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Presentation a Model for Integration of Fuzzy Data Envelopment Analysis and Goal Programming

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Abstract

Data Envelopment Analysis (DEA) has been a very popular non-parametric technique for measuring and benchmarking relative efficiency of Decision Making Units (DMUs) with multiple input and outputs. In fact, in a real evaluation problem input and output data of things evaluated often fluctuate. These fluctuating data can be represented as linguistic variables characterized by fuzzy numbers for reflecting a kind of general feeling or experience of experts. Based on the fundamental CCR model, a fuzzy DEA model is proposed to deal with the efficiency evaluation problem with the given fuzzy input and output data. Furthermore, an extension of the fuzzy DEA model to a more general form is also proposed with considering the relationship between DEA and GR (Goal Programing).

Key Words

Efficiency, Data Envelopment Analysis, Goal Programming, fuzzy situation, efficiency

I. INTRODUCTION

In this days, DEA play a powerful role for measuring the efficiencies of a group of decision making units (DMUs) that used to convert multiple inputs into multiple outputs. This method was advanced by Charnes, Cooper, and Rhodes [1] and has been used in various fields, for examples, banking[2], education [3], airline [4],stock market [5], government [6], and supply chain [7] and this large domain of use shows the importance of this issue. The previous research efforts can easily find many different types of DEA formulation, besides the ratio form, such as BCC model, an additive model, a multiplicative model, and other DEA models related to performance analysis. but this research believes that all of them are not completed for measuring the real word factors and DEA needs to be reviewed in the framework of GP and fuzzy situation and it is very important to introduce new tools in the approach that allow the model to fit into the real world as much as possible, and you know that the parameters of the problem are usually defined by the decision making in a uncertain way or by means of language statement parameters, it is useful to consider the knowledge of experts about the parameters as fuzzy data. consequently, this article can develop a new type of DEA, which has been never explored in conventional research works regarding DEA and GP.

As a literature review of goal programing, Dean and Schniederjans [8] formulated a process as a multiple criteria decision-making problem and apply a mathematical modeling approach, based on a linear goal programming method, in continue of their research Kasilingam and Lee [9] in previous research take into account various selection criteria. Babic and Plazibat [10] occupy multiple criteria analysis for ranking of enterprises, according to the achieved level of business efficiency. the business efficiency is very complex and multidimensional concept, therefore, the multicriteria analysis could be the most suitable approach to the problem. So this paper is structured as follows. Firstly, a brief literature of Goal Programming (GP) and its application on this model will be exhibited. Then, a brief review on several kinds of DEA will be presented; this is followed by a discussion on the developed DEA performance measurement model. Then, we try to mix these two models together and show the final model. The last section draws conclusions on the paper and highlights some of the important point of this model. This paper presents an improved efficiency measurement tool by modifying the existing data envelopment analysis methodology to allow the incorporation of expert knowledge.

A. Goal Programming

Lee. E. S, [11] used Goal Programing Methods for Multiple Objective Integer Programs, American Objective Integer Programs, American Institute of Industrial Engineers. Initially conceived as an application of single objective linear programming by Charns and Cooper [12]. The idea of goal programming is to establish a goal level of achievement for each criterion. a key element of a GP model is the achievement function that draws a mathematical expression of the unwanted deviation.

B. Weighted GP (WGP) Model

The achievement function of WGP model inscribes the unwanted deviation variables, each weighted according to importance. The mathematical programming of a WGP model is the following:

$$\begin{aligned} Min \quad & \sum_i (\alpha_i d_i^- + \beta_i d_i^+) \\ s.t. \quad & f(\boldsymbol{x}) + d_i^- - d_i^+ = g_i \\ & d_i^- \cdot d_i^+ = 0 \\ & d_i^- \geq 0, \quad d_i^+ \geq 0 \end{aligned} \qquad (1)$$

where

$\alpha_i = w_i / k_i$ if d_i^- is unwanted, otherwise $\alpha_i = 0$;

$\beta_i = w_i / k_i$ if d_i^+ is unwanted, otherwise $\beta_i = 0$.

The parameters w_i and k_i are the weights reflecting preferential and normalizing purposes clinging to achievement of the i-th goal.

C. Data Envelopment Analysis (DEA)

Charnes, Cooper and Rhodes (CCR) [1] introduced this ratio definition of efficiency. DEA model measures efficiency by estimating an empirical production function that represents the highest values of outputs that could be generated by relevant inputs, as obtained from observed input-output vectors for the analyzed DMUs. The inefficiency of a DMU is then measured by the distance from the point representing its input and values to the corresponding reference point on the production function.

There are a number of mathematical formulations of DEA all sharing the principle of envelopment. An output vector Y_k for DMU_k is enveloped from above when the model identifies a combination of other output vectors whose values are equal to or greater than all the elements in Y_k. Similarly, the input vector X_k is enveloped from below when the model finds a combination of other input vectors whose values are smaller than or equal to all the elements in X_k. If the pair (X_k, Y_k) cannot be enveloped simultaneously by a combination of other DMUs, then DMU_k is efficient. In general, the set of efficient DMUs selected for evaluating an analyzed DMU defines one facet of the piecewise linear empirical production function. A linear combination of these DMUs serves as a reference point for the measurement of the inefficiency of DMU_k.

Before introduce some DEA models for assessment of efficiency, common notation used in the follow up is summarized below.

Indices:

k – DMUs, $k=1,\dots,n$;

i – inputs, $i=1,\dots,r$;

j – outputs, $j=1,\dots,s$;

Data:

x_{ik} - the value of i-th input for the k-th DMU;

y_{jk} - the value of j-th output for the k-th DMU;

ε - a small positive number called non-Archimedean quantity;

Variables:

δ_i, σ_j - slacks corresponding to input i, output j respectively (≥ 0);

v_i, u_j - virtual multipliers for input i, output j respectively $(\geq \varepsilon)$;

λ_k - weight of DMU_k in the facet for the evaluated DMU (≥ 0);

h_k - relative efficiency of DMU_k.

D. The CCR Model

This model proposed by Charnes A., Cooper W.W. and Rhodes E. [1], according to their model, for each DMU_k solve.

The objective here is to find the largest sum of weighted outputs of DMU_k while keeping the sum of its weighted inputs at unit value and forcing the ratio of the sum of weighted outputs to the sum of weighted inputs for any DMU to be less than one.

$$
\begin{aligned}
&Max \quad h_k = \sum_{j=1}^{s} u_j y_{jk} \\
&s.t. \quad \sum_{i=1}^{r} v_i x_{ik} = 1, \quad \text{for } k = 1,\dots,n \\
&\sum_{j=1}^{s} u_j y_{jk} - \sum_{i=1}^{r} v_i x_{ik} \le 0, \quad \text{for } k = 1,\dots,n \\
&v_i \ge \varepsilon > 0, \quad i = 1,\dots,r \\
&u_j \ge \varepsilon > 0, \quad j = 1,\dots,s
\end{aligned}
\tag{2}
$$

E. Fuzzy DEA Model

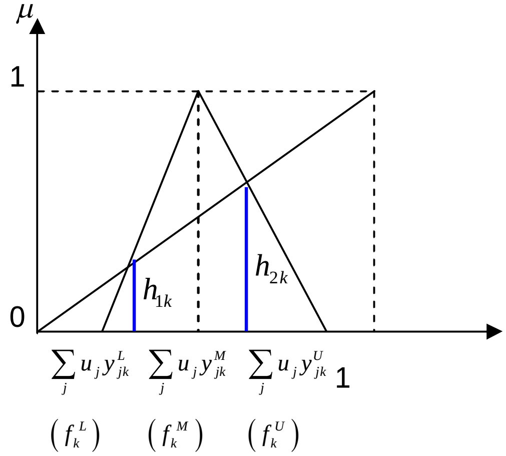
Considering the nature of fuzzy in real world problem, for example, the value of output identifying by triangular fuzzy number, the mathematics programming of fuzzy DEA based on CCR model described as:

$$
\begin{aligned}
&Max \quad \tilde{f}_k = \sum_{j=1}^{s} u_j \tilde{y}_{jk} \\
&s.t. \quad \sum_{i=1}^{r} v_i x_{ik} = 1, \quad \text{for } k = 1,\dots,n \\
&\sum_{j=1}^{s} u_j \tilde{y}_{jk} - \sum_{i=1}^{r} v_i x_{ik} \le 0, \quad \text{for } k = 1,\dots,n \\
&v_i \ge \varepsilon > 0, \quad i = 1,\dots,r \\
&u_j \ge \varepsilon > 0, \quad j = 1,\dots,s
\end{aligned}
\tag{3}
$$

This optimal value of objective function expressed by triangular fuzzy number, $\tilde{f}_k^* = \left(f_k^{L*}, f_k^{M*}, f_k^{U*} \right)$. We conduct the fuzzy objective value as follows for DMU_p with p=k

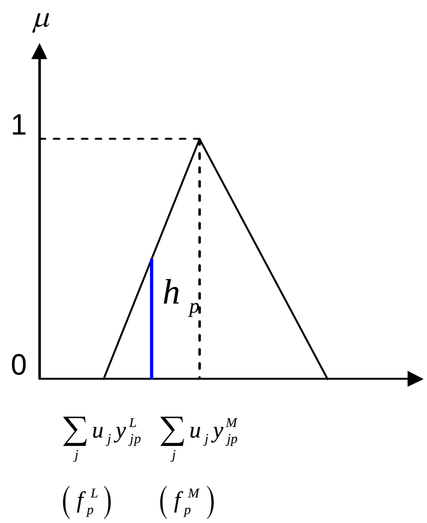
$$h_{1k}=\frac{\sum_{j=1}^{s}u_j y_{jk}^{L}}{1-\left(\sum_{j=1}^{s}u_j y_{jk}^{M}-\sum_{j=1}^{s}u_j y_{jk}^{L}\right)}$$

$$h_{2k}=\frac{\sum_{j=1}^{s}u_j y_{jk}^{U}}{1+\left(\sum_{j=1}^{s}u_j y_{jk}^{U}-\sum_{j=1}^{s}u_j y_{jk}^{M}\right)}$$



On the other hand, we conduct the fuzzy objective value as follows for DMU_p with $p \neq k$.

$$h_p=\frac{\sum_{i=1}^{r}v_i x_{ip}-\sum_{j=1}^{s}u_j y_{jk}^{L}}{\sum_{j=1}^{s}u_j y_{jp}^{M}-\sum_{j=1}^{s}u_j y_{jp}^{L}}$$



The mathematics programming of DEA with fuzzy output based on CCR model can then describe as below.

$$
\begin{aligned}
&Max \quad h_{1k} = \frac{\sum_{j=1}^{s} u_j y_{jk}^{L}}{1-\left(\sum_{j=1}^{s} u_j y_{jk}^{M} - \sum_{j=1}^{s} u_j y_{jk}^{L}\right)} \\
&Max \quad h_{2k} = \frac{\sum_{j=1}^{s} u_j y_{jk}^{U}}{1+\left(\sum_{j=1}^{s} u_j y_{jk}^{U} - \sum_{j=1}^{s} u_j y_{jk}^{M}\right)} \\
&Max \quad h_{p} = \frac{\sum_{i=1}^{r} v_i x_{ip} - \sum_{j=1}^{s} u_j y_{jk}^{L}}{\sum_{j=1}^{s} u_j y_{jp}^{M} - \sum_{j=1}^{s} u_j y_{jp}^{L}} \\
&s.t. \quad \sum_{i=1}^{r} v_i x_{ik} = 1, \quad \text{for } k = 1,\dots,n \\
&\qquad 0 \le h_{1k} \le 1,\ 0 \le h_{2k} \le 1, \quad \text{for } k = 1,\dots,n \\
&\qquad 0 \le h_{p} \le 1, \quad \text{for } p = 1,\dots,n;\ p \ne k \\
&\qquad v_i \ge \varepsilon > 0, \quad i = 1,\dots,r \\
&\qquad u_j \ge \varepsilon > 0, \quad j = 1,\dots,s
\end{aligned}
\tag{4}
$$

Because this programming is one kind of multiple objective linear programming, using max-min concept can then transfer as follows formulation,

$$
\begin{aligned}
&Max \quad \lambda_k \\
&s.t. \quad \sum_{i=1}^{r} v_i x_{ik} = 1, \quad \text{for } k = 1,\dots,n \\
&\qquad \sum_{j=1}^{s} u_j y_{jk}^{U} \ge \lambda_k \left(\sum_{j=1}^{s} u_j y_{jk}^{U} - \sum_{j=1}^{s} u_j y_{jk}^{M} + 1\right), \quad \text{for } k = 1,\dots,n \\
&\qquad \sum_{j=1}^{s} u_j y_{jk}^{L} \ge w \cdot \lambda_k \left(\sum_{j=1}^{s} u_j y_{jk}^{L} - \sum_{j=1}^{s} u_j y_{jk}^{M} + 1\right), \quad \text{for } k = 1,\dots,n \\
&\qquad \sum_{i=1}^{r} v_i x_{ip} - \sum_{j=1}^{s} u_j y_{jp}^{L} \ge \lambda_k \left(\sum_{j=1}^{s} u_j y_{jp}^{M} - \sum_{j=1}^{s} u_j y_{jp}^{L}\right), \quad \text{for } p = 1,\dots,n;\ p \ne k \\
&\qquad 0 \le \lambda_k \le 1, \quad \text{for } k = 1,\dots,n \\
&\qquad v_i \ge \varepsilon > 0, \quad i = 1,\dots,r \\
&\qquad u_j \ge \varepsilon > 0, \quad j = 1,\dots,s
\end{aligned}
\tag{5}
$$

F. Integration of Fuzzy DEA with goal programming

We change the maximum to minimum and then start to change the model to the goal programming. GP is an development form of liner programming but it is more than it, a functional model that can considering goals and estimate the deviation of goals and follow flexibility in decision making, we consider the first constrain as a goal which the decision maker want to reduce positive or negative deviation of it and continue as below(Equ 6):

$$\min\{-p_1\lambda_k + p_2(d_j^- + d_j^+)\}$$

$$\text{s.t.} \quad \sum_{i=1}^{r} v_i x_{ik} + d_j^- + d_j^+ = 1 \;,\; for\ k = 1, \dots, n \qquad (6)$$

$$\lambda_k\left(\sum_{j=1}^{s} u_j y_{jk}^U - \sum_{j=1}^{s} u_j y_{jk}^M + 1\right) - \sum_{j=1}^{s} u_j y_{jk}^U \le 0,\ \ for\ k = 1, \dots, n$$

$$\lambda_k\left(\sum_{j=1}^{s} u_j y_{jk}^L - \sum_{j=1}^{s} u_j y_{jk}^M + 1\right) - \sum_{j=1}^{s} u_j y_{jk}^L \le 0,\ \ for\ k = 1, \dots, n$$

$$\lambda_k\left(\sum_{j=1}^{s} u_j y_{jk}^M - \sum_{j=1}^{s} u_j y_{jk}^L + 1\right) - \left(\sum_{i=1}^{r} v_i x_{ik} - \sum_{j=1}^{s} u_j y_{jk}^L\right) \le 0,\ \ for\ P = 1, \dots, n; p \ne k$$

$$0 < \lambda_k < 1 \qquad fork = 1, \dots, n$$

$$v_i > 0 \qquad \text{i} = 1, \dots, \text{r}$$

$$u_j > 0 \qquad j = 1, \dots, s$$

Witch p_1 and p_2 are the variables that show the priority of the objectives;

The fuzzy DEA with considering GP is a model that covers all possible situations and can confirm with environmental and real problems and you can find it a complete method for measuring the efficiency of DMUs.

II. Conclusion

A key element of a GP model is the achievement function that shows a mathematical expression of the unwanted deviation variables that has a typical “less is better behavior”. and DEA witch introduced by Charnes, Cooper and Rhodes [1] is a ratio that define efficiency. So with considering the nature of real world problems and fuzzy numbers, for example, the value of output identifying by triangular fuzzy number, it seem to be important that considering this All and introduce a new method for responding this situation so we introduce a model for fuzzy numbers and mix it with Goal Programing for best result and reduce all unpleasant situation and be flexible for all kind of numbers.

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