

# The Design of half-subtractor Logic Function Based on Nonlinear Directional Coupler

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## Abstract

*In this paper a novel design of all-optical half-subtractor based on nonlinear directional coupler is proposed. By using four waveguides and appropriately adjusting the refractive indices and selecting the proper length of waveguides, half-subtractor function can be obtained. The operation of this function is simulated by RSoft CAD-Layout (BeamPROP) simulator. The simulation results confirm the all-optical half-subtractor circuit without any optoelectronic conversions.*

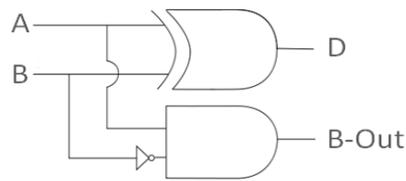
**Keywords:** *all-optical half subtractor, coupling, nonlinear directional coupler, refractive index, waveguide.*

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## 1. Introduction

In conventional optical communication and computation which is still performing in the electrical domain, optoelectronic conversion is inevitable. Optoelectronic conversion not only requires extra power, but also induces speed bottleneck in data transmission [1]. In recent years, the requirement for high speed and high bandwidth information processing has provided more attention to the all-optical devices and circuits. An all-optical signal processing has capability of handling large bandwidth signals and large information flows and it is able to work with transmissions rates up to hundreds  $\text{Gb.s}^{-1}$ . According to these advantages, all-optical signal processing is expected to have many applications such as binary addition, header recognition, parity checking, addressing, demultiplexing, regenerating and switching with very high speed [2].

Many researches propose various schemes for implementing of all-optical devices by means of semiconductor optical amplifier [3], Mach-Zehnder Interferometer, micro-ring resonators [4], Ultra Nonlinear Interferometer [5], cross gain modulation (XGM), four-wave mixing and Kerr effect at high-Q cavity [6]. Many all-optical logic gates and all-optical arithmetic circuits have been proposed using above mentioned methods. Among these arithmetic circuits, a binary half-subtractor is one of the most importance operations of two digits. The half-subtractor can be implementing a full subtractor. In electrical domain this circuit constructed by use of AND, XOR and NOT gates. Figure 1 illustrated a particular half-subtractor circuit.



**Figure 1. Half-subtractor**

All-optical half-subtractor reported in the literatures could be achieved with a Dark-bright Soliton [7], Phase Encoding principle [8], PPLN waveguides [9, 10] and high-Q bacteriorhodopsin [11]. However, the searching of new techniques remain, in this paper we propose the operation of half-subtractor based on nonlinear directional coupler and use of linear and nonlinear mediums.

## 2. Design and Simulation Results

### a. Basic Review of Half-Subtractor

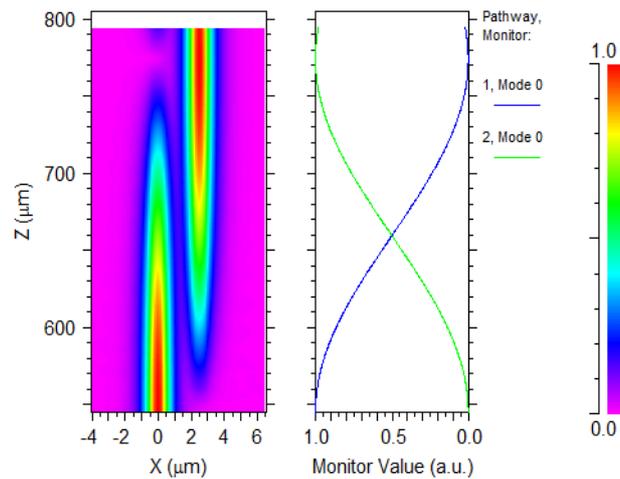
The half-subtractor is a combinational circuit which is used to perform subtraction of two bits (A-B). It has two inputs, A (minuend) and B (subtrahend) and two outputs D (difference) and B-OUT (borrow). The borrow bit is a '1' when subtraction '1' from '0' otherwise is '0'. The difference bit is '1' if either but not both inputs are '1' and is '0' if the inputs are both '0' or both '1'. The truth table of the half-subtractor function is shown in Table 1.

**Table 1. Truth table of half subtractor function**

Input		Output	
A	B	D	B-OUT
0	0	0	0
0	1	1	1
1	0	1	0
1	1	0	0

### b. Coupled-Mode Theory

In this section, nonlinear directional coupler theory is presented and discussed using mathematical principles. If two waveguides are sufficiently close each other, light beam can be coupled between the waveguides and total power or partial power can be exchanged between them. Figure 2 illustrated this state of coupled mode theory.



**Figure 2. Coupling between two waveguides**

The coupled equations for the field amplitude in each waveguide can be derived as follows [12]:

$$\frac{da_1}{dz} = -jk_{21} \exp(j\Delta\beta z)a_2(z) \quad (1)$$

$$\frac{da_2}{dz} = -jk_{12} \exp(-j\Delta\beta z)a_1(z) \quad (2)$$

In the above equations,  $\Delta\beta = \beta_1 - \beta_2$  is the phase mismatch per unit length and  $a_1(z)$  and  $a_2(z)$  are amplitudes of the modes of waveguides 1 and 2 respectively, also  $k_{21}$ ,  $k_{12}$  are coupling coefficients in each waveguide (equations (3), (4)):

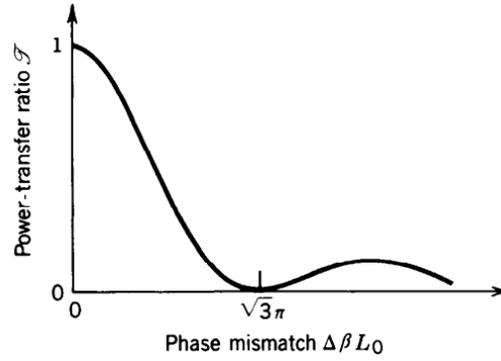
$$k_{21} = \frac{1}{2}(n_2^2 - n^2) \frac{k_0^2}{\beta_1} \int_a^{a+d} u_1(y)u_2(y)dy \quad (3)$$

$$k_{12} = \frac{1}{2}(n_1^2 - n^2) \frac{k_0^2}{\beta_2} \int_{-a-d}^{-a} u_2(y)u_1(y)dy \quad (4)$$

In which  $n_1$  and  $n_2$  are refractive indices embedded for each waveguide in a medium with refractive index  $n$ . A power transfer ratio is expressed as a  $P_2/P_1$ , which written as equation (5):

$$\frac{P_2}{P_1} = \left(\frac{\pi}{2}\right)^2 \sin^2 c^2 \left\{ \frac{1}{2} \left[ 1 + \left( \frac{\Delta\beta L_0}{\pi} \right)^2 \right]^{\frac{1}{2}} \right\} \quad (5)$$

Where  $P_1$  and  $P_2$  are powers at the end of waveguides 1 and 2 respectively and  $L_0$  is coupling length. This term depends on the phase mismatch parameter,  $\Delta\beta L_0$ , so the power transfer ratio decreases with increasing in  $\Delta\beta L_0$  as shown in Figure 3.



**Figure 3. Power transfer ratio vs. Phase mismatch**

In a Kerr-type material the refractive index,  $n$ , is described by the Kerr law,  $n = n_0 + n_2 I$  where  $n_0$  is the linear refractive index,  $n_2$  is the third-order nonlinear coefficient, and  $I$  is the field intensity.

### c. Operation Principles Of Half-Subtractor

The proposed structure for an all-optical half-subtractor circuit is depicted in Figure 4. This scheme consists of four waveguides, ( $I_1, I_2$ ) are input channels and ( $O_1, O_2$ ) are output ports. The value of refractive indices is shown in Table 2. In fact the refractive index of linear mediums set to this value by the aid of equivalent layers theory [13] and the used material for input channels is polydiacetylene PTS with  $n_0 = 1.66$  that have the kerr nonlinearity about  $2 \times 10^{-4} \mu\text{m}^2/\text{W}$  [14].

**Table 2. The value of refractive indices of waveguides**

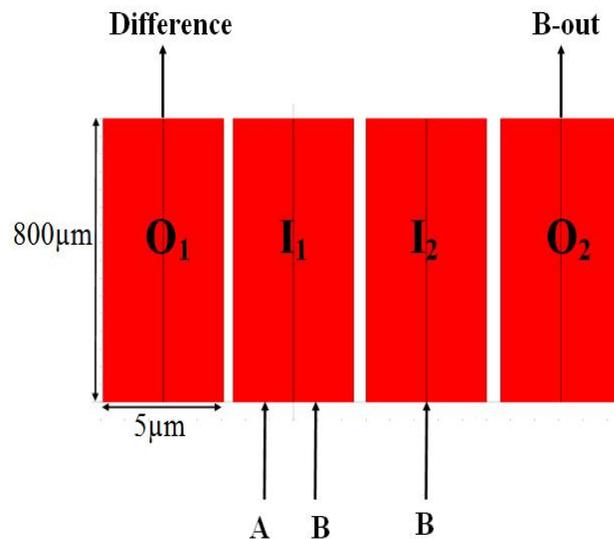
Waveguides	Refractive Index	Behavior
$I_1$	1.66	Nonlinear
$I_2$	1.664	Nonlinear
$O_1$	1.66	Linear
$O_2$	1.664	Linear

Actually, operation of the circuit is done based on changing the refractive index by applying optical input signals to the nonlinear waveguides. As the input power increases, the nonlinear contribution to the refractive index increases and the phase match between the input channels and their corresponding output will be destroyed. In this scheme 3Watt in power considered as logic one and no light in inputs considered as zero logic. In half-subtractor function the sequence of operands is considerable. The Boolean functions for the two outputs can be obtained directly from the truth table (Table 1) as:

$$D = A \oplus B$$

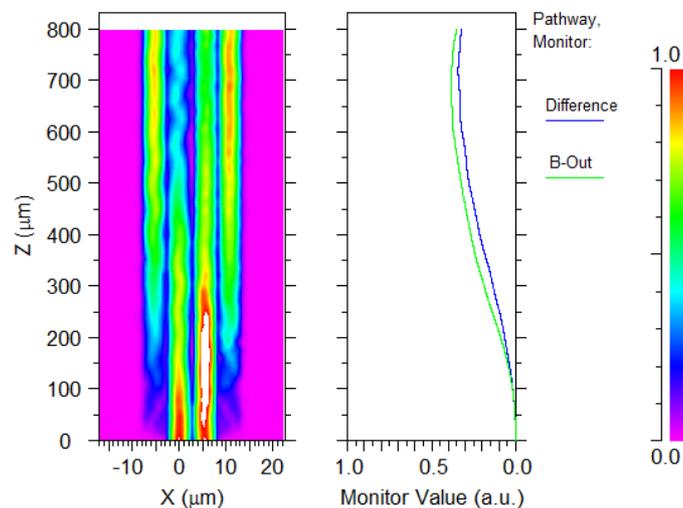
$$B - \text{Out} = \overline{A} \cdot B$$

If the second operand greater than the first one, the borrow bit must be set, because of that we propose to inject the second operand to the both input channels.



**Figure 4. Proposed structure for all optical half-subtractor**

In first state, when there is no light in inputs, no signal emerges from the output ports, D and B-out, obviously. In the second state, when the value of subtrahend is logic one ( $B=1$ ) and the minuend value is zero ( $A=0$ ), the beam with 3Watt in power is launched into the both input channels. Since the entrance power is insufficient, the refractive indices of input channels will not change significantly, therefore the input light will be coupled from  $I_1$  and  $I_2$  to the port D and B-out, respectively (Figure 5).

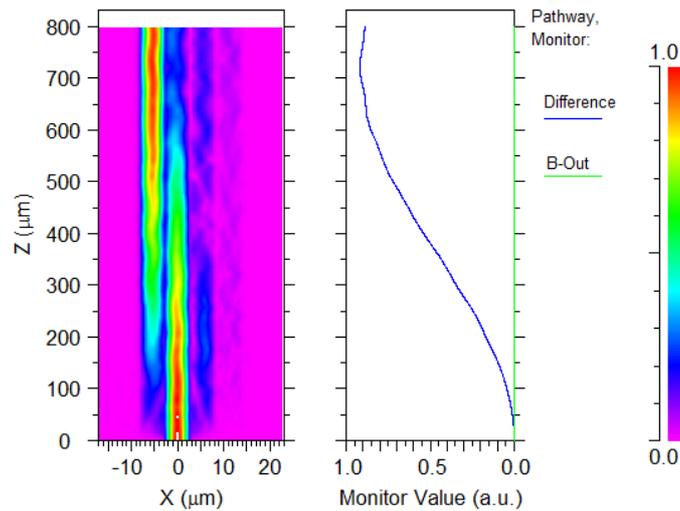


**Figure 5. Output power of port O1 and O2 when ( $A=0$ ,  $B=1$ )**

For the third state, only the minued is logic one ( $A=1$ ) and the second operand is zero ( $B=0$ ). In this case  $I_2$  channel is empty and the signal beam is launched to the  $I_1$  channel. As well as first state, the power of the input is not sufficient to change the refractive index of input guide, hence the majority of the power will be coupled to  $O_1$  port and exit from D output. On the other hand the input power can not compensate the difference

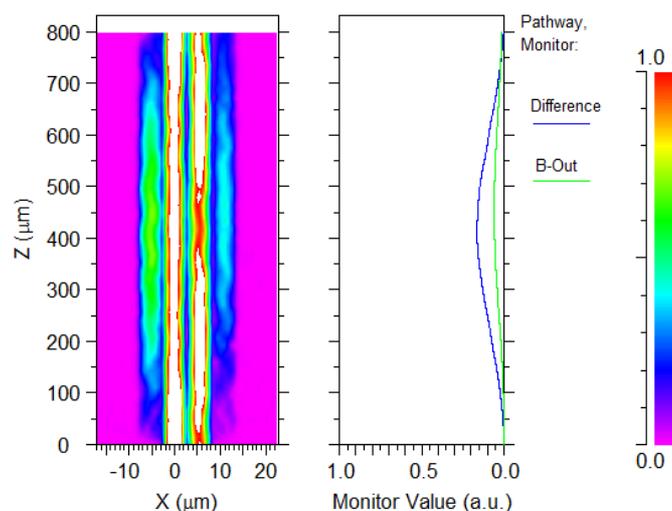
between index profile of  $I_1$  and  $I_2$  guides. Therefore, the optical power is not transferred between the input channels and so, B-out port which shows borrow bit has no light. This situation is represented by Figure 6.

In these two states when the incident power is low, it means that only one operand is logic one, no nonlinearities are excited and only linear coupling is present. Therefore, according to equation (5), the optical power can transport between input channels and output ports.



**Figure 6. Output power of port O1 and O2 when (A=1, B=0)**

For the case where both optical input signals are logic one ( $A=1$  and  $B=1$ ), the signal with 6Watt power enters the  $I_1$  and 3Watt power is applied to  $I_2$ . For the nonlinear guides, when the input power is high enough the propagation constant mismatch ( $\Delta\beta$ ) is so large that power does not exchange between the two adjacent guides. In this state, the input light with 6Watt in power is sufficiently intense to change the refractive index of  $I_1$  channel to such an extent that no light can transmit to the  $O_1$  port. At this time, the increment of the refractive index of  $I_1$  leads to transfer the percentage of light from  $I_1$  to  $I_2$ . This amount of power which disseminates to the  $I_2$  channel destroyed the equivalence of refractive indices between  $I_2$  and its corresponding output ( $O_2$ ) thus, no power transfer occurs and there is no light will appear in the B-out. Figure 7 is a graph of this case.



**Figure 7. Output power of port O1 and O2 when ( $A=1$ ,  $B=1$ )**

The operation of this circuit is based on absence or presence of light in the output ports. However if the output optical signals could be managed in another fashion, cascading several optical logic of this circuit would be possible. For this aim we propose the use of SOA (Semiconductor Optical Amplifier) at the end of output ports to amplify power of output signals.

### 3. Conclusion

This paper has proposed and simulated an all-optical half-subtractor function based on nonlinear directional coupler. This circuit is compact and potentially applicable for photonic integrated circuit. The operation speed of proposed scheme is faster than using the type of electrical in order to avoid inefficient optoelectronic conversion.

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