The Effect of Out-of-Plane Flexural Stiffness of Boundary Frame on Buckling Stress Patterns of Steel Plate Shear Wall

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Abstract: - When buckling occurs in the infill plate of Steel Plate Shear Wall (SPSW), a diagonal tension field is formed through the plate. It seems that the lateral displacements of plate due to buckling may have a role in formation of the tension field. Meanwhile revealing this role, this paper investigates the influence of out-of-plane flexural stiffness of boundary members (i.e. beams and columns) on the buckling coefficients and the tension field behaviour of SPSW associated with the symmetric and anti-symmetric buckling modes. The linear buckling equations in the sense of von-Karman have been solved in conjunction with various boundary conditions, by using the Ritz method.

Key-Words: - Steel shear wall, Thin plate, Shear buckling, Symmetric, Anti-symmetric, Ritz method, Principal stresses

1 Introduction

The steel plate shear wall is a lateral load resisting system consisting of an infill plate located within a frame. While performing experimental tests on thin aluminium shear panels of an aircraft, Wagner [1] found out that in thin-webbed structures with stiff boundary members a diagonal tension field would be formed when buckling occurs. Then he developed the pure tension theory stating that the formation of the tension field is the primary mechanism for shear resistant. Later, Kuhn et al. [2] presented the incomplete tension field theory. On the basis of Kuhn's theory the shear resistance capacity is a combination of pure shear and inclined tension field. None of these theories represented any descriptions of the extent, uniformity or non-uniformity as well as the angle of inclination of the tension field and the affecting factors.

In order to assess response of SPSWs using conventional analysis softwares, Thorburn et al. [3] developed an analytical simple model termed the strip model in which the tension field behaviour of SPSW was simulated by a series of tension-only strips at the same angle of inclination simply suppured to a boundary frame. They derived the angle of inclination for the strips, θ, from the principle of Least Work as a function of axial stiffness of boundary members. Timler and Kulak [4] derived another equation for θ in terms of axial and in-plane flexural rigidities of surrounding members.

The Canadian Steel Design Standard [5] suggests the application of the strip model as a design tool for steel plate shear wall (CAN/CSA 516-01) and the equation derived by Timler and Kulak [4] for the calculation of θ (clause 20.3.1). However, researchers are still searching for an increase in the precision of the prediction of the overall behavior of the shear wall.

This paper investigates the effect of different parameters on buckling loads as well as on the distribution and orientation patterns of the tension field principal stresses. These parameters include out-of-plane flexural stiffness of boundary members as well as symmetric and anti-symmetric buckling modes.

2 Theory

2.1 Modelling of SPSW

The boundary members of the SPSW are modelled by the springs.

![Fig. 1. General scheme for a section of the model](image-url)

To define logical parameter for the amount of out-of-plane stiffness of boundary members, the non-dimensional stiffness parameter β is introduced as
follows:

$$\beta = \frac{K_{flx}}{D}$$  \hspace{1cm} (1)

where $K_{flx}$ is the unit length out-of-plane flexural stiffness of surrounding members and $D$ is the flexural rigidity of plate. A model is defined for studying the effect of the stiffness parameter $\beta$ shown in Fig. 1.

### 2.2 Ritz method

This paper utilizes the Ritz method to analyze the buckling of infill plate of a SPSW under an applied in-plane shear loading (Fig. 2).

$$U_s = \int \left[ \frac{K_{flx}}{2} \left( w_{,x}^2 \left|_{y=a}^{y=b} \right. + w_{,y}^2 \left|_{x=a}^{x=b} \right. \right) \right] \, dy$$

$$+ \int \left[ \frac{K_{flx}}{2} \left( w_{,x}^2 \left|_{y=a}^{y=b} \right. + w_{,y}^2 \left|_{x=a}^{x=b} \right. \right) \right] \, dx$$  \hspace{1cm} (5)

in which $D$ is the flexural rigidity of plate. In the use of the Ritz method, an appropriate displacement function for $w$ must be chosen. That used herein is the polynomial-based displacement function which consists of a boundary polynomial specifying the geometric and kinematic boundary conditions multiplied by a complete simple polynomial. This displacement function is written by:

$$w = \phi_{b_1}(\xi, \eta) \sum_{q=0}^{p} \sum_{r=0}^{q} a_{lm} \phi_{i_m}(\xi, \eta)$$

$$+ \phi_{b_2}(\xi) \sum_{n=1}^{l} a_{2n} \phi_{2n}(\xi) + \phi_{b_3}(\eta) \sum_{j=1}^{l} a_{3j} \phi_{3j}(\eta)$$  \hspace{1cm} (6)

where $p$ is the degree of a two-dimensional polynomial and $a_{lm}, a_{2n}, a_{3j}$ are the arbitrary Ritz coefficients. $\phi_{i_m}(\xi, \eta)$ is the m-th term of a two-dimensional polynomial as below [6]:

$$\phi_{i_m}(\xi, \eta) = \xi^m \eta^r$$  \hspace{1cm} (7)

in which $\xi = 2x/a, \eta = 2y/b$. The value of the m-th term is given by:

$$m = \frac{(q+1)(q+2)}{2} - r$$  \hspace{1cm} (8)

$\phi_{2n}(\xi)$ and $\phi_{3j}(\eta)$ are one-dimensional polynomials expressed by:

$$\phi_{2n}(\xi) = \xi^n, \hspace{1cm} \phi_{3j}(\eta) = \eta^j$$  \hspace{1cm} (9)

The number of degrees of freedom in the two-dimensional polynomial $\phi_{i_m}(\xi, \eta)$ is given by:

$$f_i = \frac{(p+1)(p+2)}{2}$$  \hspace{1cm} (10)

and also, $f_2$ and $f_3$ are the numbers of degrees of freedom for the one-dimensional polynomials $\phi_{2n}(\xi)$ and $\phi_{3j}(\eta)$ which can be arbitrarily chosen,
respectively. The terms \( \varphi_{b1}(\xi, \eta), \varphi_{b2}(\xi) \) and \( \varphi_{b3}(\eta) \) are the boundary polynomials describing the boundary conditions defined as below:

\[
\varphi_{b1}(\xi, \eta) = (\xi - 1)^1(\xi + 1)^1(\eta - 1)^1(\eta + 1)^1, \\
\varphi_{b2}(\xi) = (\xi - 1)(\xi + 1)^1, \\
\varphi_{b3}(\eta) = (\eta - 1)^2(\eta + 1)^2
\]  

(11)

In the buckling analysis, the kinematic and geometric boundary conditions are specified when the boundary polynomial \( \varphi_b(\xi, \eta) \) is multiplied by the corresponding internal interpolation polynomial.

2.3 Linear Eigenvalue Analysis

The total potential energy \( \Pi \) of the system is given by:

\[
\Pi = U + V_p
\]  

(12)

Based on the principle of minimum potential energy, the total potential \( \Pi \) in Equation (12) is minimized with respect to the unknown Ritz coefficient \( a_m \). Because \( \Pi \) is a function of the product of Ritz coefficients \( a_m a_n \), minimization by formal differentiation leads to a set of simultaneous linear independent equations. The solution of these equations produced the eigenvalues (buckling loads) and substituting of the corresponding eigenvectors into the displacement function \( \psi \) in Equation (6) as the Ritz coefficients gives the buckling modes.

2.4 Stress Analysis

Since the buckling modes of a plate specify the proportional values of transverse deflections, the corresponding values of strains and stresses will be calculated proportionally. Using the transverse deflection \( w \), the stresses in the mid-plane of plate can be written by:

\[
\sigma_x = \frac{E}{2} \left( w_{,x}^2 + v w_{,y}^2 \right) \\
\sigma_y = \frac{E}{2} \left( w_{,y}^2 + v w_{,x}^2 \right) \\
\tau_{xy} = G w_{,x} w_{,y}
\]  

(13)

where \( G \) is the shear modulus of elasticity. Using the Mohr’s circle, the state of stresses can be represented in the principal coordinates. Also the angle of inclination of the tension field can be calculated by determining the orientation of the principal stresses. Then, it is possible to plot the distribution and orientation patterns of the principal stresses in the tension field of plate.

3 Numerical Parametric Studies

3.1 Shear Buckling Analysis

A computer program has been developed based on the von-Karman theory and the Ritz method. The numerical analyses were performed by the computer program. In these buckling analyses, the value of \( p \) was selected equal to 8. To compare the various buckling analyses, the non-dimensional buckling coefficient was employed as follows:

\[
k_s = \frac{N_{xy} b^2}{\pi^2 D}
\]  

(14)

By plotting the various buckling mode shapes, it will be specified which modes are symmetric or anti-symmetric. Typically, the symmetric and anti-symmetric buckling modes of plate are depicted in three-dimension views (Fig. 3).

![Fig. 3. Symmetric (a) and anti-symmetric (b) shear buckling modes for plate with aspect ratio 3](image-url)

The “first” symmetric and anti-symmetric modes are corresponding with the minimum values of the symmetric and anti-symmetric buckling loads, respectively. However, in this paper the word “first” is omitted for brevity.

On the purpose of verifying the validity of buckling analyses, the results are compared with the available references. So, the stiffness of spring is selected infinite for modelling simply support edges (S). In Table 1, the results from the present analyses are compared with
those reported in references. As seen, the results are in good agreement.

Table 1. Comparison the present results with available reference

<table>
<thead>
<tr>
<th>β = \frac{K_{\text{fc}}}{D}</th>
<th>Boundary Condition</th>
<th>a/b</th>
<th>Present Symmetric</th>
<th>Present Anti-symmetric</th>
<th>Timoshenko</th>
</tr>
</thead>
<tbody>
<tr>
<td>\beta = \infty</td>
<td>SSSS</td>
<td>1.0</td>
<td>9.3245</td>
<td>11.5424</td>
<td>9.34</td>
</tr>
<tr>
<td></td>
<td></td>
<td>1.5</td>
<td>7.0701</td>
<td>7.9563</td>
<td>7.10</td>
</tr>
<tr>
<td></td>
<td></td>
<td>2.0</td>
<td>6.5458</td>
<td>6.5767</td>
<td>6.60</td>
</tr>
<tr>
<td></td>
<td></td>
<td>3.0</td>
<td>5.9531</td>
<td>5.8460</td>
<td>5.90</td>
</tr>
</tbody>
</table>

Fig. 4. Shear buckling coefficient v.s. \( \beta \) for the model (line for symmetric and dashed for anti-symmetric buckling)

Fig. 4 shows the effect of varying the stiffness parameter \( \beta \) on the symmetric and anti-symmetric buckling coefficients of plate. The following results can be concluded by attending to these figures:

- The symmetric and anti-symmetric buckling coefficients of a plate with aspect ratio equal or greater than 1.5 are close together.
- For plates with aspect ratios close to 1, the anti-symmetric mode may be critical.
- The variation of amounts of the stiffness parameter \( \beta \) may change the shear buckling mode of plate.

3.2 Stress Analysis

3.2.1 Principal Stress Distribution Pattern (PSDP)

By comparing the PSDPs with the corresponding buckling modes, the areas where the amounts of principal stresses are peak, may be specified. Fig. 5 illustrates these comparisons for two extreme values of zero and infinite for the stiffness parameter \( \beta \). This figure shows that the peak(s) of principal stresses occurs at the slope(s) of buckling mode shapes for both symmetric and anti-symmetric modes. Therefore, in symmetric buckling modes, the principal stresses peaks are being at both sides of the plate centre, while in anti-
<table>
<thead>
<tr>
<th></th>
<th>Symmetric buckling mode</th>
<th>Distribution of principal stresses</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Zero $\beta$</strong></td>
<td><img src="image" alt="Symmetric buckling mode" /></td>
<td><img src="image" alt="Distribution of principal stresses" /></td>
</tr>
<tr>
<td><strong>Infinite $\beta$</strong></td>
<td><img src="image" alt="Symmetric buckling mode" /></td>
<td><img src="image" alt="Distribution of principal stresses" /></td>
</tr>
<tr>
<td><strong>Anti-symmetric buckling mode</strong></td>
<td><img src="image" alt="Symmetric buckling mode" /></td>
<td><img src="image" alt="Distribution of principal stresses" /></td>
</tr>
<tr>
<td><strong>Zero $\beta$</strong></td>
<td><img src="image" alt="Symmetric buckling mode" /></td>
<td><img src="image" alt="Distribution of principal stresses" /></td>
</tr>
<tr>
<td><strong>Infinite $\beta$</strong></td>
<td><img src="image" alt="Symmetric buckling mode" /></td>
<td><img src="image" alt="Distribution of principal stresses" /></td>
</tr>
</tbody>
</table>

Fig. 10. Symmetric and anti-symmetric buckling mode shapes and PSDPs for two extremes of $\beta = 0$ and $\beta = \infty$ (Aspect ratio 1.5).

For symmetric buckling this peak would be in centre of the plate. Also, Fig. 5 shows that the PSDPs are symmetric for both symmetric and anti-symmetric buckling modes.

**3.2.2 Principal Stress Orientation Pattern (PSOP)**

For showing some patterns simultaneously, it is advantageous that the patterns are putted together and combined as shown in Fig. 6. The orientations of principal stresses can be determined at each point of the plate by using the Mohr’s circle. Fig. 7 shows the combined PSOPs related to various values of stiffness parameters $\beta$. In this figure, the orientation of each depicted line represents the orientation of the related principal stress. By careful observation, it is realized that, there are areas in the plate where the orientations of related principal stresses will not be changed by varying the value of the stiffness parameter $\beta$. These areas of the plate in symmetric buckling are more extended than those in anti-symmetric buckling.

Fig. 6. Scheme for combination of some patterns
4 Conclusions

The observations reveal that the variation of the out-of-plane flexural stiffness of boundary members may change the critical shear buckling mode of plate; moreover, although the critical shear buckling mode of plate is often symmetric, sometimes the anti-symmetric mode would be critical.

Based on this research, the symmetric buckling load of a plate with an aspect ratio equal or greater than 1.5 is close to that for its anti-symmetric mode. For such a plate, the postbuckling mode may be a function of its initial imperfection; because initial imperfection of plates due to their fabrication causes that the plates do not experience the buckling bifurcation point and hence initial imperfections may play a role in determining which postbuckling modes of symmetric or anti-symmetric would occur. This role would be significant for plates with close symmetric and anti-symmetric buckling loads. As a result, it seems that the initial imperfection of plate should be considered as a parameter in developing the strip model, especially for a plate with aspect ratio equal or greater than 1.5. This parameter has not been included in any analytical models presented so far.

It is also shown that the orientation patterns of principal stresses corresponding to the symmetric and anti-symmetric buckling modes of a plate are different, relatively. Since the angle of inclination of the tension field of a SPSW is an effective parameter on development of the strip model, this result may be vital in modifying the strip model.

References:


