$6+8+9+10 = 33$ points total

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HW 3
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Problem 1

a) By symmetry, internal inductance of both conductors are same

2 points $\lambda_{int} = \frac{\mu i}{8\pi} \Rightarrow \mathcal{L}_{int} = \frac{\mu}{8\pi}$

Since copper is non-magnetic, $\mathcal{L}_{int} = \frac{\mu_0}{8\pi} = 5\times10^{-8}$ H/m $= \boxed{0.05 \text{ mH/km}}$

b) $\mathcal{L}_1 = \mathcal{L}_2$ by symmetry

3 points $\mathcal{L}_1 = \frac{\mu_0}{2\pi}\ln\left(\frac{R}{r'}\right)$; $r' = re^{-1/4}$ since $\mu_r = 1$

$= \frac{\mu_0}{2\pi}\ln\left(\frac{0.5}{0.0075e^{-1/4}}\right)$

$= 8.899\times10^{-7}$ H/m

$= \boxed{0.89 \text{ mH/km}}$ for each conductor

If units do not match those specified in problem, −0.5 point

ie. problem ask for mH/km, you give H/km,

1 point c) $\mathcal{L}_{tot} = \mathcal{L}_1 + \mathcal{L}_2 = 0.89 + 0.89 = \boxed{1.78 \text{ mH/km}}$

Problem 2 [8 points]

a) $r_{new} = 0.9$ cm

4 points (1 point for comparison, 3 points for answers)

$\mathcal{L}_{int} = \frac{\mu_0}{8\pi} = \boxed{0.05 \text{ mH/km}}$ [No change]

$\mathcal{L}_1 = \mathcal{L}_2 = \frac{\mu_0}{2\pi}\ln\left(\frac{0.5}{0.009e^{-1/4}}\right) = 8.53\times10^{-7}$ H/m $= \boxed{0.853 \text{ mH/km}}$ [decreased]

$\mathcal{L}_{tot} = \mathcal{L}_1 + \mathcal{L}_2 = \boxed{1.706 \text{ mH/km}}$ [decreased even more]

b) $r_{new} = 0.6$ cm

4 points

$\mathcal{L}_{int} = \boxed{0.05 \text{ mH/km}}$ [No change]

$\mathcal{L}_1 = \mathcal{L}_2 = \frac{\mu_0}{2\pi}\ln\left(\frac{0.5}{0.006e^{-1/4}}\right) = 9.34\times10^{-7}$ H/m $= \boxed{0.934 \text{ mH/km}}$ increase

$\mathcal{L}_{tot} = \mathcal{L}_1 + \mathcal{L}_2 = \boxed{1.868 \text{ mH/km}}$ [increased even more]

Problem 2

Conclusions: Inductance due to internal flux linkages stay same

As r increased by 20%, inductance of conductor and total inductance decreased by 4.15%.

As r decreased by 20%, inductance of conductor and total inductance increased by 4.94%.

9 points

Problem 3

a) $GMR_A = \sqrt[N^2]{D_{11} \cdot D_{12} \cdot D_{21} \cdot D_{22}}$; $D_{11} = D_{22} = re^{-1/4}$, $D_{12} = D_{21} = 0.3$, $N = 2$

Since $D_{11} = D_{22}$ and $D_{12} = D_{21}$

$\therefore GMR_A = \sqrt{D_{11} \cdot D_{12}} = \sqrt{0.0074 e^{-1/4} \cdot 0.3} = 0.04158 \text{ m}$ (1)

$D_{AB} = \sqrt[mn]{D_{13} D_{14} D_{23} D_{24}} = \sqrt[4]{6 \times 6.3 \times 5.7 \times 6} = 5.996 \approx 6 \text{m}$

$D_{AC} = \sqrt[mn]{D_{15} D_{16} D_{25} D_{26}} = \sqrt[4]{12 \times 12.3 \times 11.7 \times 12} = 11.998 \approx 12 \text{m}$

$D_{BC} = D_{AB} = 5.996 \approx 6 \text{ m}$

$D_{eq} = \sqrt[3]{D_{AB} D_{AC} D_{BC}} = \sqrt[3]{6 \times 12 \times 6} = 7.56 \text{ m}$ (2)

8 points

$\mathcal{L}_A = \frac{\mu_0}{2\pi} \ln\left(\frac{D_{eq}}{GMR_A}\right) = 2 \times 10^{-7} \ln\left(\frac{7.56}{0.04158}\right) = 1.04 \times 10^{-6} \text{ H/m} = \boxed{1.04 \text{ mH/km}}$

$1 \text{Km} = 0.6214 \text{ miles}$

$\therefore \mathcal{L}_A = 1.04 \text{ mH/Km} \times \frac{1 \text{ Km}}{0.6214 \text{ mi}} = \boxed{1.675 \text{ mH/mile}}$

1 point

b) $X_A = \omega \mathcal{L}_A = (2\pi 60)(1.675 \times 10^{-3}) = \boxed{0.631 \,\Omega/\text{mile}}$

Since line is transposed, $\mathcal{L}_A = \mathcal{L}_B = \mathcal{L}_C$

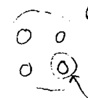
Note: The above solution used notation of book.

For part a), we could also have solved the problem by the following steps. (1), (2) and the rest are same. This way of solving is quicker and follows notation presented in lecture.

Problem 4

10 points

$f = 60\,Hz$, $T = 70°C$



$$\boxed{R = \frac{0.12}{4} = 0.03\,\Omega/\text{mile}}$$ for each phase

$D_{AB} \approx \sqrt{40^2 + 12^2} = 41.76$ ft

$D_{AC} = 80$ ft

$D_{BC} \approx 41.76$ ft

$GMD = \sqrt[3]{D_{AB} D_{AC} D_{BC}} = \sqrt[3]{(41.76)^2 \times 80} = 51.86$ ft

$GMR = \sqrt[4]{r' d_{12} d_{13} d_{14}}$

Using table A.4, $r' = 0.0403$ ft $= 0.4836$ in

$\therefore GMR = \sqrt[4]{0.4836 \times 20 \times 20 \times 20\sqrt{2}} = 8.6$ in $= 0.717$ ft

$$\mathcal{L}_a = \frac{\mu_0}{2\pi} \ln\left(\frac{GMD}{GMR}\right) = 2\times10^{-7} \ln\left(\frac{51.86}{0.717}\right) = 8.56\times10^{-7}\,H/m = 0.001377\,H/\text{mile}$$

$$X_a = \omega \mathcal{L}_a = (2\pi 60)(0.001377) = \boxed{0.519\,\Omega/\text{mile/phase}}$$

From Table A4, Current capacity is such that each conductor can carry 1010A ⇒ total line capacity is

$4 \times 1010\,A = \boxed{4040\,A}$

Note: To use Table A.4, we need to find out code name of the conductor. Table only specifies resistances for 25°C and 50°C. Hence, use equation 4.2.3 in book. Assume aluminum conductor.

$$R_{50} = 0.12\left(\frac{50 + 228.1}{70 + 228.1}\right) = 0.1119$$

⇒ Cardinal with GMR of 0.0403 ft.