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Manager perceptions of decision-making for evaluation indicators: a centralized data envelopment analysis based approach

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Abstract

Purpose – This paper aims to propose an integrated centralized data envelopment analysis (CDEA)-balanced scorecard (BSC) model to provide a selective approach to determine the most efficient indicators for evaluating the four perspectives of the BSC.

Design/methodology/approach – An integer linear programming model based on the efficiency concept of the CDEA method is presented to select the best indicators for evaluating four perspectives of the BSC. The basis for selecting indicators in this method is to maximize the overall performance of each BSC perspective. The modeling is performed on a real case. The considered model is solved using a general algebraic modeling system software for the data set of the real case.

Findings – A real-world case is solved using the proposed method. The integration of the CDEA and the BSC seems to be advantageous because it sheds more light on the complexity and tradeoffs inherent in actual performance measurement. It is important to note that there cannot be a unique and universal model of performance measurement applicable in every situation, in every organization and at any time.

Research limitations/implications – The data set of a single organization in the manufacturing industry is used to show the performance of the proposed mathematical model; therefore, generalization of the results should be done cautiously. This framework is based on the Iranian community and experts' viewpoints; therefore, different results may be obtained if it is applied elsewhere, and the importance of perspectives and their indicators might show different results in other populations and other countries. In addition, because the data is collected in a specific period of time, the results cannot be extended to other periods of time.

Originality/value – The main contribution of this paper lies in the adaption of a new integrated CDEA-BSC model to performance measurement in the industrial sector that technology improves the ultimate results

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of performance measurement and provides wider opportunities for decision-makers. This paper aids managers and decision-makers to control the efficient indicators in perspectives.

Keywords Centralized data envelopment analysis (CDEA), Balanced scorecard (BSC), Centralized decision-maker, Efficiency

Paper type Research paper

1. Introduction

To remain in today's competitive environment, organizations need to apply more efficient performance measurements and revise them according to the needs of their customers and the market. As an operational research tool, data envelopment analysis (DEA) is used for evaluating the relative efficiency of many decision-making units (DMUs) and takes both qualitative and quantitative measures into account. Charnes, Cooper, and Rhodes proposed the first DEA model in 1978. Unlike traditional DEA models assuming that DMUs are independent units, the new DEA model, known as the centralized or intraorganizational resource allocation approach, assumes that all DMUs are under the control of a high-level authority, namely, the centralized decision-maker who makes an attempt to optimize the resource consumption of all the units that belong to the same organization, whether it is private or public. In other words, the centralized decision-maker designs an intraorganizational scenario to maximize the efficiency of individual units at the same time that total input consumption is minimized or total output production is maximized (Lozano and Villa, 2004). An efficient performance measurement system is one that provides a balanced picture of current operating performance as well as the drivers of future performance, includes both financial and nonfinancial measures and improves strategic management and decision-making to achieve the specific objectives of an organization. Recent studies provide considerable evidence that organizations are increasingly adopting balanced scorecard (BSC) in their strategic process (Gumbus, 2005). This approach is more than just a performance measurement tool. It is also a way to manage new change and increase a company's effectiveness, productivity and competitive advantage (Keyes, 2005). The BSC is a conceptual framework that takes the multidimensional nature of performance into account. By moving beyond financial measures of performance, the BSC integrates a set of performance measures distributed among four perspectives: financial, customer, internal business processes and learning and growth.

Although the BSC has been one of the most practically successful performance measurement systems used by organizations all over the world, the method has some limitations to be resolved. Some researchers and experts have mentioned some limitations of the BSC. It does not specify how tradeoffs are to be made between different scorecard criteria (Otley, 1999). It does not specify an objective quantitative weighting scheme for the performance measures, and it may fail to identify inefficiency in the use of resources (Werner and Brokmper, 1996). BSC indicates that all strategic objectives have the same importance, weighted the same, which may be not true in real contexts (Chytas *et al.*, 2011). The selection process of indicators from all the possible important ones that can exist in an organization does not receive enough attention (Quezada and López-Ospina, 2014). The identification of appropriate targets for each of the performance indicators without a benchmarking exercise is difficult in a real context (Amado *et al.*, 2012). On the contrary, DEA allows each unit to identify a benchmarking group, that is, a group of units that try to perform better by following the same objectives and priorities. It means that DEA let each DMU choose the weight structure for inputs and outputs that most benefit its evaluation according to its vision and objectives. Another advantage of DEA is that it does not require the specification of a cost or production function that makes the model more complex. According to the abovementioned points, the integration of DEA with the BSC model seems to be a good option to tackle some weaknesses of the BSC. The BSC is usually applied for quality measurement of performance, whereas the DEA is more suitable to perform quantity measures because DEA models can integrate unlike inputs and outputs to make simultaneous comparisons of DMUs (Avkiran, 2002).

Although the integrated use of DEA and BSC has been investigated in some studies, to the best of my knowledge, none of them apply centralized data envelopment analysis (CDEA) and BSC together. The present article extends the study strand of Lozano and Villa (2004) and Amado *et al.* (2012) and presents a new integrated CDEA-BSC model to determine the most efficient indicators for evaluating the four perspectives of the BSC. The proposed model is equivalent to finding inputs that independently maximize the efficiency of a total DMU in each perspectives of the BSC. Our goal is to show that the integration of these two methods can technically improve the ultimate results of performance measurement and provide wider opportunities for decision-makers.

The remainder of this article is organized as follows. Section 2 provides a literature review on BSC, DEA, CDEA and the integrated use of DEA and BSC. Section 3 provides the methodological issues. Data collected from Esfahan Steel Company as the case study are presented in Section 4. Empirical results are discussed in Section 5. Section 6 concludes the study.

2. Literature review

This section reviews the literature on the BSC, DEA, the research that applied the BSC and CDEA together and the CDEA.

2.1 Balanced scorecard

Unlike traditional performance measurement tools, the BSC, devised by Kaplan and Norton (1996, 2004), has acquired more popularity in practice and has been used efficiently by organizations all over the world. The BSC is designed to improve managers' decision-making and problem-solving. It provides a holistic performance measurement system that takes into account the nonfinancial measures and the financial measures simultaneously (Kaplan and Norton, 1996). BSC is among a few measures of performance indicators that need to be checked periodically (Neely and Hii, 1998). BSC, as a strategic management system, improves the management of information in organizations (Huang, 2009). The BSC framework has been applied in many types of research and various management fields such as supply chain management (Aliakbari Nouri *et al.*, 2019), research and development projects (Purnomo and Sutanto, 2019), commerce (Rickards, 2007; Shan *et al.*, 2019), marketing (Abedian *et al.*, 2021a), enterprise resource planning (Kajtazi and Holmberg, 2019; You and Wu, 2019), business (Bénet *et al.*, 2019; Hamamura, 2019), quality function deployment (Dincer *et al.*, 2019) and research and development (R&D) (Salimi and Rezaei, 2018).

In general, the BSC framework consists of four perspectives according to which the organization is evaluated: the financial, the customer, the internal process and the learning and growth perspectives. Organizations determine the performance indicators, the target levels of the indicators and the actions to be taken to achieve these indicators according to their visions, strategies and objectives. According to Basso *et al.* (2018), indicators of the financial perspective reflect profitability (return on investment, cash flow, operating income, etc.) and the ability to meet the needs of the shareholders. The customer perspective refers to the way in which the company should be perceived by customers (loyalty, customer satisfaction, the number of new customers, etc.). The internal process refers to actions to be taken to meet the expectations of shareholders and customers (quality of the services and the information systems, the costs of the service production, the ability to differentiate the product, etc.). The learning and growth perspective reflects the ability to develop continuous improvement, innovation and learning to deal successfully in the medium- and long term (investments in training, equipment, the ability to increase empowerment and the staff satisfaction). The required data is presented in the following Figure 1. The original formulation of BSC has been proposed for the evaluation of profit-oriented organizations. The application of the BSC to nonprofit organizations requires some adjustments with respect to the original setting of the BSC to fulfill the social purpose of their activity (Basso *et al.*, 2018). Despite the popularity and practicality of the BSC, the method has some deficiencies in terms of implementation on a quantitative basis (Kádárová *et al.*, 2014). Some authors have proposed significant changes of traditional perspectives and indicators or prefer to use the original perspectives combined with other methods such as knowledge-based system, analytic network process, control objectives for information technology and game theory to produce in an improved approach to strategic planning and decision-making as well as to enhance quality control and human aspects. A new approach based on the BSC and game theory has been proposed for evaluating the performance of an Iranian company to determine the most appropriate combination of BSC indicators and to build an equilibrium point between financial and nonfinancial performance measures (Abedian *et al.*, 2021b).

2.2 Data envelopment analysis

DEA is a linear programming-based approach used to identify best practices or efficient frontier DMUs, in the presence of multiple inputs and outputs (Charnes *et al.*, 1989). The performance or efficiency of a DMU is expressed in terms of a set of measures classified as DEA inputs and outputs. In conventional DEA, each DMU is treated as a black box and its internal structures and operations are ignored (Cook *et al.*, 2014). DEA methodology provides an overall multidimensional measure of performance that is simultaneously open to multiple input and multiple output situations. So DEA has a wide range of applications both in profit and nonprofit institutions to identify efficient and inefficient units. A significant amount of research exists on both the theory and applications of the DEA approach. Seiford and Zhu (1999) use a two-stage DEA process to measure the profitability and marketability of the top 55 US commercial banks. Chen *et al.* (2006) develop a DEA nonlinear programming model to evaluate information technology investment. Yu and Lin (2008) use a multiactivity network DEA model to evaluate efficiency and effectiveness in railway performance in Germany. Wu *et al.* (2013) propose a DEA-based approach for fair reduction and reallocation of emission permits. Cetin and Bahce (2016) measure the

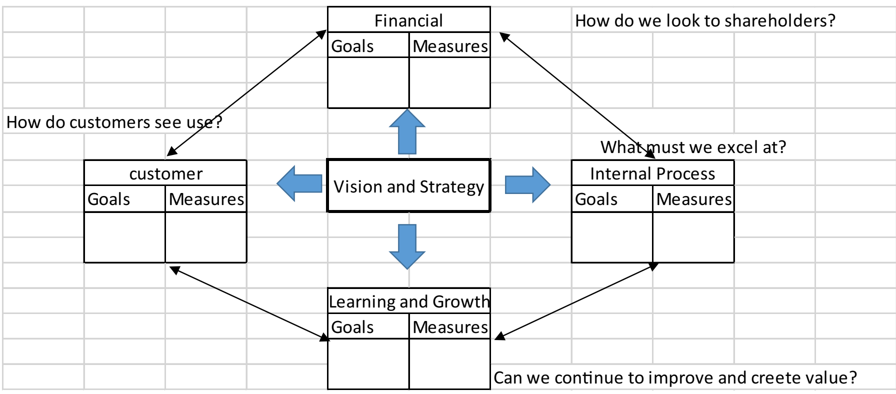

Figure 1.
Four perspectives of balanced scorecard

efficiency of health systems of OECD countries by data envelopment analysis. Wu *et al.* (2018) survey allocation of emission permits based on DEA and production stability. Lee and Park (2019) develop a two-stage DEA analysis for efficiency analysis of eight fields of weapon systems in Korea. An integrated DEA-based approach has been implemented to suggest an appropriate structure to the actual public pharmaceutical supply chain in Morocco (Chorfi *et al.*, 2019). Contreras (2020) proposed a systematic revision of the existing literature regarding the procedures to determine a common set of weights in the DEA context. A fuzzy preference programming – DEA (FPP-DEA) was employed to handle missing, unavailable and imprecise data for output values, where the outputs are expressed as fuzzy numbers and the inputs are conveyed in their actual crisp values (Mirasol-Cavero and Ocampo, 2021).

2.3 The integrated use of data envelopment analysis and balanced scorecard

Over the past few years, many researchers apply a combination of multiple methodologies to obtain enhanced performance assessment frameworks and tackle the weaknesses of applying only one approach (Franco and Lord. 2011; Xu and Yeh, 2012; Amado *et al.*, 2012). The BSC-DEA hybrid method has several features, including ensuring linkage of strategic goals and key performance indicators; balancing key performance indicators in different dimensions of the organization; evaluating the comparative spatial organization experienced from previous years or similar organizations; determining the path of improvement and improvement of indicators based on the scientific results of performance evaluation; and determining the goals of the indicators based on the path of improvement.

The integrated use of DEA and BSC has been investigated in some studies. In some studies, the classic DEA model of efficiency evaluation has been used to survey the specific number of relationships between the four perspectives of BSC, all possible relationships between the four perspectives of BSC or to obtain the efficiency score for the DMU. Rickards (2007) developed a DEA model to set benchmarks and evaluate the four perspectives of the BSC in a comparison of 69 units of a multinational company. Rickards considered the cash flow, customer commitment and internal service quality and employee motivation as outputs. Eilat *et al.* (2006) used a DEA-based methodology to construct and evaluate balanced portfolios of research and development projects with interactions. Chen *et al.* (2008) use DEA and BSC for performance evaluation of a credit cooperative bank. Min *et al.* (2008) apply a DEA-BSC integrated model for Korean hotels in which the financial perspective of BSC is evaluated with the efficiency results obtained from DEA analysis. Macedo *et al.* (2009) apply a DEA to indicators of BSC perspectives for performance evaluation of bank branches in Brazil. Amado *et al.* (2012) presented an integration of the DEA and the BSC for organizational performance to identify where there is room for improving organizational performance and points out the opportunities for reciprocal learning between DMUs. They recommend that moving away from a unique all-embracing DEA model toward multiple complementary models is advantageous, leading to enhanced performance assessment. Shafiee *et al.* (2014) propose a general framework to evaluate the overall performance of the supply chain by means of the BSC and DEA models. Basso *et al.* (2018) propose a new two-stage DEA-BSC approach to the empirical analysis of the municipal museum of Venice. In the first stage, they defined a proper DEA model for each perspective of BSC to compute the DEA efficiency scores for every perspective. In the second stage, they defined a DEA model to combine the efficiency scores of various BSC perspectives into an overall performance indicators. Tan *et al.* (2017) integrate the DEA with BSC to measure the service performance. The use of DEA to analyze the efficiency of automotive industry, as well as to sort out the inefficient units to make them efficient using BSC. Khalili and Alinezhad (2018) evaluate the performance of the green supply chain by using an integration of BSC, Malmquist productivity index and decision tree models. The proposed model was investigated in the form of a case study in the automotive parts manufacturing industry. Sarraf and Nejad (2020) propose DEA and grey relational analysis approaches based on a BSC for improving performance evaluation of water and wastewater companies. The fuzzy set theory and data employment analysis were employed to propose an objective assessment model for evaluating the performance of internal and external capabilities of firms (Lin *et al.*, 2021). The BSCs and network data envelopment analysis were used to conduct a performance evaluation task from multiple perspectives (Hsu and Lin, 2021). The relative efficiency of the supply chain operations reference and the BSC were compared for measuring the supply chain performance (El-Garaihy, 2021). The results provide empirical support for the DEA network model concerning its preference and improved performance measurement as compared to the BSC model.

2.4 Centralized data envelopment analysis

The CDEA model proposed by Lozano *et al.* (2004) and Lozano and Villa (2004) is a useful decision-making technique to optimize the combined resource consumption of all operating units. Lozano and Villa (2004) proposed two rather simple DEA models for resource allocation that project all DMUs onto the efficient frontier in a jointed manner, whereas conventional DEA models set targets separately for each DMU. One type of their model seeks radial reduction of the total consumption of every input, whereas the other type seeks separate reduction for each input according to a preferred structure and at the same time, none of these models do decrease the total output production. The first phase of the output oriented for the centralized resource allocation (CRA) can be formulated as follows:

$$\varphi^* = \underset{s.t}{Max}\,\varphi \tag{1}$$

$$\sum_{j=1}^{n}\sum_{l=1}^{n}\lambda_{lj}x_{ij} \leq \sum_{j=1}^{n}x_{ij} \tag{2}$$

$$\sum_{j=1}^{n}\sum_{l=1}^{m}\lambda_{lj}y_{rj} \geq \varphi\sum_{j=1}^{n}y_{rl}, r = 1, 2, \ldots, s, \tag{3}$$

$$\sum_{l=1}^{n}\lambda_{lj} = 1, \forall_j \tag{4}$$

$$\lambda_{lj} \geq 0, \tag{5}$$

$$\varphi \geq 1 \tag{6}$$

where $j, l = 1\ldots n$ indicate the indices of DMUs. Figure 2 shows that, unlike noncentralized models, DMUs in the centralized model are not projected along the output axis, and the input and output of each DMU can be increased or decreased. As an example, $\mathrm{DMU_B}$ is an efficient DMU and therefore is on the efficient frontier. Using the BCC model, the outputs of a DMU have no effect on its input and output projection, but the results of the output centralized model indicate that the input and output projection of this DMU both increase.

Asmild *et al.* (2012) modify the centralized models introduced by Lozano and Villa (2004) to consider adjustments of previously inefficient DMUs. They show how their new model formulation relates to standard BCC models, which is used to analyze the mean inefficient point and provide a procedure to generate alternative optimal solutions, which enables a decision-maker to select the preferred solution among all alternative possibilities. Lotfi *et al.* (2010) proposed a CRA for the enhanced Russell model to project all DMUs onto an efficient frontier by solving only one model. Lozano *et al.* (2011) propose a number of nonradial, output-oriented, CDEA models to target setting and resource allocation in the Spanish national port system. They show that by adopting a centralized perspective and allowing input reallocations, it is possible to achieve higher system output levels and more cost-effective utilization of resources. Liu and Tsai (2012) present serial slacks-based models to manage the interaction between two decision-making levels, governing decision-maker and DMUs, to provide the reallocated targets of inputs/outputs for DMUs in the next operating period of 25 branches of a commercial bank in Taiwan. Mar-Molinero *et al.* (2014) simplify the CRA model and study the model using real data of Spanish public schools to find the best way to reallocate resource among the schools and the most desirable operating unit. Yu *et al.* (2013) studied the human resource rightsizing problem for Taiwan's airports using CDEA. Yu *et al.* (2013) modified a single-phase slack-based model by considering transfer-in and transfer-out slacks to extend the CDEA model of Yu *et al.* (2013) and to remove its inconsistency problem regarding the intensity variables. Fang (2016) developed a generalized centralized resource allocation (GCRA) model that extends Lozano and Villa's (2004) and Asmild *et al.*'s (2012) models. He has also applied the structural efficiency by Li and Ng (1995) and Li and Cheng (2007) to propose a GCRA-BCC model to incorporate the nonadjustable input variables and nontransferable outputs. Chih-Ching Yang (2017) proposed two models to demonstrate the practical feasibility of the local resource reallocation plans applied to an empirical analysis of a hospital system in Taiwan. The first one considers an acceptable percentage of changes in resources to ensure practical feasibility. The second one further considers a situation where resources can only be allowed to shift within specific units. Yang has pointed out that sometimes a central authority may just seek to improve efficiency through a minor local change of resources, whereas both the models of Asmild *et al.* (2012) and Lozano and Villa (2004) pursue an allocation plan that makes the whole system perfectly efficient through a drastic change in resource allocation that is difficult to carry out in practice. Sadeghi and Dehnokhalaji (2019) propose two new CRA methods extending Lozano and Villa's (2004) method to allocate planned future resources across a set of DMUs.

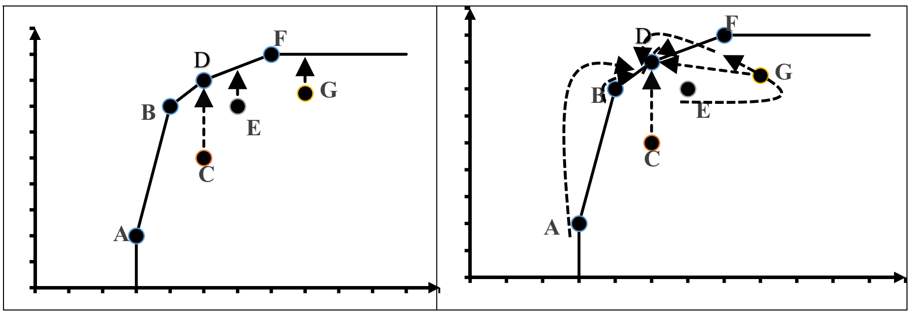

Figure 2.
Projection of DMUs in BCC-O model and projection of DMUs in centralized DEA

From the literature review, it is noted that the researchers believe a combination of the DEA and BSC maybe be more advantageous in improving performance measurement. Although the BSC has been proved to be successful in practice and used by organizations all over the world, the method has some limitations to be resolved. So, this research has applied a CDEA approach for determining perspectives of the BSC that has received very little attention in previous studies. The integrated use of CDEA and BSC for enhancing efficiency and removing inefficiency of organizational performance evaluation that is a new area of research is the main focus of the present study.

3. The proposed methodologies

The purpose of this paper is to propose a CDEA-BSC integrated model to determine the number of input indices in each BSC perspective by the central decision-maker. A review of related studies using the BSC-DEA integrated method indicates that the researchers have used different approaches in this regard, in that all of them have identified a number of DEA models after identifying and explaining the performance indicators based on the BSC method. Given the limitation of the DMUs and the independence of the specified criteria of each perspective on criteria used in other perspectives, four independent CDEA models are considered to use in this study. The relationship between BSC and CDEA in the proposed model is shown in Figure 3. To develop the CDEA models to assess the performance of the six separate production units in the company from multiple perspectives, it was necessary to develop a BSC for the company. Several workshops were undertaken with the heads of the department and other managers to discuss the vision and strategy for the company.

3.1 Assumptions

The model in this study has been proposed on the basis of some assumptions that seem necessary to be specified to run the model in real-world situation. These assumptions are presented here:

- all the parameters are deterministic;
- the number of perspectives and indicators in the company is finite;
- all BSC indicators are prespecified;

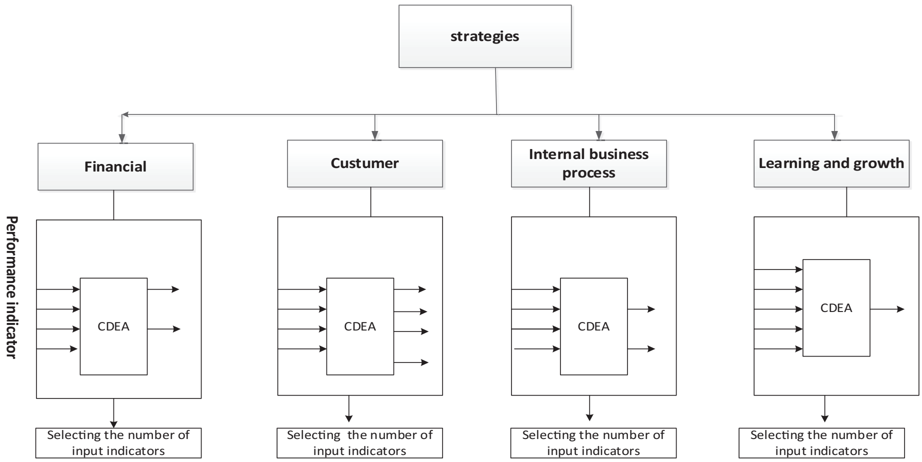

Figure 3. Relationship between BSC and CDEA in the proposed model

- the inputs and the outputs are selected for the CDEA over the two-year period; and
- the DMUs are not independent of each another.

3.2 Sets and indexes

$i \in \{1, 2, \cdots, m\}$ – Index for inputs;
$r \in \{1, 2, \cdots, n\}$ – Index for outputs;
$j, l \in \{1, 2, \cdots, n\}$ – Indexes for existing DMUs;

3.3 Parameters

n – Number of existing DMUs;
m – Number of inputs;
r – Number of outputs;
x_{ij} – Amount of input i consumed by DMU_j; and
y_{rj} – Quantity of output r produced by DMU_j.

3.4 Decision variables

φ – Radial expansion of total output vector;
λ_{lj} – Vector for protecting DMUr; and
μ_i – Is 1 if indicator i is chosen otherwise 0.

3.5 Mathematical model

$$\varphi^* = \underset{s.t}{Max}\, \varphi \tag{1}$$

$$\sum_{j=1}^{n} \sum_{l=1}^{n} \lambda_{lj} x_{ij} \leq \sum_{j=1}^{n} x_{ij} + M_i \mu_i \tag{2}$$

$$\sum_{j=1}^{n} \sum_{l=1}^{m} \lambda_{lj} y_{rj} \geq \varphi \sum_{j=1}^{n} y_{rl}, r = 1, 2, \ldots, s, \tag{3}$$

$$\sum_{l=1}^{n} \lambda_{lj} = 1, \forall_j \tag{4}$$

$$\sum_{i} \mu_i = \mathrm{t} \tag{5}$$

$$\lambda_{lj} \geq 0, \mu_i \in \{0, 1\} \qquad \varphi \ \mathit{free} \tag{6}$$

In this model, the objective function is to maximize the total efficiency score. Constraint (2) determines the indicators selected for each perspective and imposes the fact that only one combination of indicators must be selected. Constraint (3) seeks to radially increase each output as much as possible and ensures that each output remains in the feasible aggregated output set. The constraints in equation (3) ensure that these n projected points cannot lie outside the aggregated output set. Constraint (4) shows that variable returns-to-scale is adopted in this model and any intensity variable used to project DMU cannot be less than 0, as shown in equation (6). Constraint (5) determines the number of inputs selected for each perspective. Finally, constraint (6) determines the type of the variables.

Suppose among m indicators known as input indicators in each BSC perspective, the selection of t indicators is considered and the aim is to use only t as the main indicator in that BSC perspective in performance evaluation. In this case, the above model can be used from any perspective:

The above model is a combination of a CDEA model and zero-one planning. The above model has some advantages. The model itself determines the most efficient indicators that do not need any judgment or analysis. It is enough to solve a model in each perspective.

A brief review of model properties is also described in Appendix.

The centralized decision-maker can use this general model to determine the most efficient inputs and outputs. In this model, t_1 is the number of efficient inputs and t_2 is the number of efficient outputs specified by the centralized decision-maker:

$$\varphi^* = \underset{s.t}{\mathrm{Max}(\varphi - \theta)} \tag{1}$$

$$\sum_{j=1}^{n}\sum_{l=1}^{n}\lambda_{lj}x_{ij} \leq \theta\sum_{j=1}^{n}x_{ij} + M_i\mu_i \tag{2}$$

$$\sum_{j=1}^{n}\sum_{l=1}^{m}\lambda_{lj}y_{rj} \geq \varphi\sum_{j=1}^{n}y_{rl} + N_r\gamma_r, r = 1, 2, \ldots, s, \tag{3}$$

$$\sum_{l=1}^{n}\lambda_{lj} = 1, \forall_j \tag{4}$$

$$\sum_{i}\mu_i = t_1 \tag{5}$$

$$\sum_{i}\gamma_r = t_2 \tag{6}$$

$$\lambda_{lj} \geq 0, \mu_i, \gamma_r \in \{0, 1\} \qquad \varphi \ \mathit{free} \tag{7}$$

In this section, the proposed model is solved for a set of data with specific inputs and outputs. The numerical example consists of three inputs and four outputs. Table 1 displays the numerical example for six DMUs.

The CDEA model can be considered as an integrated set of DMUs that enhances the whole efficiency of DMUs though the efficiency of some DMUs may be reduced. Therefore, DMUs and their efficiencies are dependent on each other and they are under the influence of the whole set. Also, DMUs are assumed to be homogeneous because they convert the same kinds of resources/inputs to the same kinds of products/outputs. We select three inputs ($X1$,

$X2$ and $X3$) and four outputs ($Y1$, $Y2$, $Y3$ and $Y4$) such that their reliable data is available. It should also be noted that our proposed model can consider other inputs and outputs.

It is assumed that two inputs could be selected among three inputs ($X1$, $X2$ and $X3$). The inputs ($X1$ and $X2$) as the most efficient inputs would be selected by solving the 3.5 model. Then it is assumed that two inputs and three outputs could be selected among all inputs and outputs. The inputs ($X1$ and $X2$) and outputs ($Y2$, $Y3$ and $Y4$) as the most efficient inputs and outputs would be selected by solving the second generalized model.

4. A case study

Amado *et al.* (2012) have mentioned that one way to broaden DEA to incorporate the paradoxes and tradeoffs inherent in real-life organizations is to move away from the "black box" type of evaluation. Incorporating the DEA exercise into case studies and context-driven research projects can facilitate the interpretation and implementation of the results in practice. To understand our proposed model, this real-world case study is provided. Esfahan Steel Company is the first and largest construction steel and rail producer in Iran and the biggest producer of long products in the Middle East with 3 million tons capacity per year, producing various constructional and industrial steel sections. Esfahan Steel Plant started operation in 1971; this factory that is located in the Southwest of Esfahan has an important role in the formation of other steel industries. The proposed model was applied in this company. The required data is collected in the years 2018 and 2019. The BSC has already been applied in this company at the business level. The required data is presented in Tables 2–6. There are six separate production units in the company, with input and output indicators specified in each perspective. During a period of evaluation by the plant manager, due to time- and cost-consuming matters, he decides to select three inputs for each perspective. First, it is necessary to explore how the four separate goals, each related to one perspective of the BSC (learning and growth, internal processes, customers and finance), are combined in an integrated evaluation framework. For this purpose, the inputs of the learning and growth perspective include the cost of personnel training, increasing personnel proficiency, the number of improvement committees and the number of personnel and motivational expenses. The ratio of skilled workers, electronic services, speeding up the services and the increase in the European Foundation for Quality Management (EFQM) assessment score are inputs for the internal process perspective. Competitive pricing, stop-working (days), energy (kWh/ton) and production (ton/no. of people) are the inputs from the customer perspective. Additionally, income ratio, export (%), cost (monetary units) and delivery cost are used from the finance perspective. As mentioned above, in the integrated model, the inputs of each stage are the outputs of the previous stage. Inputs and outputs are visible in Tables 2–6.

Table 1. Numerical example of three inputs and four outputs

DMUs	$X1$	$X2$	$X3$	$Y1$	$Y2$	$Y3$	$Y4$
1	3	5	26	0.5	1	9	5
2	8	2	95	0.3	1.8	5	3
3	5	6	15	0.4	1.8	5	8
4	6	9	85	0.1	1.3	7	6
5	5.3	5	73	0.3	0.7	34	7
6	7	3	14	0.6	0.5	10	3

Financial			
Inputs		Outputs	
I1	Income ratio	O1	Capital growth rate
I2	Export	O2	Returns of the capital
I3	Cost	O3	Interest margin
I4	Delivery cost		
Customer			
Inputs		Outputs	
I1	Competitive pricing	O1	Customer satisfaction
I2	Stop-working (days)	O2	High-quality service
I3	Energy (kWh/ton)	O3	Customer attraction rate
I4	Production(ton/no. of people)	O4	Quick service
Internal process			
Inputs		Outputs	
I1	Speeding up the services	O1	Customer satisfaction
I2	Increase in the EFQM	O2	High-quality service
I3	Ratio of skilled workers		
I4	Electronic services		
Learning and growth			
Inputs		Outputs	
I1	Number of improvement committees	O1	Increasing personnel skills
I2	Number of personnel	–	–
I3	Motivational expenses	–	–
I4	Increasing personnel proficiency	–	–
i5	Cost of personnel training	–	–

Table 2.
CDEA-BSC indices

DMU	INPUT1	INPUT2	INPUT3	INPUT4	INPUT5	OUTPUT1
DMU1	10	200	23.03	12	45	58.54
DMU2	4	263	18.72	12	10	30.8
DMU3	3	102	18.5	12	3	46.25
DMU4	2	224	5.3	12	5	18.55
DMU5	3	219	17	12	20	39.1
DMU6	3	210	30	14	15	69

Table 3.
CDEA-BSC data in learning and growth perspective

DMU	INPUT1	INPUT2	INPUT3	INPUT4	OUTPUT1	OUTPUT2
DMU1	800	80	289	1,205	1,376	91
DMU2	692	20	104	1,806	1,896	57
DMU3	718	30	120	1,658	1,842	58
DMU4	682	10	87	1,400	1,315	37
DMU5	643	40	102	645	787	34
DMU6	555	50	115	417	510	10

Table 4.
CDEA-BSC data in internal process perspective

5. Results

The four output-orientation CDEA models were run following the BCC formulation to obtain the relative performance scores for each BSC perspective of DMUs. The general algebraic modeling system (GAMS) software is used to run the CDEA models. Table 6 presents the performance scores obtained for each department of six separate production units in the company, based on output-oriented models. The four CDMU models have been solved with the GAMS package that is used to solve integer zero-one programming. The top manager of the company decides to select three inputs $t = 3$ for each perspective of the BSC with four model solutions. In each BSC perspective, the input indicators are specified and the results are listed in Table 7.

According to Table 7, the selection of indicators in each perspective is determined by the central decision-maker. From four indicators of learning and growth perspective, three indicators of the number of personnel, motivational expenses and increasing personnel proficiency are selected. From four indicators of internal process perspective, three indicators of speeding up the services, ratio of a skilled worker and electronic services are selected. From four indicators of customer perspective, three indicators of competitive pricing, energy and production are selected. From four indicators of financial perspective, three indicators of income ratio, export and delivery cost are selected.

The $\frac{1}{\varphi}$ relation can be used to calculate the total efficiency of each perspective of the BSC. It is worth mentioning that indicators of cost, stop-working and energy can be considered undesirable and should be minimized (Alizadeh Bidgoli *et al.*, 2021). In that respect, and following the suggestion of Dyson *et al.* (2010), we have transformed these indicators by using the following expression: $\tilde{y_{rj}} = \left(\text{Max} \;\; y_{rj}\right) - y_{rj} + 1$. The most efficient perspective among these six DMUs is the customer perspective, and the least efficient perspective among these six DMUs is the learning and growth perspective.

Table 5.
CDEA-BSC data in customer perspective

DMU	INPUT1	INPUT2		INPUT3		INPUT4	OUTPUT1	OUTPUT2	OUTPUT3	OUTPUT4
DMU1	16	31	62	1,045	40	1,105	3.21	3.19	22.91	3.13
DMU2	19	23	70	1,084	1	2,401	3.22	3.61	25.8	3.41
DMU3	34	51	42	1,032	53	1,549	3.43	3.34	29	3.25
DMU4	33.5	92	1	1,055	30	1,420	3.14	3.41	34.5	3.32
DMU5	30.4	89	4	1,064	21	999	3.45	3.39	21.8	3.25
DMU6	13	15	78	1,039	46	1,969	3.66	3.5	13	3.37

Table 6.
CDEA-BSC data in financial perspective

DMU	INPUT1	INPUT2	INPUT3		INPUT4	OUTPUT1	OUTPUT2	OUTPUT3
DMU1	62	30	266,791	1	200	17	5	2
DMU2	42.77	90	222,358	44,434	320	12.98	7.16	2.62
DMU3	60	43	240,000	26,792	1,230	47.59	7	8
DMU4	60.2	109	164,811	101,981	520	18.9	1.4	2.7
DMU5	57.5	39	249,974	16,818	263	20.13	1.23	3
DMU6	96	69	175,647	91,145	311	10.29	10.2	4

Table 7.
Estimated results of Model (5.1)

Perspectives/indicator	μ_1	μ_2	μ_3	μ_4	μ_5	φ
Learning and growth		√	√	√		1.57
Internal process	√		√	√		1.17
Customer	√		√	√		1.00
Financial	√	√		√		1.31

6. Conclusion

In the conventional DEA model, separate models are used to evaluate the performance of each DMU, whereas in the CDMU model, only a linear programming model is used to optimize the sum of inputs and outputs in the BSC. The main contribution of this paper is that it provides a systematic and centralized attitude to apply a selective approach for determining input indicators of the BSC based on DEA models. In this paper, a centralized data envelopment model is proposed, which aims at optimizing the total output production and determining the number of input indicators in the BSC. In this way, the basic CDEA model were integrated with the 0–1 programming model to achieve the above objective. Our proposed model considers all manufacturing units seamlessly, and each scorecard model is implemented separately. In this model, all DMUs are simultaneously depicted on the efficient border.

The integration of the CDEA and the BSC seems to be advantageous because it shed more light on the complexity and tradeoffs inherent in actual performance measurement. It is important to note that there cannot be a unique and universal model of performance measurement applicable in every situation, in every organization and at any time. The results showed that this method has several advantages. It is easy to use, and implementation, and there is no computational complexity. It also does not need experts' judgment, so it is a cost-effective way. Wholey (1996) has mentioned that performance is socially constructed and means different things for different stakeholders. It seems that a good performance measurement approach, whether it consists of a single model or multiple complementary models, stems out of the real-life situation, customer's needs and interest of stakeholders, and priorities, visions and strategies of top decision-makers. Researchers can use the proposed method in various aspects of future research. Considering different datasets, such as ambiguous, game theory, negative or fuzzy data, can be interesting research topics. Other DEA models with correlated variables can also be studied.

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Further reading

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Appendix. Model properties

Theorem 1: Model (3.5) can always be feasible.

Proof: put

$$\varphi = 1 \ , \ \begin{cases} \lambda_{jl} = 1 & l = i \\ \lambda_{jl} = 0 & l \neq i \end{cases}$$

Clearly (φ, λ) is a feasible answer of Model (3.5).

Theorem (2): Model (3.5) is unit invariant to input transitions.

Proof: Suppose that the *i*th input of all DMUs is increased by α_i. Then, we have to apply the relation $\tilde{x_{ij}} = x_{ij} + \alpha_i \quad i = 1 \ldots m$ in Model (3.5). As a result, we have the following input restrictions:

$$\sum_{l=1}^{n}\sum_{j=1}^{n} \lambda_{jl}(x_{ij} + \alpha_i) \leq \sum_{j}^{n}(x_{ij} + \alpha_i) + M_i\mu_i \qquad i = 1, \ldots, m$$

$$\Rightarrow \sum_{l=1}^{n}\sum_{j=1}^{n} \lambda_{jl}x_{ij} + n\alpha_i) \leq \sum_{j}^{n}x_{ij} + n\alpha_i + M_i\mu_i \Rightarrow$$

$$\sum_{l=1}^{n}\sum_{j=1}^{n} \lambda_{jl}x_{ij} \leq \sum_{j}^{n}x_{ij} + M_i\mu_i$$

As can be seen, because μ_i has a value of 0 or 1, by transferring the inputs as large as α_i, the constraints of Model (3.5) remain, so the above model is unit invariant to the input transition.

Theorem 3: Model (3.5) is not unit invariant to output transitions.

Proof: Suppose that the output of all DMUs is increased by β_r. Then, we apply $\tilde{y_{rj}} = y_{rj} + \beta_r \quad r = 1 \ldots s$ to Model (3.5). Similar to the proof of Theorem 2, by applying the above relationship to the constraints of Model (3.5), the model is not stable with respect to the output transfer as β_r.

Theorem 4: Model (3.5) is unit invariant.

Proof: Suppose we multiply the *i*th input and the *j*th output of each DMU to α_i and β_r, respectively.

Putting the above relation in (3.5), we have the new model as follows:

$$\varphi^* = \text{Max}\,\varphi$$
$$\sum_{j=1}^{n}\sum_{l=1}^{n}\lambda_{lj}(\alpha_i x_{ij}) \leq \sum_{j=1}^{n}\alpha_i x_{ij} + M_i\mu_i$$
$$\sum_{j=1}^{n}\sum_{l=1}^{m}\lambda_{lj}(\beta_r y_{rj}) \geq \varphi\sum_{j=1}^{n}\beta_r y_{rj}, r = 1, 2, \ldots, s,$$
$$\sum_{l=1}^{n}\lambda_{lj} = 1, \forall_j$$
$$\sum_i \mu_i = \mathrm{t}$$
$$\lambda_{lj} \geq 0, \mu \in \{0, 1\} \qquad \varphi \ \textit{free}$$

Now consider the input and output constraints:

$$\sum_{j=1}^{n}\sum_{l=1}^{n}\lambda_{lj}(\alpha_i x_{ij}) \leq \sum_{j=1}^{n}\alpha_i x_{ij} + M_i\mu_i \Rightarrow \sum_{j=1}^{n}\sum_{l=1}^{n}\lambda_{lj}x_{ij} \leq \sum_{j=1}^{n}x_{ij} + M_i\mu_i$$
$$\sum_{j=1}^{n}\sum_{l=1}^{m}\lambda_{lj}(\beta_r y_{rj}) \geq \varphi\sum_{j=1}^{n}\beta_r y_{rj} \Rightarrow \sum_{j=1}^{n}\sum_{l=1}^{m}\lambda_{lj}y_{rj} \geq \varphi\sum_{j=1}^{n}y_{rj},$$

As you can see, the new model is equivalent to (3.5) model. Thus, the performance efficiency value in both models are the same and Model (3.5) is unit invariant.

By multiplying the ith input and rth output of all DMUs with α_i and β_r, respectively, and applying Model (3.5) to these new data, the results show that the projection point may be different from the original data for each DMU, but the total input consumption and output production from Model (3.5) is the same for the original data and the new data.

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