت

نام درس: مدل سازی ریاضی

استاد درس: مسعود براتی

D.I 6.7-3. Consider the following problem.

$$\text{Maximize } Z = 2x_1 - x_2 + x_3,$$

subject to

$$3x_1 + x_2 + x_3 \leq 60$$

$$x_1 - x_2 + 2x_3 \leq 10$$

$$x_1 + x_2 - x_3 \leq 20$$

and

$$x_1 \geq 0, \quad x_2 \geq 0, \quad x_3 \geq 0.$$

Let  $x_4$ ,  $x_5$ , and  $x_6$  denote the slack variables for the respective constraints. After we apply the simplex method, the final simplex tableau is

Basic Variable	Eq.	Coefficient of:							Right Side
		Z	$x_1$	$x_2$	$x_3$	$x_4$	$x_5$	$x_6$	
Z	(0)	1	0	0	$\frac{3}{2}$	0	$\frac{3}{2}$	$\frac{1}{2}$	25
$x_4$	(1)	0	0	0	1	1	-1	-2	10
$x_1$	(2)	0	1	0	$\frac{1}{2}$	0	$\frac{1}{2}$	$\frac{1}{2}$	15
$x_2$	(3)	0	0	1	$-\frac{3}{2}$	0	$-\frac{1}{2}$	$\frac{1}{2}$	5

Now you are to conduct sensitivity analysis by *independently* investigating each of the following six changes in the original model. For each change, use the sensitivity analysis procedure to revise this final tableau and convert it to proper form from Gaussian elimination for identifying and evaluating the current basic solution. Then test this solution for feasibility and for optimality. If either test fails, reoptimize to find a new optimal solution.

(a) Change the right-hand sides

$$\text{from } \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix} = \begin{bmatrix} 60 \\ 10 \\ 20 \end{bmatrix} \quad \text{to} \quad \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix} = \begin{bmatrix} 70 \\ 20 \\ 10 \end{bmatrix}.$$

(b) Change the coefficients of  $x_1$

$$\text{from } \begin{bmatrix} c_1 \\ a_{11} \\ a_{21} \\ a_{31} \end{bmatrix} = \begin{bmatrix} 2 \\ 3 \\ 1 \\ 1 \end{bmatrix} \quad \text{to} \quad \begin{bmatrix} c_1 \\ a_{11} \\ a_{21} \\ a_{31} \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 2 \\ 0 \end{bmatrix}.$$

(c) Change the coefficients of  $x_3$

$$\text{from } \begin{bmatrix} c_3 \\ a_{13} \\ a_{23} \\ a_{33} \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 2 \\ -1 \end{bmatrix} \quad \text{to} \quad \begin{bmatrix} c_3 \\ a_{13} \\ a_{23} \\ a_{33} \end{bmatrix} = \begin{bmatrix} 2 \\ 3 \\ 1 \\ -2 \end{bmatrix}.$$

(d) Change the objective function to  $Z = 3x_1 - 2x_2 + 3x_3$ .

(e) Introduce a new constraint  $3x_1 - 2x_2 + x_3 \leq 30$ . (Denote its slack variable by  $x_7$ .)

(f) Introduce a new variable  $x_8$  with coefficients

$$\begin{bmatrix} c_8 \\ a_{18} \\ a_{28} \\ a_{38} \end{bmatrix} = \begin{bmatrix} -1 \\ -2 \\ 1 \\ 2 \end{bmatrix}.$$

D.I 6.7-4. Consider the following problem.

$$\text{Maximize } Z = 2x_1 + 7x_2 - 3x_3,$$

subject to

$$x_1 + 3x_2 + 4x_3 \leq 30$$

$$x_1 + 4x_2 - x_3 \leq 10$$

and

$$x_1 \geq 0, \quad x_2 \geq 0, \quad x_3 \geq 0.$$

By letting  $x_4$  and  $x_5$  be the slack variables for the respective constraints, the simplex method yields the following final set of equations:

$$(0) \quad Z + x_2 + x_3 + 2x_5 = 20$$

$$(1) \quad -x_2 + 5x_3 + x_4 - x_5 = 20$$

$$(2) \quad x_1 + 4x_2 - x_3 + x_5 = 10.$$

Now you are to conduct sensitivity analysis by *independently* investigating each of the following seven changes in the original model. For each change, use the sensitivity analysis procedure to revise this set of equations (in tableau form) and convert it to proper

form from Gaussian elimination for identifying and evaluating the current basic solution. Then test this solution for feasibility and for optimality. If either test fails, reoptimize to find a new optimal solution.

(a) Change the right-hand sides to

$$\begin{bmatrix} b_1 \\ b_2 \end{bmatrix} = \begin{bmatrix} 20 \\ 30 \end{bmatrix}.$$

(b) Change the coefficients of  $x_3$  to

$$\begin{bmatrix} c_3 \\ a_{13} \\ a_{23} \end{bmatrix} = \begin{bmatrix} -2 \\ 3 \\ -2 \end{bmatrix}.$$

(c) Change the coefficients of  $x_1$  to

$$\begin{bmatrix} c_1 \\ a_{11} \\ a_{21} \end{bmatrix} = \begin{bmatrix} 4 \\ 3 \\ 2 \end{bmatrix}.$$

(d) Introduce a new variable  $x_6$  with coefficients

$$\begin{bmatrix} c_6 \\ a_{16} \\ a_{26} \end{bmatrix} = \begin{bmatrix} -3 \\ 1 \\ 2 \end{bmatrix}.$$

(e) Change the objective function to  $Z = x_1 + 5x_2 - 2x_3$ .

(f) Introduce a new constraint  $3x_1 + 2x_2 + 3x_3 \leq 25$ .

(g) Change constraint 2 to  $x_1 + 2x_2 + 2x_3 \leq 35$ .