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A mathematical tri-level programming model for designing an integrated dynamic petroleum product supply chain

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Abstract

Purpose – The purpose of this paper is to design petroleum products' supply chain management, which includes efficient integration of suppliers, manufacturers, storehouses and retailers.

Design/methodology/approach – This paper proposes that a three-level supply chain will be turned into a bi-level supply chain of petroleum products by simultaneous integration of the middle level with the upstream and downstream levels. Also, it is integrally optimized by considering the multiple managerial flows' mutual results at various supply chain levels. Also, it is integrally optimized by considering the multiple managerial flows' mutual results at various supply chain levels.

Findings – The concepts of the design, structure and outputs are led by the model's solution. The model also responds to the variations in the market via coordination in the related decisions to the distribution, production and inventory issues, and also coordinating between the demands and production.

Research limitations/implications – This paper has limited its analysis to definite values due to the over-expansion of calculations and analysis. Future works can study other aspects of the proposed model for a multi-level petroleum product supply chain in different states of certain parameters and time zones.

Practical implications – The designed model can directly and transparently help the oil managers and decision-makers lower the costs of manufacturing, distribution and sales with respect to the determined criteria.

Originality/value – This paper establishes that effectiveness of the dynamic petroleum materials supply chain design will increase by considering maintained and increased production costs and coordinate management flows at all levels by supply chain creation's integration.

Keywords Multilevel supply chain, Optimization, Integration, Petroleum products, Maintained and increased production

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1. Introduction

By virtue of the increase in the complexity of the petroleum products supply chain, which is caused by the variations in the customers' characteristics, the economy's integration and the competition between companies, the managerial decision that is often on the basis of the vision and experience is far from the optimal state. Another significant part of the petroleum products supply chain is to coordinate such activities so as to improve its function; for instance, lowering the costs, enhancing the service level, bullwhip effect mitigation, better deployment of resources and response to the variations in the market via coordination in the related decisions to the distribution, production and inventory issues and also coordinating between the demands and production.

The petroleum products' supply chain has invariably been of paramount importance in macro-level decision-making issues (Fazli *et al.*, 2015). Because most of the existing technologies in human societies directly or indirectly depend on petroleum products (Shah *et al.*, 2010). Thereby, a coordinated structure should be designed in the supply chain form to satisfy the industrial demands of this product. Various techniques for petroleum products' supply chain modeling have been suggested and categorized into four general groups. The first group includes certain models in which all the parameters' values will be determined. The probable models are considered in the second group in which at least one parameter is not defined, but it follows the probability distribution. The third and fourth groups involve probable game theory models and those based on the simulation assessing the function of the petroleum supply chain's various strategies (Wisner *et al.*, 2014). Most of them are steady models on the basis of the average performance or steady conditions. The stationary models with the dynamic properties of the petroleum products supply chain are not sufficient when leading to the fluctuations in demand, lead time (LT) delay, sales estimation, etc. (Wang *et al.*, 2016).

Generally, controlling the supply chain of petroleum products has a process structure, and the cross-relationships between the levels lead to the formation of the information and products transformation structure. These processes involving the organization, planning, providing and codifying the implementation methods and procedures in the supply chain have been created to achieve the aims. The petroleum products' supply chain control objectives are various factors related to the chain's economic activities; it includes technique, production, performance, personnel, organization, financial activities, information, efficiency, capital, etc. These objectives can be observed in the form of workflow from the suppliers to the users, and commodities, information and capital flow. The main objective of controlling the supply chain of petroleum products is considered the operations flow, financial activity, information, organization and personnel (Tong *et al.*, 2014). Optimizing the supply chain is the greatest chance for most companies to substantially mitigate their costs and enhance their performance. The optimization aims at achieving the most efficient and optimal technique for supply chain management to satisfy the customer's demands at the lowest cost. Satisfying the needed demands, orderly supply of crude oil, decreasing the delivery time and reducing the generation and distribution cost are the chief objectives of the petroleum products supply chain (Attia *et al.*, 2019).

2. Literature review

According to the published data by British Petroleum Institute, the global production and consumption of oil together with the global trade of crude oil and refined products have risen ever since 2010. Nevertheless, as stated by the American Petroleum Institute, the revenues in the petroleum industry have been equal to the total costs, including extraction, production and preparing crude oil and the costs of refining, distributing and marketing the refined products (Lima *et al.*, 2016). Stamatti *et al.* (2019) proposed a dynamic supply chain network for diesel fuel consumption in petroleum and gas reservoirs. By presenting an integer linear mathematical programming model, the authors were to determine the position of the storage tanks (ST) to satisfy the diesel fuel demands, and on the other hand, minimize the current net total costs in the planning horizon. An integrated supply chain network is considered by taking all the specified properties into account. Indeed, a general approach is provided, creating flexible flows all over the network and keeping interwoven natures of decision. The economies of scale that govern the investment and operating costs, including various storage tanks, are reflected for optimal selection by model. In the end, the service level was assessed by combined criteria consisting of the flow magnitude and the total time required for satisfying the fuel demands. A case research was investigated under various scenarios to indicate the proposed model's potential and the efficiency of the obtained plans.

Attia *et al.* (2019) provided a multi-objective optimization model for the medium-term planning of the petroleum and gas hydrocarbon supply chain. This model provides robustness and the environmental facet of the hydrocarbon supply chain's performance planning. By minimizing the reduction of natural resources, robustness is considered an objective. By restraining the CO_2 emissions, the environmental factors are taken as a constraint in the model. Furthermore, the model consists of lowering the total costs and maximizing the revenue as two objectives. The proposed model helps the decision-makers research the transactions regarding the substitute decisions. The adopted decisions influence the satisfied demand, natural resources reduction, costs decrease and/or maximizing the revenues. Case research based on the actual data for demonstrating the practicality of the proposed model is provided, and the sensitivity analysis is executed to obtain the managerial visions. For instance, oil and gas reserves can be maintained for a longer time without decreasing the market share, and at the same time, cover the total cost-efficiency. Beiranvand *et al.* (2018) stated that the petroleum industry plays a pivotal part in today's global economy and has an extreme influence on pioneer commercial environments, particularly in oil-producing countries. In order to overcome the complexity of the crude oil supply network, a mathematical programming model is prepared to manage the crude oil supply chain. A robust optimization model is developed to maximize the profitability of the entire chain and the uncertainty of the price and demand. According to the actual case research data, the results demonstrated that the robust optimization model enhances the profitability of the crude oil supply chain. Wang *et al.* (2020) provided a framework by analyzing the performance and emission cost for optimizing the downstream oil supply chain. Three scenarios consisting of the standard condition, distribution pipeline and demand enhancement were examined. The provided model supported the downstream decision-making in the oil supply chain by reducing pollutant emissions. The results demonstrated that decision-making must accord a high priority to reducing the emission of greenhouse gases, dangerous contaminations and emission factors from transportation barges that leads to a lot of environmental benefits.

By analyzing 23 published articles from 2010 to 2019, Abdussalam *et al.* (2021) indicated that employing nonlinear mathematical models for analyzing the environmental effects of different objectives of the oil supply chain will lead to making more accurate and realistic decisions. Also, the integration of strategic and tactical decisions will increase economic and environmental performances, which will be followed by robustness enhancement in the oil supply chain. Lima *et al.* (2021a) employed an integer linear programming model for strategic and operational design and planning of downstream oil supply chain with various uncertainty resources. The uncertainty of resources was considered to consist of logistic cost and demand and was defined by fuzzy mathematical programming. The results obtained from solving the model indicated that transportation cost is the most crucial factor in downstream oil network costs, which makes the tactical plan to be costlier than the strategic plan. Lima *et al.* (2021b) proposed a certain mixed-integer linear programming model aiming at designing and planning the downstream petroleum industries supply chain so that in the tactical distribution plan of the optimization model, a multi-level distribution network in which road, marine and railroad, and pipeline resources might be employed, is considered. Thus, various distribution operations, such as exchanging products between storehouses, are included in the model. In the present research, what will be addressed is to provide a multi-level oil product supply chain to minimize the total supply chain costs according to the maintained and increased production costs in the production section and the existence of a multi-level programming structure. In this model, it is assumed that the producer has access to raw materials and does not face a shortage. The producer dispatches its products in the size of the batches defined by the distributor. Kumar and Barua (2021) identified the supply chain performance indices and designs an evaluation framework to assess and compare the Indian petroleum supply chain performance. They presented a case study of three Indian petroleum companies. They identified 15 performance criteria extracted from previous literature and expert inputs and classified them into four groups. ALnaqbi *et al.* (2022) described a mathematical programming approach to address potential synergistic gains after horizontal mergers in the upstream Crude Oil Supply Chain (COSC). A supply chain optimization model was used to evaluate the extent to which economies of scope and economies of scale favorably impact potential mergers. The problem was formulated as a mixed-integer linear programming (MILP) model that determines the investment level and efficient implementation of operational strategies at shared services, as well as the production and processing of oil and gas. Ebrahimi and Bagheri (2022) designed a multi-echelon network for the oil and gas supply chain, including extraction, purification, storage and shipping to the target market. Furthermore, a bi-objective mathematical model was formulated which attempts to maximize total profit from the sale of fossil fuels and to maximize the reliability of processing plants to meet the applicants' demand. It was solved using augmented ε-constraint and goal programming methods Tarei *et al.* (2022). Considered a multi-echelon, multi-product and multi-modal petroleum supply chain network design problem along with its various sources of uncertainties (demand, supply, production, etc.) and to minimize both total supply chain cost and risk simultaneously. The problem was articulated as a robust optimization problem and the results were derived under various risk attitudes (viz. risk-seeking, risk-neutral and risk-averse behaviors). The non-linear problem was proposed as a mean-variance robust optimization problem. Two-stage stochastic programming was extended to incorporate robustness and capture the risk aversion behavior. The scenario-based planning method was used for the estimation of uncertain parameters.

The most important studies in the petroleum supply chain field such as different linear planning models (Fernandes *et al.*, 2013; 2014; Gao, 2018) random linear planning models in petroleum product supply chain strategic, operational and tactical planning (Al-Othman *et al.*, 2008; MirHassani, 2008; Ribas *et al.*, 2010; MirHassani and Noori, 2011; Tong *et al.*, 2012; Leiras *et al.*, 2013; Oliveira *et al.*, 2013; Oliveira and Hamacher, 2012a; Fernandes *et al.*, 2017; Lima *et al.*, 2017, 2018a; Oliveira *et al.*, 2014) linear/non-linear complex integer planning (Guajardo *et al.*, 2013; Kuo and Chang, 2008; Pinto *et al.*, 2000; Neiro and Pinto, 2004), fuzzy linear planning (Ghatee and Hashemi, 2009), robust planning (Lima *et al.*, 2018b, 2019), two-level planning (Gao and You, 2019; Zang *et al.*, 2020; Oliveira and Hamacher, 2012b) and multi-objective planning (Gholami *et al.*, 2019). As visible, these planning models lack multi-objective functions. Most of them consist of single-level or two independent levels to avoid complex models and solution algorithm design difficulties. Obtaining the necessary information for problem-solving is another crucial problem in this field. The current research conducts supply chain planning in a multi-objective and multi-level manner optimized in a simultaneous and integrated approach.

All of the products that the producer produces are dispatched to the distributor within a period. At the producer level, the human power costs, the costs of preparing equipment and maintenance tools, the costs of repairing the parts, the costs of setting and loading the machinery and the overhead costs of the maintained and increased production section are included. The distributor receives demands from retailers and responds to them based on the inventory, or else, these demands are considered the lost sale for the distributor. The retailers also dispatch the orders to the distributor based on the demands they receive from customers. It must be noted that the mean demand of the distributor and the retailer vary. In the previous research, this hierarchical structure is considered without considering the important factors, such as the orders' sizes, which definitely leads to ambiguities in the results' implementation in the real world. On the other hand, in the proposed research, the made decisions are reported so that the decision-makers can directly implement results in the real world. It is worth mentioning that these suggestions are made by researching various review papers and concluding based on the unconsidered aspects of the previously presented studies.

3. Modeling the structure of the petroleum product supply chain

Bi-level programming is an NP-hard problem (Bard, 1991). However, many methods have practically been presented for solving it due to its wide applications. The solving methods of this problem can be categorized into five general methods, including the counting methods, second-level corresponding methods, fuzzy methods and meta-heuristic methods. In the corresponding method, the second level of the problem is substituted by either Karush–Kuhn–Tucker (KKT) optimality conditions or the penalty function so that the second-level problem is converted to constraints for the main problem. Lv *et al.* (2007) proposed a method that first converts the problem to a single-level problem by employing KKT optimality conditions and then adds the constraints pertinent to slackness complementary that cause the problem to be nonlinear to a first-level function by a penalty function. Thereby, the problem is linearized and can be solved by employing linear algorithms. Zhongping *et al.* (2011) proposed an algorithm based on the KKT optimality conditions to solve the problem when the first and second levels are nonlinear. Allende and Still (2013) proposed a method that uses the binary variables after applying KKT optimality conditions to linearize the resulting nonlinear problem, which proposes an accurate answer afterward for the problem by employing linear algorithms.

In the research method of this study, all parameters and influential criteria are first recognized in making an appropriate decision. Afterward, some variables are considered according to the major objective of the research to indicate the state and extent of the optimal decisions. However, there are some constraints in solving any problem. These constraints exist in different forms, such as available budget constraint, human power constraint, the constraint of the number of required tools and machinery, the constraints of available places, etc. These constraints are extremely influential in making ultimate decisions. Accordingly, by employing the provided parameters and variables, an effort is made to design the existing constraints in the form of mathematical equations. Nevertheless, the main objective of the present research is defined as a main mathematical model and is known as the objective function of the problem, which is cost minimization. Thus, the mathematical model of the problem is formed by taking the objective function and designed constraints into account. Afterward, the problem will be solved, and the final structure of the decisions will be provided according to the considered parameters and by employing the software for solving the mathematical models. Due to the mathematical nature of the solving method, it is guaranteed that the provided decisions are optimal.

The most significant property of bi-level programming includes two plans with a mathematical programming structure in one problem. Generally, the lower-level optimization problem is a part of the higher-level problem constraints. Some of the bi-level programming problems' properties are indicated as follows (Bard, 2013; Colson *et al.*, 2007).

(1) Assume that each of the players is aware of the acceptable objectives and choices of the other one.

(2) The leader decision-maker movement is prior to the follower decision-maker movement. The leader tries to achieve its goals in the first stage, estimating all of the probable follower reactions throughout the movement. By observing the leader's decisions and without paying attention to the external effects, the follower reacts to achieve its optimal goals.

(3) Each player's decision influences the objective function value and the probable actions (solution space) of the other one since the existing acceptable choices of each one is dependent on the other player.

(4) A particular characteristic of multi-level programming is that the decision-maker of each level can impact the behavior of another level's decision-maker. At the same time, it cannot fully control its performance.

(5) The objective function of each decision-maker is likely to be determined by the variables controlled by the same-level follower decision-makers.

(6) In general, those are non-convex and non-derivative problems.

(7) There is not any certainty in obtaining the Pareto optimal answer.

(8) Bi-level programming naturally is a difficult problem.

In the multi-level programming models, the constrained region is implicitly determined by a set of optimization problems solved based on a predetermined sequence. The multi-level programming problem can be a non-zero-sum game with complete information (at least for the higher-level decision-makers). In the first stage, how to play the game is determined, and the decision-maker's strategies are not considered separate. Accordingly, the probable choices for each decision-maker are changed during the game process, and it might be constrained using the prior decision maker's actions. Considering the assumption of the dependent sets of strategies of decision-makers increases the complexity of the whole problem substantially. The multi-level programming with the specific typical formulation, in which the prior players should move on their turn, is distinct. Due to being dependent on mathematical programming, multi-level programming dispenses the users with concentrating on the natural challenges, which is one of its merits (Bard, 2013). To formulate the problem in a mathematical form, we assume that the first-level decision-makers (leaders) control the higher-level variables' $x \in X \subseteq R^n$ vector, and the second-level decision-makers (followers) control the lower-level variables $y \in Y \subseteq R^m$ vector. First, the leader moves, choosing the vector x so that $F(x, y(x))$ is minimized by considering the existing constraints. Expression $y(x)$ indicates that the follower variables' vector is dependent on the leader variables vector (Even though y is not written in this way due to the simplicity, it is worth mentioning that y is invariably dependent on x).

By taking the leader's actions into account, the follower selects the vector y to minimize the objective function $f(x, y)$ and consider the existing constraints on the variable based on a particular value of x.

The datasets X and Y can impose more constraints like being non-negative or integer on the variables. Also, the constraint being non-integer can be put into the functions $G(x, y)$ and $g(x, y)$. According to the functions F, G, f and g, the multi-level programming problem can have various types. The simplest type can take place when all the functions are linear (Bard, 2013).

The supply chain under the research has various decision-making levels, each of which executes the related planning based on the higher-level decisions. Generally, what is addressed in this research is researching the supply chain with three levels: refinery, distributor and customer. The aim is to minimize all of the pertinent costs to the supply chain. It includes the costs related to production, maintenance, preparation, categorization at the producer level, ordering cost, product preservation in stock and product preservation. At the same time, transportation, lost sales, transportation costs at the distributor level and ordering costs, Maintained and increased production and transportation costs at the retailer level.

Nevertheless, in each supply chain of the produced product, the producer level has some operational costs of maintained and increased production in this sector. In this research, such costs are taken into consideration. These costs are indicated in the following: the costs related to the workforce, the maintenance equipment preparation, renewed repairs of the parts, adjustment and loading of machinery and the overhead costs. It is assumed that at the producer level, crude oil is available each time, and no shortage will happen. Moreover, the produced products are dispatched in the dimensions determined by the distributor. At the distributor level, the extent of the demands is received from the customers. With respect to the inventory amount, the demands are either satisfied or considered lost sales at the mentioned level. The retailers dispatch the orders to the distributors with respect to the extent of the customers' demands. However, it should be noted that lost sales at the retailer level are allowed. Also, the demand average in distributors and retailer varies. We assume that the refinery in each course has produced an extent of product, the amount of which is fully transferred to the distributor who also satisfied the retailers' demands. By considering the problem structure and the existing constraints, some assumptions will be defined in this section. These assumptions demonstrate the existing conditions in the decision-making process. The assumptions of the problem will be defined in the following.

1. The current levels in the supply chain in this research consists of the refinery (one facility), the distributor (one facility) and customers (three facilities).
2. All the produced products at the refinery level are dispatched to the distributor level. All the existing products in the mentioned level are dispatched to the retailer level.
3. The crude oil is invariably available as required.
4. The inventory shortage (of backorder type) is allowed at the distributor and retailer levels.
5. At the beginning of the course, the inventory level is zero in the distributor and customer.
6. The transportation equipment has the adequate capacity – depending on the distributor and customer – for carrying each package.
7. The extent of the batches is determined by the distributor.
8. The products are delivered to the customers in the form of one-unit amounts.
9. The average demands from the distributor and customers varies.
10. The shortage of the inventory in the distributor and customer is allowed.
11. The transportation time is negligible.
12. The refinery, distributor and the customer have adequate capacity for preserving the inventory.

(13) At the refinery level, the production system's operational costs and repairs sector are considered the production system's operational costs, and the repairs sector are considered the supply chain's costs.

(14) The maintained and increased production costs of the extraction system consist of the tools purchase, deployed workforce, machinery repairs costs and the pertinent costs to re-adjustment of machinery and equipment.

(15) All the maintained and increased production costs are calculated according to the production unit.

(16) The extraction extent is separated into two sectors: production time and overtime.

(17) There is only one workforce team expert in maintenance and repairs. If the production rate increases and the maintenance and repair operations take place over overtime, the manpower cost in such time is different from the regular cases. However, there is no constraint for the production part workforce, and their wage is constant.

(18) The maintained and increased production workforce cost merely depends on the production amount.

The employed parameters and variables for the formulation of the problem are provided in Table 1.

3.1 Tri-level programming model

The proposed model consists of three decision-making levels, including the retailer (the first level), the distributor (the second level) and the producer (the third level). The model performs in a way that the retailer reports the total demand of the system to the second level by considering the extent of demand received from the customers. The distributor responsible for supplying the demanded goods determines the level of his ordering (the batch quantity) based on the existing costs. He reports this determined extent to the producer, and the producer produces products according to the system's costs. The objective is to find the optimal extent of variables at each level so that the total cost of the chain is minimized. The general model of the problem is presented in Equations (1)–(11).

$$Min\, f_R = \sum_{i=1}^{3} A_R d_R + 0.5. \sum_{i=1}^{3} h_R(IB_R + IE_R) + \sum_{i=1}^{3} hit_R d_R + \sum_{i=1}^{3} g_R\, d_R + \sum_{i=1}^{3} Sl_R ls_R \tag{1}$$

$$Min\, f_d = A_d.\frac{d_d}{Q} + o.5\, h_d(IB_d + IE_d) + hit_d\frac{d_d}{Q} + g_d\frac{d_d}{Q} + C_{DA}\frac{d_d}{Q} + sl_d.ls_d \tag{2}$$

$$\begin{aligned} Min\, f_p = {} & C_pX + A_pX + 0.5.I.C_p.Q\frac{d_d}{X} + C_A\frac{d_d}{Q} + S_pQ\frac{d_d}{X} + Pr_pQ\frac{d_d}{X} + Re_pX + Rg_pX \\ & + Hb_pX + C1X1 + C2X2 \end{aligned} \tag{3}$$

s.t.

$$IB_d + X - d_d + Ls_d = IE_d \tag{4}$$

$$d_d - ls_d - IE_d = \sum_{i=1}^{3} d_R - \sum_{i=1}^{3} ls_R + \sum_{i=1}^{3} IE_R \cdots \forall i \tag{5}$$

$$C1X1 + C2X2 + Pr_pQ\frac{d_d}{X} + Re_pX + Rg_pX + Hb_pX \leq Budget \tag{6}$$

Table 1. The parameters and variables employed in formulating the problem

	Symbols and sets	
	i	Symbol of retailers
	The third level: The producer	
Decision variables	X	Production rate at each period
	$X1$	The extent of production at ordinary time
	$X2$	The extent of production at overtime
	f_p	The objective function of the producer level
Parameters	C_p	The production cost at the producer, including purchase costs
	A_p	The cost of production repair
	I	Percentage of inventory preservation cost per unit
	C_A	Batching cost
	S_p	The cost of human force
	Pr_p	The cost of preparing tools and necessaries of maintenance
	Re_p	The cost of repairing machinery
	Rg_p	The cost of setting and loading the machinery
	Hb_p	The overhead cost
	$Budget$	The total cost available for doing maintenance operation
	$MaxP$	The maximum extent of production at ordinary time
	$C1$	The maintenance cost pertinent to human force at ordinary time
	$C2$	The maintenance cost pertinent to human force at overtime
	The second level: The distributor	
The decision variables	D_d	The demand rate at the distributor
	IE_d	The final inventory at the distributor at each period
	Ls_d	The extent of lost sale at the distributor
	Q	The batch quantity determined by the distributor
	F_d	The objective function at the distributor level
Parameters	A_d	The ordering cost at the distributor
	H_d	The cost of inventory preservation at each unit
	Hit_d	The cost of inventory preservation during transportation from the producer to the distributor
	G_d	The cost of transportation from the producer to the distributor
	IB_d	The initial inventory at the distributor
	Sl_d	The cost of the unit's lost sale at distributor
	C_{DA}	The cost of opening the batches
The decision variables	*The first level: The retailer*	
	d_{Ri}	Order rate at the *i* retailer
	Ls_{Ri}	The extent of lost sale at the retailer *i*
	IE_{Ri}	The final inventory at the retailer *i* at each period
	f_R	The objective function at the retailer level
Parameters	A_{Ri}	The order cost at the retailer *i*
	h_{Ri}	The cost of each product's inventory preservation at each retailer
	hit_i	The cost of inventory preservation during transportation from the distributor to the retailer
	g_{Ri}	Each unit's transportation cost of from the distributor to the retailer
	Sl_{Ri}	The cost of the lost sale at the retailer *i*
	IB_{Ri}	The initial inventory at the retailer *i*

$$X1 + X2 = X \tag{7}$$

$$X1 \leq Max\,P \tag{8}$$

$$X2 \leq 2Max\,P \tag{9}$$

$$d_d, IB_d, IE_d, Q, ls_d, X, X1, X2 \geq 0 \tag{10}$$

$$d_{Ri}, IB_{Ri}, IE_{Ri}, ls_{Ri}, d_d, Q, IB_d, IE_d, ls_d \geq 0 \cdots \forall i \tag{11}$$

The objective function (1) minimized the costs at the retailer level. These costs include the ordering costs, the cost of products' preservation in the centers, the cost of products' preservation during the transportation, the transportation cost of products and the cost of the lost sale. The objective function (2) minimizes the costs at the distributor level, including the ordering costs, the cost of products' preservation in the centers, the cost of products' preservation during the transportation, the transportation cost of products, the cost of the lost sale and storing costs of patches. The objective function (3) minimized the costs at the producer level. These costs include the production costs, preparation costs, the costs of preservation at the facility, the cost of patching the products and the costs of production section's maintained and increased production (i.e. human force costs, costs of preparing maintenance necessaries and tools, costs of repairing parts, costs of setting and loading machinery and overhead costs). Given that the costs of the spare parts' preparation for repair must be defined in advance, this cost depends on the number of demands, the production rate and the batch quantity of the supplier. This definition can similarly be considered for the human force costs of the maintenance section since the human force for production must be considered based on the demand rate. However, since depending on the production rate, the machinery is depreciated and requires maintenance operation over time, other maintenance costs merely depend on the production rate. Besides, the production cost at the ordinary time and the production costs at the overtime are calculated in the final sentences of the objective function.

Constraint (4) guarantees that all the demands reported by the distributor are produced at the producer level. Constraint (5) guarantees that the distributor meets all demands satisfied by the retailer. Constraint (6) ensures that the maintained and increased production costs are not more than the available budget. Constraint (7) ensures that the sum of the production rates at the ordinary time and overtime is equal to the total extent of production. Constraint (8) guarantees that the extent of production at the ordinary time does not exceed the allowed extent of production at this time. Besides, Constraint (9) guarantees that the extent of production at overtime does not exceed the allowed extent of production at this time, the range of employed variables are presented in Constraints (10) and (11). One of the common methods for converting a problem to a conventional mathematical model is to divide the problem into bi-level problems and create a connection among different parts of the main problem. In this regard, this research first converts the particular tri-level programming problem to two bi-level programming models, and then, the model is solved generally and in an integrative way by creating a logical relationship among the levels. In the following, this method is further discussed.

3.2 Bi-level programming model

A vast majority of mathematical programming models are presented in the form of a single-level model and are employed for focused planning systems. The programming model for unfocused planning systems is developed, in which the first level is the leader and the second level is the follower. In the bi-level programming problem, each decision-maker tries to optimize its objective function without considering the objective of the other level. Nonetheless, each side's decision influences the extent of the other side's objective function as a decision-making space. The planning problem is an optimization model formulated in the form of the following model.

$$Min_x \, F(x,y) = c_1 x + d_1 y \tag{12}$$

s.t.

$$G(x,y) \leq 0 \tag{13}$$

$$Min_y f(x,y) = c_2x + d_2y \tag{14}$$

s.t.

$$g(x,y) \leq 0, \tag{15}$$

$$x,y \geq 0 \tag{16}$$

where $F(x,y)$ is the objective function of the leader and $f(x,y)$ is the objective of the follower.

The leader imposes his/her decision and receives feedback from the follower. In comparison to the single-level programming models, the bi-level programming models have more advantages. Some of its advantages are as follows:

1. Bi-level programming can be employed in analyzing two distinct and contradictory objectives simultaneously in the decision-making process.
2. Multi-criteria decision-making methods of bi-level programming can reflect the applied problems better.
3. The bi-level programming methods can clearly create an interaction between the management system and the customers.

Several methods are proposed to convert the planning problem. One of the most effective methods is to create a KKT optimality condition in the problem, based on which the problem can be converted to a single-level problem and solved. This method will be explained in the following.

3.3 Karush–Kuhn–Tucker (KKT) optimality conditions

The internal optimality problem is equivalent to the KKT optimality conditions when f and g are convex and smooth, and h on the y is linear by considering that x is constant (each $x\in$ is linear). Also, one of the first-order conditions of the first-order constraint, such as the lack of linear dependence, the KKT conditions or the convexity condition of the weak inverse in the x terms in the y^* point, should be considered. Afterwards, the necessary and sufficient condition for y^* to be an optimal solution for the internal problem is that there is an X satisfying the following conditions:

$$h_i(x,y^*) = 0 \; i\epsilon I \tag{17}$$

$$\frac{\partial f(x,y^*)}{\partial y^*} + \sum_{j=1}^{J} \lambda_j^* \frac{\partial g_j}{\partial y^*} + \sum_{i=1}^{I} \mu_i^* \frac{\partial h_i}{\partial y^*} = 0 \tag{18}$$

$$g_j(x,y^*) + s_j^* = 0, \;\; j\epsilon J, \tag{19}$$

$$\lambda_j^* s_j^* = 0 \; j\epsilon J \tag{20}$$

$$\lambda_j^*, s_j^* \geq 0, \;\; j\epsilon J \tag{21}$$

In those conditions, λ^* and μ^* are the coefficients of the equality and inequality of the KKT vector. It can be concluded that the necessary condition to make the $(x^*, y^*, \lambda^*, \mu^*)$ an optimal solution for the bi-level programming model is such that (y^*, λ^*, μ^*) satisfies the mentioned conditions in the coefficient $x = x^*$.

By considering these reasons, the bi-level programming problem is turned into a single-level problem, as indicated by:

$$Min\, F(x,y) \tag{22}$$

s.t.

$$G(x,y) \leq 0 \tag{23}$$

$$H(x,y) \leq 0 \tag{24}$$

$$h_i(x,y) = 0 \;\; i\epsilon I \tag{25}$$

$$\frac{\partial f(x,y)}{\partial y} + \sum_{j=1}^{J} \lambda_j \frac{\partial g_j}{\partial y} + \sum_{i=1}^{I} \mu_i \frac{\partial h_i}{\partial y} = 0 \tag{26}$$

$$g_j(x,y) + s_j = 0 \;\; j = J, \tag{27}$$

$$\lambda_j s_j = 0, \;\; j\epsilon J, \tag{28}$$

$$\lambda_j, s_j \geq 0 \;\; j\epsilon J \tag{29}$$

$$x\epsilon X, y\epsilon Y. \tag{30}$$

It is worth mentioning that due to the steady and supplementary conditions, this problem is non-convex. Thereby, even if the single level formulation is a linear initial bi-level problem, it is nonlinear and non-convex due to the supplementary conditions. The *f* and *g* functions must be convex to achieve the total optimum of the problem, and the problem space's *h* should have a linear structure to ensure that the KKT optimal condition is both necessary and sufficient.

3.4 Proposed model's formulation

The supply chain discussed consists of three levels: refinery, distributor and customer. For this purpose, two nonlinear bi-level models are suggested; the first one includes two levels: distributor as the leader and refinery as a follower. The second model includes two levels the customer as the upper level and the distributor as the lower level.

3.4.1 Non-linear bi-level programming model 1. This model includes two levels of distributor and refinery. In this model, the upper-level objective is to determine the optimal value of the batch size demand and the lost sales such that the total cost becomes optimal. The lower-level objective is to determine the extraction's optimal value to minimize the total cost. The non-linear bi-level programming model of the production inventory is indicated by:

$$Min f_d = A_d.\frac{d_d}{Q} + o.5\ h_d(IB_d + IE_d) + hit_d\frac{d_d}{Q} + g_d\frac{d_d}{Q} + C_{DA}\frac{d_d}{Q} + sl_d.ls_d \tag{31}$$

$$\begin{aligned} Min f_p = \; & C_pX + A_pX + 0.5.I.C_p.Q\frac{d_d}{X} + C_A\frac{d_d}{Q} + S_pQ\frac{d_d}{X} + Pr_pQ\frac{d_d}{X} + Re_pX + Rg_pX \\ & + Hb_pX + C1X1 + C2X2 \end{aligned} \tag{32}$$

s.t.

$$IB_d + X - d_d + Ls_d = IE_d \tag{33}$$

$$X1 + X2 = X \tag{34}$$

$$X1 \leq Max\,P \tag{35}$$

$$X2 \leq 2Max\,P \tag{36}$$

$$C1X1 + C2X2 + Pr_pQ\frac{d_d}{X} + Re_pX + Rg_pX + Hb_pX \leq Budget \tag{37}$$

$$d_d, IB_d, IE_d, Q, ls_d, X, X1, X2 \geq 0 \tag{38}$$

3.4.2 Bi-level nonlinear programming Model 2. This model includes two levels: 1. Retailer and 2. Distributor. In this model, the upper-level objective is to determine the optimal value of the demands and lost sales in a way that the total cost becomes optimal. The lower-level objective is also to determine the optimal value of the batch size demand and the lost sales such that the total cost becomes optimal. The nonlinear bi-level programming model of the distribution inventory is indicated by:

$$Min\,f_R = \sum_{i=1}^{3} A_R d_{Ri} + 0.5\sum_{i=1}^{3} h_R(IB_{Ri} + IE_{Ri}) + \sum_{i=1}^{3} hit_R d_{Ri} + \sum_{i=1}^{3} g_R d_R + \sum_{i=1}^{3} Sl_R ls_R \quad (39)$$

$$Min\,f_d = A_d.\frac{d_d}{Q} + o.5\ h_d(IB_d + IE_d) + hit_d\frac{d_d}{Q} + g_d\frac{d_d}{Q} + C_{DA}\frac{d_d}{Q} + sl_d.ls_d \quad (40)$$

s.t.

$$d_d - ls_d - IE_d = \sum_{i=1}^{3} d_{Ri} - \sum_{i=1}^{3} ls_{Ri} + \sum_{i=1}^{3} IE_{Ri} \quad (41)$$

$$d_{Ri}, IB_{Ri}, IE_{Ri}, ls_{Ri}, d_d, Q, IB_d, IE_d, ls_d \geq 0 \quad (42)$$

It is worth mentioning that in the two nonlinear bi-level programming models is that the distributor exists in each of the two models. Suppose each of the two models is optimized independently. In that case, the values obtained for the distributor level in one model are probably implausible in the constraints and the model solution space. By considering the mentioned fact, a solution algorithm should be employed to simultaneously obtain a plausible answer for the distributor level in both of the models.

The suggested models are nonlinear. Therefore, in the first stage, by using the KKT optimality conditions, the nonlinear bi-level programming problems are turned into optimization problems with one objective function to achieve the optimal answer. The models are in the form of relations (43)–(57).

3.4.3 Single-level model caused by applying the KKT conditions in the bi-level model 1.

$$Min\,f_d = A_d.\frac{d_d}{Q} + o.5\ h_d(IB_d + IE_d) + hit_d\frac{d_d}{Q} + g_d\frac{d_d}{Q} + C_{DA}\frac{d_d}{Q} + sl_d.ls_d \quad (43)$$

s.t.

$$IE_d = IB_d + x - d_d + ls_d \quad (44)$$

$$C_p + A_p + 0.5i\,C_p Q\frac{-d_d}{x^2} + S_p Q\frac{-d_d}{x^2} + Pr_p Q\frac{-d_d}{x^2} + Re_p + Rg_p + Hb_p + C1 + C2 + \mu = 0 \quad (45)$$

$$0.5i\,C_p\frac{d_d}{x} - C_A\frac{d_d}{Q^2} + S_p\frac{d_d}{X} + Pr_p\frac{d_d}{X} = 0 \quad (46)$$

$$C1X1 + C2X2 + Pr_p Q\frac{d_d}{X} + Re_p X + Rg_p X + Hb_p X \leq Budget \quad (47)$$

$$X1 + X2 = X \quad (48)$$

$$X1 \leq Max\,P \quad (49)$$

$$X2 \leq 2Max\,P \quad (50)$$

$$d_d, IB_d, IE_d, Q, ls_d, X, X1, X2, \mu \geq 0 \quad (51)$$

3.4.4 Single-level model caused by applying the KKT conditions in the bi-level model 2.

$$Min\,f_R = \sum_{i=1}^{3} A_R d_{Ri} + 0.5\sum_{i=1}^{3} h_R(IB_{Ri} + IE_{Ri}) + \sum_{i=1}^{3} hit_R d_{Ri} + \sum_{i=1}^{3} g_R d_R + \sum_{i=1}^{3} Sl_R ls_R \quad (52)$$

s.t.

$$d_d - ls_d + IE_d = \sum_{i=1}^{3} d_{Ri} - \sum_{i=1}^{3} ls_{Ri} + \sum_{i=1}^{3} IE_{Ri} \tag{53}$$

$$\frac{A_d}{Q} + \frac{hit_d}{Q} + \frac{g_d}{Q} + \frac{C_{DA}}{Q} - \mu_1 = 0 \tag{54}$$

$$0.5\, h_d - \mu_2 = 0 \tag{55}$$

$$sl_d - \mu_3 = 0 \tag{56}$$

$$d_R, IB_R, IE_R, ls_R, d_d, Q, IB_d, IE_d, ls_d, \mu_i \geq 0 \tag{57}$$

To validate the model, the solutions of five test problems at a smaller scale are used in the General Algebraic Modeling System (GAMS) V24.1.2 software and a search in forest optimizer (SIFO) in MATLAB software in a computer equipped with the following: CPU: Intel(R) CORE™ i5-2300 and RAM: 8 GB. The results show that the solutions obtained using the SIFO are consistent with those using GAMS. Table 2 presents the input values to the model for five problems with ($i = 3$) retailers. Table 3 presents the results of validating the model.

```
Pseudo-code of SIFO
Initialize the parameters M, N, Pr, c, Max_iteration, ;
Initialize the positions of Teams and agents using $T_{mn}$;
While ($t \leq Max\_iteraition$)
Calculate the fitness of all Teams and agents;
    $Update\ bestFitness, T_{mn}$
Team merging using Model (6) and Pr;
For $each\ search\ agent$
 $Run\ Levy\ flight$;
 $Run\ Local\ search$;
   End For
$t = t + 1$;
End While
Return $bestFitness, best\ position$;
```

The new metaheuristic algorithm “Search in Forest Optimizer” has been introduced to solve NP-Hard programming problems, By carefully searching the problem space and with the ability to exit the local optimization, this algorithm was able to provide a more optimal answer for complex spaces related to multi-level supply chain problems with multiple levels and constraints. Considering the multi-level and complex nature of petroleum products supply chains, each supply chain level contains a high number of facilities. The mathematical planning model contains different parameters and constraints for the objective functions of each level; therefore, the petroleum products planning models are considered complex NP HARD problems. The SIFO is designed for an in-depth and adequate search in the response spaces of complex problems determined by different parameters, decision variables and constraints. SIFO can easily surpass local optimums to conduct its search simultaneously in the main path alongside all other possible paths. It can also exit its local optimum to find the

Definition	Parameters and variables	Test problem no	Small value
The production cost at the producer, including purchase costs	C_p	1	0.33
		2	0.55
		3	0.25
		4	0.49
		5	0.95
The cost of production repair	A_p	1	0.66
		2	1
		3	0.98
		4	0.86
		5	0.57
Percentage of inventory preservation cost per unit	I	1	0.33
		2	0.27
		3	0.35
		4	0.96
		5	0.84
Batching cost	C_p	1	0.66
		2	0.87
		3	0.53
		4	0.88
		5	0.42
The cost of human force	S_p	1	4
		2	5
		3	2
		4	5
		5	1
The cost of preparing tools and necessaries of maintenance	Pr_p	1	16.5
		2	10
		3	8.97
		4	18
		5	12.26
The cost of repairing machinery	Re_p	1	10
		2	14
		3	19
		4	16
		5	11
The cost of setting and loading the machinery	Rg_p	1	5
		2	3
		3	1
		4	5
		5	2
The overhead cost	Hb_p	1	3.67
		2	1.78
		3	2.51
		4	1.22
		5	4.58
The total cost available for doing maintenance operation	*Budget*	1	2.66
		2	1.32
		3	3.51
		4	4.21
		5	3.74

(*continued*)

Table 2. The input values to the model for five small scale problems with (i = 3) retailers

Definition	Parameters and variables	Test problem no	Small value
The maximum extent of production at ordinary time	$MaxP$	1	2
		2	2
		3	3
		4	1
		5	3
The maintenance cost pertinent to human force at ordinary time	$C1$	1	1
		2	3
		3	1
		4	2
		5	1
The maintenance cost pertinent to human force at overtime	$C2$	1	1.64
		2	0.55
		3	1.24
		4	0.98
		5	1.83
The demand rate at distributor *d*	D_d	1	0.60
		2	0.37
		3	0.44
		4	0.79
		5	0.19
The ordering cost at distributor *d*	A_d	1	29
		2	25
		3	19
		4	16
		5	22
The cost of inventory preservation at distributor *d*	H_d	1	1.76
		2	1.11
		3	1.33
		4	1.24
		5	0.87
The cost of inventory preservation during transportation from the producer to distributor *d*	Hit_d	1	1.18
		2	1.12
		3	0.33
		4	0.57
		5	0.63
The cost of transportation from the producer to distributor *d*	G_d	1	1.76
		2	0.99
		3	0.63
		4	0.84
		5	0.45
The initial inventory at distributor *d*	IB_d	1	4.70
		2	3.26
		3	0.72
		4	0.94
		5	0.53
The cost of the unit's lost sale at distributor *d*	Sl_d	1	6
		2	3
		3	1
		4	4
		5	2

Table 2. (*continued*)

Definition	Parameters and variables	Test problem no	Small value
The cost of opening the batches	C_{DA}	1	1.17
		2	0.66
		3	0.87
		4	0.39
		5	0.48
Order rate at retailer *i*	d_{Ri}	1	0.58
		2	0.11
		3	0.17
		4	0.19
		5	0.77
The order cost at retailer *i*	A_{Ri}	1	37.71
		2	28.16
		3	22.12
		4	14.31
		5	16.48
The cost of each product's inventory preservation at retailer *i*	h_{Ri}	1	1.80
		2	0.77
		3	0.67
		4	0.72
		5	0.14
The cost of inventory preservation during transportation from the distributor to retailer *i*	hit_i	1	1.80
		2	1.11
		3	1.22
		4	0.55
		5	0.23
Each unit's transportation cost of from the distributor to retailer *i*	g_{Ri}	1	2.32
		2	1.22
		3	1.35
		4	0.68
		5	1.01
The cost of the lost sale at retailer *i*	Sl_{Ri}	1	3
		2	1
		3	0.75
		4	1.61
		5	0.77
The initial inventory at retailer *i*	IB_{Ri}	1	13.35
		2	9.25
		3	9.18
		4	4.89
		5	2.55

Table 2.

best optimum response; therefore, SIFO can present a more effective performance in linear planning model solving than other metaheuristic algorithms (Ahwazian *et al.*, 2022).

Now that the model is verified, it will be solved for oil-producing countries of OPEC. Input values to the model are close to the actual values of petroleum products in Iran. Table 4 presents the input values to the model with ($i = 3$) retailers. Table 5 presents the results of the optimizing programming model determined in this research for the Iran oil product market. According to the obtained results from the problem-solution using the SIFO algorithm in Table 5, the production cost of each petroleum materials' unit by the manufacturer should not exceed 11.6 of the monetary unit, and the distributor will deliver the produced materials to the

Table 3.
Results of model validation

Test problems	Approach	Producer cost	Distributor cost	Retailer cost
Test PROBLEM1	SIFO	4.6106	8.2723	23.508
	GAMS	4.6122	9.4301	3.508
Test PROBLEM2	SIFO	2.3824	3.6186	6.8261
	GAMS	2.6254	3.6186	7.8947
Test PROBLEM3	SIFO	5.6268	0.9576	5.8927
	GAMS	5.6668	0.9576	6.1825
Test PROBLEM4	SIFO	6.9252	1.6556	3.2127
	GAMS	7.1800	1.7556	4.0100
Test PROBLEM5	SIFO	6.7938	0.4611	0.3451
	GAMS	6.8000	0.4611	0.0551

retailer with a cost of at most 24 monetary unit. The retailer will also sell the product to the customer with a monetary unit of 67.5. Figure 1 depicts three supply chain levels.

4. Implications of research

The current research focuses on the integrated cost reduction and productivity-increasing petroleum products supply chains to increase petroleum products' production, distribution and sales profit. Productivity in valuable and high-demand commodity markets leads to increase profits. The planning results using the proposed model are visible for product suppliers and customers because this model can reduce production, distribution and supply costs that allow producer companies to create new competitive benefits and market influence while the consumers obtain this commodity with better prices. Governments, managers and experts can use such results. Cost optimization will increase productivity, reduce resource consumption and thus create savings for future generations and communities.

Maintained and increased production cost is a new concept that is added to the literature review by this paper. The necessity of continuous strategic oil products supply to the market makes the oil managers consider the maintained and increased production costs. These costs include the maintenance costs, storage and cost associated with increasing the operation and production site developments, refining and distribution of oil products. In critical conditions like downturn as appeared due to the coronavirus pandemic worldwide, oil managers spend on maintained and increased production efficiency instead of planning for new investments in supply chains, to maximize the existing potential to not only increase the sustainable production for market supply but also prevent the new investment costs.

Neglecting such costs in the current research is due to the lack of statistical data and calculation complexities associated with calculating required values for planning model parameters as well as getting the planning models more complex. In order to solve the designed model complexities, the SIFO can be used. This metaheuristic algorithm is designed to search reinforcement in the main optimum response and for simultaneous search in all routes and powerful escaping the locally optimum, the algorithm can produce the best response in complex linear planning models for future work.

Additionally, for solving the problem of the lack of data and time-consuming calculation process for the model parameters, it is suggested to analyze the effect of uncertainties associated with estimating the required parameters through different procedures such as random, fuzzy and robust methods. The maintained and increased production costs are suggested to be added as an independent objective function to the planning supply chain model and its solution space defined by constraints in the designed model. One of the outputs of this work is the optimization of the costs of maximizing operating existing supply chain facilities.

Definition	Parameters and variables	Value
The production cost at the producer, including purchase costs	C_p	1
The cost of production repair	A_p	2
Percentage of inventory preservation cost per unit	I	1
Batching cost	C_A	2
The cost of human force	S_p	12
The cost of preparing tools and necessaries of maintenance	Pr_p	50
The cost of repairing machinery	Re_p	30
The cost of setting and loading the machinery	Rg_p	15
The overhead cost	Hb_p	11
The total cost available for doing maintenance operation	$Budget$	8
The maximum extent of production at ordinary time	$MaxP$	9
The maintenance cost pertinent to human force at ordinary time	$C1$	7
The maintenance cost pertinent to human force at overtime	$C2$	5
The demand rate at the distributor	D_d	1
The ordering cost at the distributor	A_d	50
The cost of inventory preservation at each unit	H_d	3
The cost of inventory preservation during transportation from the producer to the distributor	Hit_d	2
The cost of transportation from the producer to the distributor	G_d	3
The initial inventory at the distributor	IB_d	8
The cost of the unit's lost sale at distributor	Sl_d	10
The cost of opening the batches	C_{DA}	2
Order rate at the *i* retailer	d_{Ri}	1
The order cost at retailer *i*	A_{Ri}	65
The cost of each product's inventory preservation at each retailer	h_{Ri}	3
The cost of inventory preservation during transportation from the distributor to retailer *i*	hit_i	3
Each unit's transportation cost of from the distributor to the retailer	g_{Ri}	4
The cost of the lost sale at retailer *i*	Sl_{Ri}	5
The initial inventory at retailer *i*	IB_{Ri}	23

Table 4. Input values to the model with ($i = 3$) retailers

Parameters of search in forest optimizer	Best supply chain cost		
$M = 10,\ N = 40,\ c = 0.3$	Producer 11.5712	Distributor 24.01	Retailer 67.5000

Table 5. The best cost of the optimized levels of the petroleum materials' supply chain model

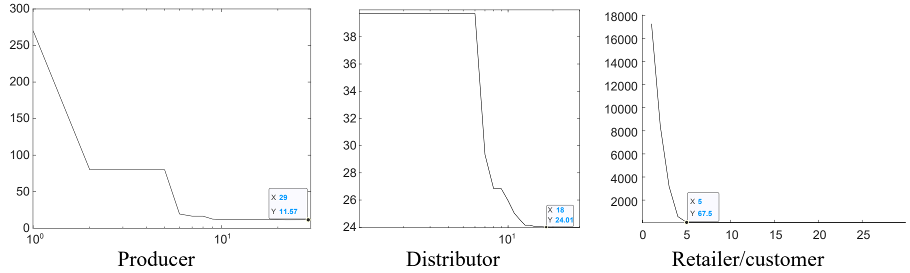

Figure 1. Graph of the petroleum materials optimized supply chain levels

5. Discussion and conclusion

This research aims at increasing the effectiveness of the dynamic petroleum materials supply chain design. Accordingly, a three-level mathematical model is employed to mitigate the total costs of the mentioned supply chain. After validating the model in small dimensions, due to the NP-Hard nature of the problem, a metaheuristic, search in forest optimizer, is employed to solve this model. The researchers emphasize the significance of supply chain creation's integration.

This designed model is non-linear; therefore, the Kuhn–Tucker optimization conditions implement the second-level objective function, so the second-level problem turns into a constraint for the first-level problem, which allows the first-level objective function to solve the problem. The KKT method is applicable for solving linear and non-linear planning problems and guarantees the existence of an accurate response. This method matches the integrated management approach of the supply chain model because it simultaneously optimizes two integrated levels in the KKT method.

Accordingly, supply chain management demonstrates an integrated view among the existing influential players and the performances of the supply chain (Attia *et al.*, 2019). Gainsborough (2006) stated that the major barrier in the industry is silo mentality, while the members of the oil supply chain and any other supply chain have a general attitude. Integration must be a powerful basis in supply chain management. Integration must occur horizontally among various sections and do the strategy coordination, planning and operational execution vertically. Analyzing is mainly focused on the downstream of the flow since this section is more potential in increasing flexibility and economizing the costs.

Besides, it must not be forgotten that first of all, the objective of the oil supply chain optimization must be clarified. Usually, this strategy is aligned with the supply chain, and its objective is to reach maximum profit through economic efficiency and customer satisfaction. Nevertheless, in the present research, the researchers optimized all levels of the petroleum product supply chain instead of one level by considering the effects of management flows of each level on other levels. Memari *et al.* (2018) stated that optimization is also an opportunity for creating oil companies and gaining a competitive advantage. Besides, the oil supply chain in an unstable field is influenced by geopolitical unrests, global competition and price fluctuations, where trade is focused on margins, and economizing is carried out by better predictions and plans with shorter planning horizons. Subsequently, an effort has been made to design and execute new tools to create a new integrated and adaptive supply chain in order to improve the decision-making process, decrease costs, decrease inventories and increase margins. Besides, as mentioned in the previous sections, integration is a powerful way to lead companies to optimize their value chain, enabling them to balance their upstream and downstream activities and decrease risks and fluctuations. Florescu *et al.* (2019) addressed the effects of robust supply chain management strategies, choosing the supplier, product management and logistics management in supply chain management performances, planning, execution, coordination and collaboration in the oil and gas distribution industry. By employing multiple regression analysis, 79 companies from Romania and the Republic of Moldova, which are active in the gas and oil industry, were studied. It was found that robust supply chain management strategies have a positive and significant influence on the under-research supply chain management performances. The authors stated that the findings of this research could generally be employed in the design of robust supply chain management strategies by the companies that are active in gas and oil distribution in order to better meet the requirements pertinent to the activities with environmental and social responsibilities in their supply chains and do their supply chain management optimization in a better way. Accordingly, in the present research, all the levels defined for the petroleum product supply chain are modeled coordinately and in the form of an integrated chain so that the feasible answers of the various levels of the chain are included in all management strategies in the whole supply chain, which increases the robustness of the proposed supply chain.

A vast majority of petroleum product's production costs are pertinent to the preservation operations and increasing the petroleum products' production, among which the maintenance costs account for the largest parts of these kinds of operations costs. Indeed, the maintenance issue can directly decrease the risk level that exists in providing service in dispatching petroleum products (Carneiro *et al.*, 2010). However, the reason behind not considering these costs in the previous research is the model's complexity and difficulty in designing solution algorithms. Besides, gathering the information required for solving the problem is another considerable problem in this realm (An *et al.*, 2011). Thereby, the companies' decision-makers at financial and macro levels will benefit from the applications of maintenance costs in the proposed model.

Another subject that captured interest in the design of the proposed model is the existence of multiple management flows in the structure of the petroleum product supply chain (Ahmad *et al.*, 2017). Therefore, it seems necessary in all stages that exist in the petroleum products chain to consider various management flows and make decisions in an integrated way. Nevertheless, there is a fundamental problem in the design of this integrated supply chain, which is the contradiction among the interests of management levels in most cases (Rowshannahad *et al.*, 2018). In the proposed model, each one of the upstream and downstream levels of the petroleum products supply chain that are obtained from the optimization of the other side level optimizes their decisions based on the feasible values of the common level's variables. These reciprocal relationships continue until both upstream and downstream levels reach an equilibrium point. Developing the proposed model for a multi-level petroleum product supply chain in different states of certain parameters and time zones is recommended for future research.

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