27 points total

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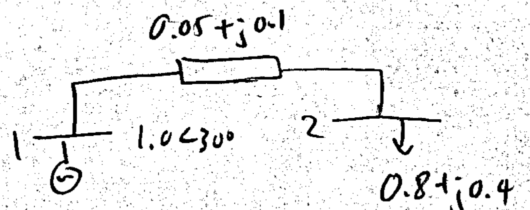
HW8

Problem 1
3 points



$Y_{21} = -4+j8 = 8.944\angle 116.56°$

$Y_{22} = 4-j8 = 8.944\angle -63.43°$

Bus 1: know $1.0\angle 30°$
find P_1, Q_1

Bus 2: know $P_2 = 0.8$, $Q_2 = 0.4$
find V_2, δ_2

Using Table 6.5 in Book, $\bar{x} = \begin{bmatrix} \delta_2 \\ V_2 \end{bmatrix}$, $J = \begin{bmatrix} \frac{dP_2}{d\delta_2} & \frac{dP_2}{dV_2} \\ \frac{dQ_2}{d\delta_2} & \frac{dQ_2}{dV_2} \end{bmatrix}$

$$\frac{dP_2}{d\delta_2} = -V_2\left[|Y_{21}|V_1 \sin(0° - 30° - 116.56°)\right]$$

use flat start $\Rightarrow V_2^0 = 1$, $\delta_2^0 = 0$

$$\therefore \frac{dP_2}{d\delta_2} = \underline{4.928}$$

$$\frac{dP_2}{dV_2} = V_2|Y_{22}|\cos(-63.43°) + |Y_{21}|V_1\cos(\delta_2 - \delta_1 - 116.56°)$$
$$+ |Y_{22}|V_2\cos(\delta_2 - \delta_2 + 63.43°)$$
$$= \underline{0.537}$$

Similarly, $\frac{dQ_2}{d\delta_2} = -7.464$

$\frac{dQ_2}{dV_2} = 11.07$

$$\therefore \boxed{J = \begin{bmatrix} 4.928 & 0.537 \\ -7.414 & 11.07 \end{bmatrix}}$$

6.43 (a) *Step 1*

By inspection:

(10 points)

$$\bar{Y}_{\text{Bus}} = \begin{bmatrix} -j12.5 & +j10.0 & +j2.5 \\ +j10.0 & -j15.0 & +j5.0 \\ +j2.5 & +j5.0 & -j7.5 \end{bmatrix}$$

$\delta_2(0) = \delta_3(0) = 0°$ $V_2(0) = 1.0$

Compute $\Delta y(0)$

$$P_2(X) = 1.0[10\cos(-90°) + 5\cos(-90°)] = 0$$
$$P_3(X) = 2.5\cos(-90°) + 5\cos(-90°) = 0$$
$$Q_2(X) = 1.0[10\sin(-90°) + 15 + 5\sin(-90°)] = 0$$
$$\Delta y(0) = \begin{bmatrix} P_2 - P_2(X) \\ P_3 - P_3(X) \\ Q_2 - Q_2(X) \end{bmatrix} = \begin{bmatrix} -2.0 - 0 \\ 1.0 - 0 \\ -0.5 - 0 \end{bmatrix} = \begin{bmatrix} -2.0 \\ 1.0 \\ -0.5 \end{bmatrix}$$

(b) *Step 2* Compute $\underline{J}(0)$ (see Table 6.5 text)

Must have J both Analytically and numerically

$$J1_{22} = \frac{\partial P_2}{\partial \delta_2} = -V_2[Y_{21}V_1 \sin(\delta_2 - \delta_1 - \theta_{21}) + Y_{23}V_3 \sin(\delta_2 - \delta_3 - \theta_{23})]$$
$$= -1.0[10(1)\sin(-90°) + 5(1)\sin(-90°)] = 15.$$
$$J1_{23} = \frac{\partial P_2}{\partial \delta_3} = V_2 Y_{23} V_3 \sin(\delta_2 - \delta_3 - \theta_{23}) = (1.0)(5)\sin(-90°) = -5.$$
$$J1_{32} = \frac{\partial P_3}{\partial \delta_2} = V_3 Y_{32} V_2 \sin(\delta_3 - \delta_2 - \theta_{32}) = (1)(5)(1)\sin(-90°) = -5$$
$$J1_{33} = \frac{\partial P_3}{\partial \delta_3} = -V_3[Y_{31}V_1 \sin(\delta_3 - \delta_1 - \theta_{31}) + Y_{32}V_2 \sin(\delta_3 - \delta_2 - \theta_{32})]$$
$$= -1.0[(2.5)(1)\sin(-90°) + (5)(1)\sin(-90°)] = 7.5$$
$$J2_{22} = \frac{\partial P_2}{\partial V_2} = V_2 Y_{22}\cos(\theta_{22}) + [Y_{21}V_1\cos(\delta_2 - \delta_1 - \theta_{21}) + Y_{22}V_2\cos(-\theta_{22})$$
$$+ Y_{23}V_3\cos(\delta_2 - \delta_3 - \theta_{23})] = 0$$

$$J2_{32}=\frac{\partial P_3}{\partial V_2}=V_3Y_{32}\cos(\delta_3-\delta_2-\theta_{32})=0$$

$$J3_{22}=\frac{\partial Q_2}{\partial \delta_2}=V_2\left[Y_{21}V_1\cos(\delta_2-\delta_1-\theta_{21})\right]+Y_{23}V_3\cos(\delta_2-\delta_3-\theta_{23})=0$$

$$J3_{23}=\frac{\partial Q_2}{\partial \delta_3}=-V_2Y_{23}V_3\cos(\delta_2-\delta_3-\theta_{23})=0$$

$$J4_{22}=\frac{\partial Q_2}{\partial V_2}=-V_2Y_{22}\sin\theta_{22}+\begin{bmatrix}Y_{21}V_1\sin(\delta_2-\delta_1-\theta_{21})+Y_{22}V_2\sin(-\theta_{22})\\ +Y_{23}V_3\sin(\delta_2-\delta_3-\theta_{23})\end{bmatrix}$$

$$J4_{22}=(-1)(15)\sin(-90°)+\left[(10)(1)\sin(-90°)+15(1)\sin(90°)+5(1)\sin(-90°)\right]$$
$$=15$$

$$\underline{J}(0)=\left[\begin{array}{c|c}\underline{J1}&\underline{J2}\\ \hline \underline{J3}&\underline{J4}\end{array}\right]=\left[\begin{array}{cc|c}15&-5&0\\ -5&7.5&0\\ \hline 0&0&15\end{array}\right]\text{ per unit}$$

Step 3 Solve $J\,\Delta x=\Delta y$

$$\begin{bmatrix}15&-5&0\\-5&7.5&0\\0&0&15\end{bmatrix}\begin{bmatrix}\Delta\delta_2\\ \Delta\delta_3\\ \Delta V_2\end{bmatrix}=\begin{bmatrix}-2.0\\1.0\\-0.5\end{bmatrix}$$

Using Gauss elimination, multiply the first equation by (−5/15) and subtract from the second equation:

$$\begin{bmatrix}15&-5&0\\0&5.833333&0\\0&0&15\end{bmatrix}\begin{bmatrix}\Delta\delta_2\\ \Delta\delta_3\\ \Delta V_2\end{bmatrix}=\begin{bmatrix}-2.0\\0.33333\\-0.5\end{bmatrix}$$

Back substitution:

$$\Delta V_2=-0.5/15=-0.033333$$
$$\Delta\delta_3=0.33333/5.833333=0.05714285$$
$$\Delta\delta_2=\left[-2.0+5(0.05714285)\right]/15=-0.1142857$$
$$\Delta x=\begin{bmatrix}\Delta\delta_2\\ \Delta\delta_3\\ \Delta V_2\end{bmatrix}=\begin{bmatrix}-0.1142857\\0.05714285\\-0.033333\end{bmatrix}$$

Step 4 Compute $x(1)$

$$\underline{x}(1)=\begin{bmatrix}\delta_2(1)\\ \delta_3(1)\\ V_2(1)\end{bmatrix}=x(0)+\Delta x=\begin{bmatrix}0\\0\\1\end{bmatrix}+\begin{bmatrix}-.1142857\\.05714285\\-.0333333\end{bmatrix}=\begin{bmatrix}-0.1142857\\0.05714285\\0.96666667\end{bmatrix}\begin{array}{l}\text{radians}\\ \text{radians}\\ \text{per unit}\end{array}$$

Check Q_{G3} using Eq. (6.5.3)

$$Q_3 = V_3\left[Y_{31}V_1 \sin(\delta_3 - \delta_1 - \theta_{31}) + Y_{32}V_2 \sin(\delta_3 - \delta_2 - \theta_{32}) + Y_{33}V_3 \sin(-\theta_{33})\right]$$

$$= 1\left[(2.5)(1)\sin\left(\underbrace{0.05714}_{\text{radians}} - \frac{\pi}{2}\right) + 5(0.966666)\sin\left(0.05714 + 0.11429 - \frac{\pi}{2}\right) + 7.5(1)\sin\left(\frac{\pi}{2}\right)\right]$$

$$Q_3 = 1[-2.4959 - 4.7625 + 7.5] = 0.2416 \text{ per unit}$$

$$Q_{G3} = Q_3 + Q_{L3} = 0.2416 + 0 = 0.2416 \text{ per unit}$$

Since $Q_{G3} = 0.2416$ is within the limits [−5.0, +5.0], bus 3 remains a voltage-controlled bus. This completes the first Newton-Raphson iteration.

~~Problem 4~~

Problem 3

Solution The Newton-Rhapson method is described by the following equation

7 points

$$\underline{x}(i+1) = \underline{x}(i) - \underline{J}^{-1}\underline{f}[\underline{x}(i)],$$

where the Jacobian, $\underline{J}$, is evaluated at $\underline{x}(i)$ and defined as

$$\underline{J} = \left.\frac{d\underline{f}}{d\underline{x}}\right|_{\underline{x}=\underline{x}(i)} = \left.\begin{bmatrix} \frac{df_1}{dx_1} & \frac{df_1}{dx_2} \\ \frac{df_2}{dx_1} & \frac{df_2}{dx_2} \end{bmatrix}\right|_{\underline{x}=\underline{x}(i)}$$

If we consider the given equations to be f_1 and f_2 respectively, the Jacobian is found to be

$$\underline{J} = \begin{bmatrix} 4x_1 & 2x_2 \\ 2x_1 + x_2 & -2x_2 + x_1 \end{bmatrix}.$$

The first iteration can then be computed as follows

$$\underline{x}(1) = \underline{x}(0) - \underline{J}^{-1}\underline{f}[\underline{x}(0)],$$

where $\underline{x}(0)$, $\underline{J}|_{\underline{x}(0)}$ and $\underline{f}[\underline{x}(0)]$ are given respectively as

$$\underline{x}(0) = \begin{bmatrix} 1 \\ 1 \end{bmatrix} \qquad \underline{J}|_{\underline{x}(0)} = \begin{bmatrix} 4 & 2 \\ 3 & -1 \end{bmatrix} \qquad \underline{f}[\underline{x}(0)] = \begin{bmatrix} -5 \\ -3 \end{bmatrix}.$$

Continuing the same process, it is seen that the values converge to within 3 decimal places ($\epsilon = 0.001$) in 4 iterations.

Table 1: Newton-Rhapson Results After 4 Iterations

Iteration	**0**	**1**	**2**	**3**	**4**
x₁	1	2.1	1.8284	1.8092	1.8091
x₂	1	1.3	1.2122	1.2061	1.2060

Must have matLab code

Problem 4

7 points must have Matlab code

$$P_2 = -\frac{V_1 V_2}{X_l}\sin(\theta_2)$$

$$Q_2 = -\frac{V_2^2}{X_l} + \frac{V_1 V_2}{X_l}\cos(\theta_2)$$

Newton Raphson: $\bar{x}^{k+1} = x^k - J^{-1} f(x^k)$

where J is the Jacobian.

See Matlab code.

For first part, if $\theta_2^0 = 0°$, then $\theta_2^1 = -18.1°$ $\boxed{\theta_2^2 = -18.4°}$

if $\theta_2^0 = 60°$, then $\theta_2^1 = -75°$, $\theta_2^2 = 73°$...

$\boxed{\theta_2^7 = -161.6°}$

if $\theta_2^0 = 90°$, then N-R $\boxed{\text{doesn't work}}$

For second part, $\begin{cases} \theta_2^0 = 0° \\ V_2^0 = 1 \end{cases}$, $\boxed{\begin{cases} \theta_2^5 = -19.2° \\ V_2^5 = 0.911 \end{cases}}$

```
%Problem ~~4~~ 3

%solve for x1 x2
syms x1 x2
f1=x1^2+3*x2^2-31;
f2=x1+x1*x2^2-20;
f = [f1; f2];
x=[x1, x2];
J = jacobian(f, x);
solution=[1; 1]; %initial guess
maxiteration=8;
epsilon=0.001;
x_old=solution(:,1);
Jinv=inv(J);

i=0;
while (i<maxiteration)
    x_new = x_old+(subs(Jinv,x,x_old)*(-subs(f,x,x_old)));
    error=max(abs(x_new-x_old));
    if (error<epsilon)
       i=maxiteration;
    end
    i=i+1;
    x_old=x_new;
    solution=[solution x_new];

end
%%
%Problem ~~5a~~ 4a
clear all
syms theta;
V1=1;
V2=0.95;
P=1.5;
X=0.2;
f1=P+V1*V2*sin(theta)/X;
J = jacobian(f1, theta);
solution=[pi/2]; %initial guess
maxiteration=8;
epsilon=0.005;
theta_old=solution(1);

Jinv=inv(J);

i=0;
while (i<maxiteration)
    theta_new = theta_old+(subs(Jinv,theta,theta_old)*(-subs(f1,theta,theta_old))
    error=max(abs(theta_new-theta_old));
    if (error<epsilon)
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```
        i=maxiteration;
     end
     i=i+1;
     theta_old=theta_new;
     solution=[solution theta_new];

end
solution=solution./pi*180

%%
%Problem 5b 4b
clear all
clc
syms theta2 V2;
V1=1;
P=1.5;
Q=0.2;
X=0.2;
f1=P+V1*V2*sin(theta2)/X;
f2=Q+V2*V2/X-V2*cos(theta2)/X;
J = jacobian([f1; f2], [theta2; V2]);
solution=[0; 1]; %initial guess
maxiteration=20;
epsilon=0.0001;
x_old=solution(:,1);

Jinv=inv(J);

i=0;
while (i<maxiteration)
    x_new = x_old+(subs(Jinv,[theta2, V2],x_old)*(-subs([f1;f2],[theta2, V2],x_ol
    error=max(abs(x_new-x_old));
    if (error<epsilon)
       i=maxiteration;
    end
    i=i+1;
    x_old=x_new;
    solution=[solution x_new];

end
solution(1,:)=solution(1,:)./pi*180
```