

Fast Local Semi Definite Programming-based Localization for Large Wireless Sensor Networks

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Abstract—Recently, Wireless Sensor Networks (WSNs) are deployed inevitably in various technologies. By increasing the demand of deploying WSNs, some ambiguous parts of them should be cleared enough. One the bottlenecks of WSNs, is the localization of each node in WSN. In special applications, the location of sensors is mandatory. In this paper, the speed of localization is taken into account and a fast algorithm is developed to localize the sensors, accurately in a large WSN. Local semi definite programming (LSDP) is used to localize each sensor. The simulation results, has shown tremendous efficiency, from speed point of view.

Keywords—WSN, Localization, LSDP, Fast.

I. INTRODUCTION

Wireless sensor networks (WSNs) provide a special infrastructure to revolutionize the process of communication in different applications. Recently, WSN-based services have a critical role in building flexible services [1,2]. Service based applications such as location-based application, condition monitoring have been addressed in many researches in environment monitoring [3-5]. Since in real application most services are related to the correct knowledge of node location, the localization became an important issue in WSNs, especially large WSNs. Inasmuch as, there is no sufficient condition for a WSN to be localizable; the localization became a difficult problem.

Recently, many localization schemes have been proposed to deal with this problem [6-11]. This problem can be dealt with different ways which are not always cost effective such as Global Positioning System (GPS)-aided methods. As an alternative approach, we can use reference nodes in the WSN and the location of unknown nodes can be derived based on the distances between reference nodes and unknown nodes. Since, the distance is estimated by nodes itself; we always suffer from measurement noise. To decrease this noise effect and increase the accuracy of GPS-independent localization scheme, Biswas and Ye [12] developed a Semi definite programming (SDP)-based approach to model the localization problem into a convex optimization problem.

In large WSNs, the number of sensors is relatively high, and each of them should be located fast enough to assign the services. Using SDP to increase the accuracy of large WSNs would decrease the speed of localization. This problem is arisen because of the optimization time exponential relation considering the size of optimization problem. In [13], local SDP-based approach is used to increase the accuracy of localization in a smaller size and then merging these localized WSNs. Since, the speed of localization is not considered; there is no benefit for large WSNs from speed point of view. In this paper, we developed a localization algorithm based on LSDP to increase the speed of localization in large WSNs.

In this paper, we used the LSDP approach to increase the accuracy and speed of localization in WSN. In fact, the accuracy of localization is maintained by using LSDP methods and the speed of the localization is maintained in clustering approach. In other words, the large WSN is clustered into some sub-WSNs, based on the adjacency matrix. Since, there are a lot of sensors in large WSNs; we tried to limit the sub-WSN sizes. By this approach, the exponential manner of time increasing in localization, changed to linear form and the order of time increasing considering the size of clusters is really decreased.

The remainder of this paper is organized as follows. In next section the system model is represented. The proposed clustering model is explained in section III. Section IV includes the simulation results. Concluding remarks is represented in section V.

II. SYSTEM MODEL

Consider a network of sensors and anchors where we labeled sensors as 1 to s and anchors $s + 1$ to n . Three types of distances could be defined for any point in the network including the distance between two sensors, the distance between a sensor and an anchors and the distance between two anchors. Since, the latter one is not desirable we exclude it from our measurements. From now on, we define the sensor indexes by i and j , and anchor indexes by k . Considering two

applicable distances, sets N_1 , N_2 , \bar{N}_1 and \bar{N}_2 are defined. N_1 is involved of pairwise sensors (i, j) if $i < j$ with known distance measurement d_{ij} . \bar{N}_1 is involved of pairwise sensors (i, j) if $i < j$ with unknown distance measurements d_{ij} . N_2 and \bar{N}_2 is defined according to the N_1 and \bar{N}_1 except for the type of the node which is anchor. We consider a large WSN where the nodes included are all distance-measured. Thus, the sets \bar{N}_1 and \bar{N}_2 are empty in our model, so could be omitted.

The whole WSN with its nodes and edges can be modeled by graph $G = \{V, E\}$, where $V = \{1, 2, \dots, s, s+1, \dots, n\}$ and $E = \{N_1, N_2\}$.

A. Euclidean distance model

Let α_{ij} be the difference between the measured distance and the real Euclidean distance of nodes x_i and x_j . Besides, let β_{ik} be the difference between the measured distance and the real Euclidean distance between sensor i and anchor k . Biswas and Ye [12] formulate the sensor localization problem as minimizing the ℓ_1 -norm of the distance errors α_{ij} and β_{ik} subject to mixed equality constraints as follows:

$$\begin{aligned} \min_{x_i, x_j, \alpha_{ij}, \beta_{ik}} \quad & \sum_{(i,j) \in N_1} |\alpha_{ij}| + \sum_{(i,k) \in N_2} |\beta_{ik}| \\ \text{s. t.} \quad & \|x_i - x_j\|^2 - \alpha_{ij} = (d_{ij})^2 \quad \forall (i, j) \in N_1 \\ & \|x_i - a_k\|^2 - \beta_{ik} = (d_{ik})^2 \quad \forall (i, k) \in N_2 \\ & x_i, x_j \in \mathcal{R}^2; \alpha_{ij}, \beta_{ik} \in \mathcal{R}; i, j = 1, 2, \dots, s; k = s+1, \dots, n \end{aligned}$$

This is a non-convex constrained optimization problem. Biswas and Ye [12] represent a relaxed model for solving this problem approximately.

B. SDP relaxed model

The relaxation approach which is represented by Biswas and Ye [12] is to relax the constraint $Y = X^T X$ to be $Y \succeq X^T X$, which can be reformulated into matrix form as

$$Z_I \equiv \begin{pmatrix} Y & X^T \\ X & I \end{pmatrix} \succeq 0.$$

We define

$$A_I = \begin{pmatrix} \mathbf{0} & \mathbf{0} & \mathbf{0} \\ 1 & 0 & 1 \\ 0 & 1 & 1 \end{pmatrix}, b_I = \begin{pmatrix} 1 \\ 1 \\ 2 \end{pmatrix}$$

where $\mathbf{0}$ in A_I is a zero column vector of dimension s . By these definition the above problem can be relaxed and reformulated into matrix form as

$$\begin{aligned} \min \quad & \sum_{(i,j) \in N_1} |\alpha_{ij}| + \sum_{(i,k) \in N_2} |\beta_{ik}| \\ \text{s. t.} \quad & \text{diag}(A_I^T Z A_I) = b_I \\ & \begin{pmatrix} e_{ij} \\ \mathbf{0} \end{pmatrix}^T Z \begin{pmatrix} e_{ij} \\ \mathbf{0} \end{pmatrix} - |\alpha_{ij}| = d_{ij}^2 \quad \forall (i, j) \in N_1 \\ & \begin{pmatrix} e_i \\ -a_k \end{pmatrix}^T Z \begin{pmatrix} e_i \\ -a_k \end{pmatrix} - |\beta_{ik}| = d_{ik}^2 \quad \forall (i, k) \in N_2 \end{aligned}$$

where $X = (x_1, x_2, \dots, x_s)$ is a $2 \times s$ matrix; e_{ij} is a s dimensional zeros column vector except for 1 in position i and

-1 in position j to define $\|x_i - x_j\|^2 = e_{ij}^T X^T X e_{ij}$; e_i is a zero column vector except for 1 in position i to define $\|x_i - a_k\|^2 = \begin{pmatrix} e_i \\ -a_k \end{pmatrix}^T (X \ I)^T (X \ I) \begin{pmatrix} e_i \\ -a_k \end{pmatrix}$ and constraint $\text{diag}(A_I^T Z A_I) = b_I$ ensures that the matrix variable Z 's lower right corner is a 2-dimensional identity matrix I , so that Z can take the form Z_I .

III. CLUSTERING MODEL

As mentioned previous, a WSN can be modeled by a graph $G\{V, E\}$, in which V consists of two kinds of nodes: normal nodes and Cluster Heads (CH) [14]. CHs are selected according to some factors such as application requirements, residual energy and node degree. In this paper, CHs are selected by a factor called importance index which is defined later in this chapter.

In general, Let $I(x_i x_j)$ represent the intersected area of the two nodes x_i and x_j as follows:

$$\begin{aligned} I(x_i x_j) = 2 \int_{r_i \cos \theta}^{r_{ii}} \sqrt{r_i^2 - x^2} dx \\ + 2 \int_{d_{ij}-r_j}^{r_j \cos \theta} \sqrt{r_j^2 - (x - d_{ij})^2} dx \end{aligned}$$

where d_{ij} represent the distance between node x_i and node x_j and $\theta = \cos^{-1}(d_{ij}^2 - r_i^2 - r_j^2 / 2r_i r_j)$; r_i and r_j are the communication range of nodes x_i and x_j , respectively. To describe the weight of a path from node x_i to node x_j a weighting metric is defined as

$$W_1(x_i x_j) = \frac{I(x_i x_j)}{\sum_{j=1}^{|N_i|} I(x_i x_j)}.$$

In the case of x_l as the intermediate node from x_i to x_j , the two hop weight can be defined as the multiplication of the two one hop weights as

$$W_2(x_i x_j) = W_1(x_i x_l) W_1(x_l x_j)$$

The n -hop path weight is defined, similarly. Furthermore, let $\lambda_{x_i x_j}$ represents the shortest path hop count from x_i to x_j and $\lambda_{x_i x_j}(x_k)$ counts the shortest path nodes which intermediate by node x_k . By these assumptions, importance index of node x_j is defined as

$$NI(x_j) = \sum_{i \neq j, l} \frac{\sum_h^{hop} W_h(x_i x_k) \lambda_{x_i x_k}(x_j)}{\lambda_{x_i x_k}}.$$

Apparently, the value of $NI(x_j)$ represents the difficulty of node x_j to reach others. In other words, larger values of $NI(x_j)$ indicate the shortest path to others intermediating by x_j would be shorter. In each cluster, the node with highest NI value would be selected as the CH.

Considering the graph $G = \{V, E\}$, bidirectionality of the links is assumed. $N_1(x_i)$ represent the set of neighbors of node x_i and $N_2(x_i)$ represents two-hop nodes. In defining $N_2(x_i)$,

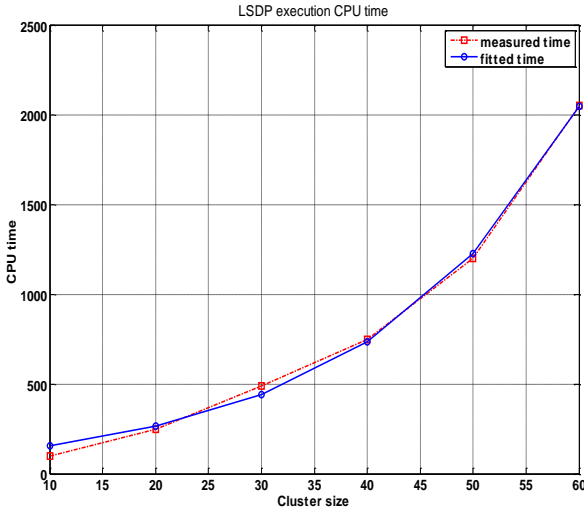


Fig. 1. LSDP execution CPU time vs. cluster sizes

the effect of the second hop on the coverage and connectivity is not considered. By combining these two sets, we consider $N_{12}(x_i)$ as the sub-graph the cluster.

This type of clustering and CH defining is represented in [13]. For more information about it, one can refer to [13]. In this clustering method, there is no limitation on the number of the nodes to be in one cluster. Large number of the nodes would cause SDP problem to take time, exponentially with respect to number of the nodes.

A. Proposed Clustering Algorithm

Speed related studies of the method of localization in [13], represented that the size of clusters is related to time of simulation, exponentially. To decrease the time of the simulation, we consider limiting the number of nodes in each cluster to increase the speed of localization. By limiting the number of the nodes, the number of clusters would increase which affect the time of simulation, linearly; thus, the localization time would be done faster than before. As a measure of the speed of localization respect to cluster sizes, a diagram is represented on Fig. 1. The exponential increase of the CPU time is apparent.

In Fig. 1, the real measurements are fitted on an exponential function and the result represents that execution time of LSDP with respect to the cluster size is an exponential function. The proposed clustering algorithm is represented in the following, which make the cluster size to a fixed number, therefore the LSDP localization would increase linearly with the number of clusters.

Algorithm 1. WSN Clustering

Input: ClusterSize, WSN adjacency matrix

Output: Clusters

1. One of the nodes selected randomly and called x_i
2. $N_1(x_i)$ is collected
3. Importance index of each node is derived

4. Size of $N_1(x_i)$ is compared to ClusterSize,

If it is smaller $N_2(x_i)$ is collected till cluster size is equal to ClusterSize, this collecting is done by considering the higher importance index.

If it is bigger, some of the nodes with smaller importance index will be omitted till cluster size is equal to ClusterSize.

5. The importance index is derived again for all the nodes in a cluster.

Using the proposed approach in clustering the WSN, would limit the cluster sizes; thus, LSDP approach would result faster in each of the clusters.

IV. SIMULATION RESULTS

In this section, the simulation results of a large WSN localization are represented. The simulated WSN is a large WSN with different number of nodes from 400 to 1000. These nodes are randomly distributed in a rectangular space with dimensions $1200 \times 700(m^2)$. All these nodes have the same communication range 2 meter. Here, 5% of nodes are selected as reference nodes. The distance measurements noise is modeled as $d_{ij} = \hat{d}_{ij}(1 + 0.1N(0,1))$, where \hat{d}_{ij} is the accurate distance between two nodes and $N(0,1)$ is a normal distribution with zero mean and unit variance.

To compare the speed of algorithm with the previous one, we run the localization problem using two different clustering methods, on large WSN with 400 to 1000 nodes. The CPU time after localization, is represented in Table 1 and Fig. 2. As it is obvious, by increasing the number of nodes in each cluster, we would have more nodes, and cluster size would increase. We select 40 nodes in each cluster as the maximum nodes in the proposed approach. The CPU time in this table is based on hour. SDP calls which are related to each of methods represent that how many clusters are formed by each of them. Each of the WSNs is run for ten times and the average of ten times is represented here.

TABLE I. CPU TIME COMPARISON

WSN size	Ref [1] approach CPU time (h)	Proposed approach CPU time (h)	Ref [1] approach SDP calls	Proposed approach SDP calls
400	8.1247	2.0241	9	12
500	9.2476	2.5475	11	15
600	10.0575	3.1547	13	18
700	10.9975	3.7665	12	20
800	12.1024	4.1176	15	24
900	13.0541	4.7885	14	27
1000	13.8974	5.1004	15	30

V. CONCLUSION

In this paper, we have developed a kind of clustering method to increase the pace of localization in large WSNs. This clustering was done before based on the one hop and two hop neighborhood without any limitation on the size of

clusters. Since, the time of SDP increases exponentially, we limit the size of clusters to change the exponential increasing into linear increasing. Simulation results demonstrate the efficiency of our method rather than the previous method, obviously.

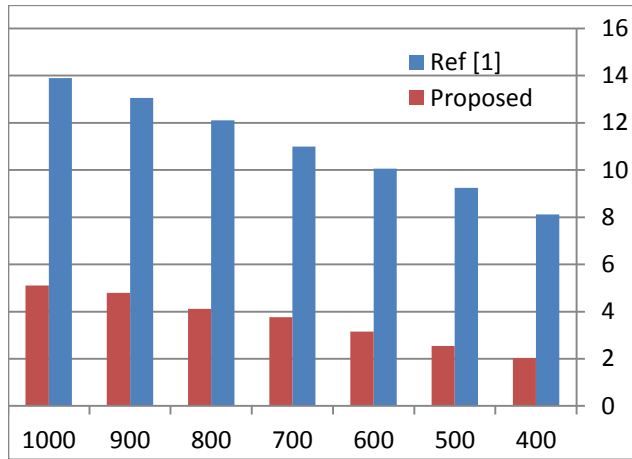


Fig. 2. CPU run times for two different approaches

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