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Development of A Multi-objective Robust Probabilistic Programming Mathematical Model Using Improved ε - Constrained Method in Gas Distribution Network Projects Implementation

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A B S T R A C T

Optimizing time, cost, quality and risk are the four main factors in gas project management. By considering the significance of gas projects in the country's gas development as one of the top sources of revenue and economic advantages, efficient and targeted management in gas distribution network projects implementation is believed essential. The objective of this research is to develop a multi-objective robust probabilistic programming model using the improved ε - constraint method while adhering to the objectives of minimizing time, cost and risk functions and maximizing the expected quality function. To this end, a sample size of 36 experts and specialists in the gas industry has been selected. The results of this research enable gas project managers to select the best solution among various scenarios based on effective management with different values between the four objective functions of time, cost, quality and risk, thus achieving strong optimization. Additionally, the results indicate that the two objective functions derived from the Improved ε - constraint method have 99% confidence in better exploring the problem-solving space in producing superior solutions and Pareto charts of the risk and quality objective functions exhibit the optimal solution. The findings demonstrate the accuracy of the model and the efficiency of the proposed method.

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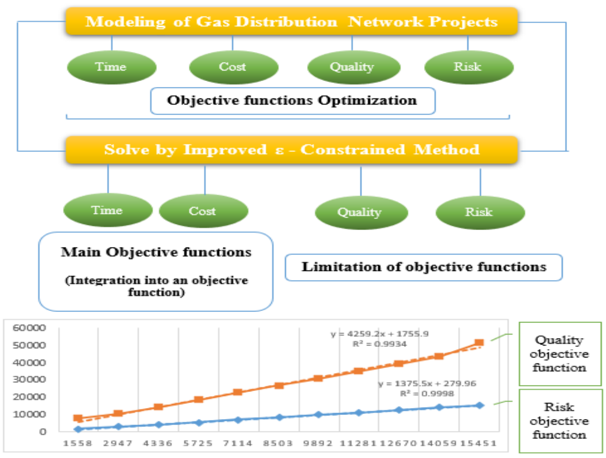
Graphical Abstract



Comparative Pareto chart of the two obtained objective functions

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1. INTRODUCTION

In project-oriented organizations today, the execution and completion of plans and large projects in the gas industry, considering the competitive economic conditions and the importance of gas projects in the country's development, are considered essential in both project implementation and project operation and maintenance in gas distribution network projects (1, 2). Project management and project planning and control are carried out through the method of exchange to determine and manage interdependent and simultaneous activities and resource allocation within a network called a project. This method aims to determine the optimal amount and balance between time, cost and quality of a project, which should be allocated to the project's activities to achieve the best outcome (3-5).

Gas project managers often encounter multi-objective or multi-criteria projects in gas distribution network projects implementation. The objective of project managers is to complete their projects in the shortest possible time, with the lowest cost and the highest possible quality (5, 6). Often, some projects fail to achieve the desired time, budget, and quality. The main reason for these deviations is the occurrence of risks during project execution. Therefore, it is necessary to identify, assess, and respond to the risks that gas projects may encounter, to minimize negative risks and various detrimental consequences in projects. On the other hand, ignoring the risks of project failure can result in additional costs or time increases. Therefore, the risk factor must also be considered (7-9). From an effective management perspective, given the complex and variable environments in projects and the necessity of appropriate planning, especially in gas projects, gas project managers strive to simultaneously consider the four project management criteria, including time, cost, quality, and risk, in successful project execution. They aim to select the best path and solution among several possible scenarios to achieve strong optimization (10, 11).

In general, the necessity of conducting this research can be attributed to the effectiveness, strengthening, and improvement, as well as informed decision-making and strong planning in gas distribution network projects implementation, to select the best possible scenario among the four criteria of time, cost, quality and risk in the gas industry.

Therefore, considering the challenges encountered in the implementation of large gas projects and by executing companies, including constraints on executive and physical resources, full supervision of operational and executive sections for coordination and effective communication between client and contractor sections, uncertainty and lack of sufficient information availability, the existence of different methods in design, implementation and operation and failure to adhere to the limits of time, cost, quality, and risk in projects, as well as the importance and necessity of implementing such projects in the gas industry, it is necessary to devise management measures to address these challenges and increase efficiency, strengthen, improve and facilitate the performance of project activities. By making appropriate, systematic, and goal-oriented decisions and planning, systematic optimization of project management can lead to the successful implementation of projects. The present study aims to optimize and model the relationship between the four factors of time, cost, quality, and risk to minimize or compress the duration, executive and operational costs, project risks and maximize or enhance the expected level of quality in gas distribution network. The projects implementation through the development of a multi-objective robust probabilistic programming mathematical model using the ε -constrained method. The main structure of the present research is summarized as follows: the first section introduces the issue under study and the second section addresses the literature review. The third and fourth sections describe the problem and present the mathematical model, along with the practical findings of the research. Finally, the last section is dedicated to the results, discussion, and conclusion.

2. LITERATURE REVIEW

Due to the importance of the research topic, several related research studies have been conducted in the field of construction management. For example, Dulebenets (12) discusses optimizing a mathematical model for scheduling activities at marine terminals, considering the handling resources available at the terminal. This is achieved through the development of the diffused memetic optimizer algorithm (DMOA). Additionally, hybridization techniques are employed to ensure precise optimization and an ideal solution search space. Haghighi et al. (13)explored an optimization model for balancing cost, risk and quality in oil and gas projects under uncertain conditions. Their research involved developing a fuzzy ε-constraint method to generate optimal solutions along the Pareto. The findings indicate that this proposed method empowers managers to effectively navigate energy projects within uncertain environments. Son and Khoi (4) focused on finding operational and practical solutions and the ability to optimize civil engineering projects. This research, using the Slime Mold Algorithm, compares the results with five other metaheuristic algorithms (OMOSOS, MOABC, MODE, MOPSO and NSGA-II). Elkliny et al. (14) in their research in the field of construction projects, discuss balancing the three factors of time, cost and quality considering resource constraints. They introduce an optimization model for construction resource management to minimize project time and cost and maximize project quality. This research is conducted by using the genetic algorithm. Chen and Tan (15) in the research, self-adaptive fast fireworks algorithm (SF-FWA) proposed to effectively conduct large-scale black-box optimization in order to computationally efficient search distributions to cope with different function landscapes while tuning the hyperparameters of the search distributions in an online fashion. To achieve this, the expressive fast explosion (EFE) mechanism is designed to achieve effective and efficient sampling, and the inter-fireworks competitive cooperation (IFCC) mechanism is designed to adapt hyperparameter distributions. Eirgash et al. (16) in their study focus on optimizing and balancing simultaneously between the three criteria of time, cost and quality in civil projects by using the generalized differential evolution algorithm MDOLTLBO. Nguyen et al. (3) in their article introduce a fuzzy logic approach for balancing and optimizing the three factors of time, cost and quality in civil and construction projects using the co-evolutionary organism search algorithm. Safaeian et al. (17) introduced a novel tri-objective optimization model aimed at minimizing total travel time, total carbon dioxide emissions produced during transportation and total delay in reaching designated destinations for decision-making related to routing and scheduling within a ridesharing system. This study employs a multi-objective red deer algorithm to intelligently find efficient pareto solutions. Haghighi et al. (18) have introduced a new fuzzy Bayesian network-based approach for optimizing the three factors of time, cost and quality in projects considering the conditions of uncertainty. Singh and Pillay (19) in this research, expands to the existing body of research by presenting a novel ant-based generation constructive hyper-heuristic and then investigates how different pheromone maps affect its performance. The analysis indicated that the different pheromone maps work most optimally for different types of optimization problems. Masoomi et al. (20) in their study for selecting a strategic energy supplier in the renewable energy supply chain under green conditions, utilized the fuzzy BWM, VIKOR and COPRAS approaches to optimize resource consumption rates and reduce negative environmental impacts. Dulebenets (21) proposed the adaptive polyploid memetic algorithm (APMA) to improve the quality and solve the scheduling problem for inbound and outbound trucks at a cross-docking terminal (CAT). This approach can significantly aid in planning CAT operations. The research also employs hybridization techniques that directly consider the specific characteristics of the problem. In other study conducted by Celik and Gul (22) a risk management approach for building a safe dam and optimal use of water resources and hydroelectric power generation using the BWM and MARCOS approach is presented. Lopez-Ospina et al. (23) addressed the optimization of location and transportation to minimize costs and increase service quality indices to consumers using a repetitive combinatorial optimization model based on genetic algorithms and muti-objective constraints. Jeunet and Orm (24) in their article discuss optimizing and simultaneous balancing between the three main criteria of project management: time, cost and quality as a case study of manufacturing projects in France, conducted using MILP. Bischiniotis et al. (25) introduced an evaluation of time, cost and quality based on risk management actions and predicting flood reduction effects for protective strategies using the EWEAS method. Moradi et al. (26) aimed to provide a strong programming model with limited resources for projects and contractors under uncertain conditions in large and complex construction projects to minimize time and cost. Rahmanniyay et al. (27) in their study referred to optimizing cost and quality in multi-period planning with uncertain parameters in resource allocation by using a multi-stage stochastic programming model. Pérez-Cañedo et al. (28) in a research linear ranking functions are used to transform fuzzy multi-objective linear programming (MOLP) problems into crisp ones. In this study used of an ε -constraint method for fully fuzzy multi-objective linear programming. Khalil et al. (29) to a multi-objective optimisation approach with improved pareto-optimal solutions to enhance economic and environmental dispatch in power systems will pay with the method particle swarm optimization (PSO) meta-heuristic algorithm. Pishvaee et al. (30) introduced the aim of their research as minimizing costs and increasing efficiency and performance of social responsibilities in private and government organizations through the introduction of a robust probabilistic programming model. In other studies, Nikas et al. (31), Shojatalab et al. (32), Shadkam et al. (33) and Davoodi et al. (34) have solved a multi-objective optimization model with the types of meta-heuristic algorithms and developed or augmented ε- constraint method.

A review of the relevant theoretical literature shows that in most studies mentioned above, research has been conducted in the field of optimization and balancing between the three factors of time, cost and quality; however, research simultaneously optimizing and balancing the four project management factors of time, cost, quality and risk in gas distribution network projects implementation has not been observed. Finally, by considering the objectives introduced in this research, it focuses on introducing the development of a multi-objective robust probabilistic programming mathematical model and solving it by using the ε -constrained method.

3. PROBLEM DESCRIPTION AND MATHEMATICAL MODEL

3. 1. Assumptions of the Problem

The assumptions of the mathematical model according to the

degree of simplification of the real world (35) in this study are as follows:
- Utilization of real-world data and numerical examples in various dimensions.
- Examination of objectives in both quantitative and qualitative dimensions.
- Project scheduling.
- Completion of all projects within the specified time frame.
- Execution of activities in three states: earliest possible time, latest possible time and normal time.
- Optimization of time, cost, quality, and risk of gas distribution network projects in different periods.

3. 2. Mathematical Model The indices, parameters, and variables used in the mathematical model of this research problem are described as follows:

Indices:

J	Defined gas distribution network projects $j = 1,2,\dots,N, j \in J$
T	planning horizon (will include period T) $t = 1,2,\dots,T$
J_i	The i phase of the gas distribution network of the j total project $i = 1,2,\dots,n$
Z	Problem objectives $z = 1,2,\dots,Z$
P	Prerequisite projects for implementation $p \in S_p$
Q	Interdependent projects for implementation $q \in S_d$
e,f	Single track projects for gas distribution network $e, f \in j$
a,b	Multi-route projects to select the gas distribution network $a, b \in j$
m	Project implementation status (earliest time, latest time and normal time) $m \in l, m, u$

Parameters:

C_{jm}	Fixed costs of construction and gas distribution network of project j in the state of m
C j, L-t+1,m	The level of unanticipated operational costs required for the construction of project j in period L in the state of m
Ccapexjm	Implementation costs of project j in each period in the state of m
Copexjm	Operating costs of project j in each period in the state of m
W'_{jim}	The costs of each day of the implementation of the multi-route gas distribution network of phase i of project j in the state of m
W_{jim}	The costs of each day of the implementation of the single-route gas distribution network of phase i of project j in the state of m
R_{jtm}	The risk of project j in period t in the state of m
Q_{jtm}	The quality of implementation of project j in period t in the state of m
d_{jm}	The duration of time required to complete project j in the state of m
T_{qm}	The construction initiation duration of interdependent projects (to carry out interdependent projects) in the state of m
d_{qm}	The construction duration of interdependent projects (to carry out interdependent projects) in the state of m
B_L	Funds available for the construction of projects at the beginning of the L period
S_m	Mandatory projects of the gas company that must be implemented in the program
S_p	A set of prerequisite projects
S_d	A set of dependent projects
S_j	Prerequisite projects of project j ($S_j \in S_p$)
D_{ji}	Projects directly related to stage i of project j ($D_{ij} \in S_d$)

Variables:

$x_{j,t,m} = 1$	Choosing project j in period i in the state of m
$x_{j,t,m} = 0$	Not choosing project j in period i in the state of m
T_{jm}	The construction initiation duration of project j in the state of m

The mathematical model is presented as follows:

$$\text{Min } Z1 = \sum_{j=1}^{N}\sum_{t=1}^{T}\sum_{m} x_{j,t,m} * \left(\sum_{i=1}^{n}((W'_{jim})(T - (\sum_{t=1}^{T}(T_{qm} + d_{qm}) . x_{qtm}) - (T_{jm} + d_{jm})) + (\sum_{i=1}^{n}((W_{jim})(T - (T_{jm} + d_{jm})))\right) \quad (1)$$

$$Min\ Z2 = \sum_{j=1}^{N}\sum_{t=1}^{T}\sum_{m} x_{j,t,m}\ (Ccapex_{jm} + Copex_{jm}\left(T - (T_{jm} + d_{jm})\right)) \quad (2)$$

$$Min\ Z3 = \sum_{j=1}^{N}\sum_{t=1}^{T}\sum_{m} x_{j,t,m}\ . R_{jtm} \quad (3)$$

$$Max\ Z4 = \sum_{j=1}^{N}\sum_{t=1}^{T}\sum_{m} x_{j,t,m} \quad (4)$$

Subject to:

$$\sum_{t=1}^{T}\sum_{m} x_{j,t,m} \le 1 \qquad \forall j \quad (5)$$

$$\sum_{t=1}^{T}\sum_{m_T} x_{j,t,m} = 1 \qquad \forall j \epsilon Sm \quad (6)$$

$$\left(\sum_{m}\sum_{t=1}^{T} t \cdot x_{j,t,m}\right) - 1 \le T_{jm} < \sum_{m}\sum_{t=1}^{T} t \cdot x_{j,t,m} \qquad \forall j x_{j,t,m} \ne 0 \quad (7)$$

$$\sum_{m}\sum_{t=1}^{T} x_{j,t,m}(T_{jm} + d_{jm}) \le T \qquad \forall j \quad (8)$$

$$\sum_{j=1}^{N}\sum_{t=1}^{T}\sum_{m} x_{j,t,m} \cdot C_{L-t+1} \le B_L \qquad \forall L = 1,2,\dots,T \quad (9)$$

$$\sum_{I\epsilon Sj}\sum_{t=1}^{T}\sum_{m} x_{j,t,m} \ge |Sj|\sum_{t=1}^{T}\sum_{m} x_{jtm} \qquad \forall j \epsilon Sd \quad (10)$$

$$\sum_{t=1}^{T}\sum_{m} x_{j,t,m} + x_{f,t,m} \le 1 \qquad \forall e, f \quad (11)$$

$$\sum^{T}\sum x_{a,t,m} - x_{b,t,m} = 0 \qquad \forall a, b \quad (12)$$

$$Mj = Max[\sum_{t=1}^{T} \sum_m x_{j,t,m}(T_{jm} + d_{jm}) \quad for\ peSj] \quad \forall j \epsilon Sd \quad (13)$$

$$\sum_{t=1}^{T} \sum_m x_{j,t,m}(T_{jm} + d_{jm}) \geq Mj \quad \forall j \epsilon Sd$$

$$x_{j,t,m} \in [0,1], Integer\ Numbers \quad T_{jm} \in N \quad (14)$$

The objective Functions 1, 2 and 3 indicate the minimization of time, cost and risk in gas distribution network projects implementation, while objective Function 4 represents maximizing the quality of projects. Constraint 5 ensures that each project can be selected at most once during period (T). Constraint 6 mandates the selection of projects by the end of period (T), known as mandatory single-path projects. Constraint 7 denotes the start time of each project, represented by (T_j) during period ($X_{j,t,m}$). Constraint 8 guarantees the completion of all selected projects by the end of period (T). Constraint 9 relates to the available capital for implementation gas distribution network projects in each period. Constraint 10 includes respecting the prerequisites and dependencies of some projects, where a dependent project occurs when all its prerequisites are fulfilled. Constraint 11 indicates non-parallel (single-path) projects. Constraint 12 ensures parallel (multi-path) projects, where selecting any of these projects requires them to be chosen during period (T). Constraint 13 signifies the completion of constructing dependent units after the completion of all their prerequisite projects. Constraint 14 represents the variable ($X_{j,t,m}$), which is binary, and the variable (T_{jm}) is a subset of natural numbers.

Therefore, to address issues and ambiguities such as the lack of guarantee of reliability of final values at confidence levels of probabilistic constraints, increasing the number of required experiments to determine appropriate confidence level values, imposing significant costs from the unintended outcome of the focal response and deviations from probabilistic constraints and imposing high risks on decision-makers regarding deviations of the objective function from the expected value, a robust probabilistic programming optimization model is established by considering the two concepts of justifiability and efficiency robustness. This approach was proposed by Pishvaee et al. (30).

In the above-presented mathematical model, some parameters are inherently uncertain., such as (T), (B_l) (D_{jm})and (T_{jm}), which pertain to time and budget and are subject to uncertainty. This is because various factors influence the time required for activities and the budget, some of which are identifiable and impactful, while others are unidentifiable (such as project execution conditions, material quality, etc.). Therefore, to address the uncertainty of these influential parameters and to represent uncertain parameters in the presented model, a trapezoidal possibility distribution is used. Furthermore, the expected value for the trapezoidal fuzzy number f is calculated based on Equation 15. Subsequently, the robust probabilistic programming model with chance constraints involving probabilistic parameters (λ), (φ), (γ) and (ζ) (representing the minimum confidence level for satisfying the chance constraints) is implemented.

$$E[f] = Expected\ value = \frac{f_1+f_2+f_3+f_4}{4} \quad (15)$$

Finally, the developed final model of the robust probabilistic programming optimization approach, considering the presented mathematical model, is expressed as follows:

$$\text{Min} \quad Z_1: \quad E[O] + \eta(Z1_{MAX} - E[O]) + \tau \sum_h \sum_k [T_{jm} - (1-\gamma)T_{jm} - \gamma T_{jm})] + \delta \sum_h \sum_k [(\widetilde{T_{jm}} + \widetilde{d_{jm}}) - (1-\lambda)(\widetilde{T_{jm}} + \widetilde{d_{jm}}) - \lambda(\widetilde{T_{jm}} + \widetilde{d_{jm}}))] + \upsilon \sum_h \sum_k [(B_L - (1-\varphi)B_L - \varphi B_L)] + \rho \sum_h \sum_k [(Mj - (1-\zeta)Mj - \zeta Mj)] \quad (16)$$

Subject to:

$$\left(\sum_m \sum_{t=1}^{T} \tilde{t} \,.\, x_{j,t,m} \right) - 1 \leq \widetilde{T_{jm}} < \sum_m \sum_{t=1}^{T} \tilde{t} \,.\, x_{j,t,m} \leq (1-\gamma)T_{jm} + \gamma T_{jm} \quad \forall j \quad (17)$$

$$\sum_m \sum_{t=1}^{T} x_{j,t,m} \leq M * [(1-\lambda)(\widetilde{T_{jm}} + \widetilde{d_{jm}}) \ \tilde{T} + \lambda(\widetilde{T_{jm}} + \widetilde{d_{jm}}) \ \tilde{T}\ ] \quad \forall j \quad (18)$$

$$\sum_{j=1}^{N} \sum_{t=1}^{T} \sum_m x_{j,t,m} \,.\, C_{L-t+1} \geq (1-\varphi)\widetilde{B_L} + \varphi \widetilde{B_L} \quad \forall\ l \quad (19)$$

$$\sum_{t=1}^{T} \sum_m x_{j,t,m} \left(\widetilde{T_{jm}} + \widetilde{d_{jm}}\right) \leq M * [(1-\zeta)Mj + \zeta Mj] \quad \forall j \quad (20)$$

The objective function of the presented model minimizes the average total cost. Additionally, it pertains to robustness, which represents the difference between the maximum possible value of the objective function (Z1) and its average value (E[O_1]) with a given degree of importance (**η**). This serves as a criterion for interpretability and effectiveness in accommodating uncertainties in the objective function (Z1) with non-deterministic parameters. ($Z1_{MAX}$) is defined as follows:

$$Z1_{MAX} = \sum_{j=1}^{N} \sum_{t=1}^{T} \sum_m x_{j,t,m} * (\sum_{i=1}^{n}((W'_{jim})(T - (\sum_{t=1}^{T}(T_{qm} + d_{qm}) \,.\, x_{q,t,m}) - (T_{jm} + d_{jm})) + (\sum_{i=1}^{n}((W_{jim})(T - (T_{jm} + d_{jm}))) \quad (21)$$

This model ensures that the objective function only restricts positive deviations, meaning deviations related to values of the objective function greater than its average value are included. Additionally, in the objective function of the above model, it controls the attainable robustness of the response and minimizes the difference between the worst-case value of non-deterministic parameters and their selected value in chance constraints. The values of (δ), (υ), (τ) and (ρ), along with the penalty for each violation of chance constraints, can be properly determined based on the nature of the problem.

3. 3. The Enhanced ε -Constrained Method

There are various techniques for solving multi-objective problems, one of which is the ε -constraint method. The ε -constraint method can be considered as one of the multi-objective optimization techniques based on robust mathematical modeling and generates a set of non-dominated solutions. This method transforms optimization problems into several single objective optimization subproblems, each solution of which can be considered as one of the non-dominated solutions of the overall problem.

The ε -constraint method was first introduced by Haimes (36). The main idea of this method is to initially select the multiple objectives with the highest priority as the main objective function in optimization problems, while the remaining objectives are treated as constraints, considering upper and lower bounds. Consequently, all possible pareto solutions for multi-objective optimization problems may be generated by shifting the constraints of these objectives from their upper bounds towards their lower bounds and iteratively solving the problem. In this method, one of the objective functions is considered as the main objective function and the other objective functions are applied to the problem as constraints. In other words, the main advantage of this method over other multi-objective optimization techniques is its applicability to non-convex solution spaces. This is because methods such as the weighted sum of objectives lose their efficiency in non-convex spaces (36-38).

Various developments have been proposed to enhance the efficiency of the ε -constrained method, among which the enhanced ε -constrained method developed by Mavrotas (38) and Copado-Méndez et al. (39) is notable. The enhanced ε -constrained method comprises six steps as follows:

- **Step 1:** Select one of the objective functions as the primary objective function.
- **Step 2:** Solve the problem each time considering one of the objective functions and obtain the optimal value for each objective function.
- **Step 3:** In this method, using the lexicographic method, the best and worst values for each objective function are obtained. The best value of the first objective function is equal to its optimum value in the optimization problem considering the objective function individually. Then, by optimizing the second objective function with the constraint that the first objective function remains at its optimum value, the worst value of the second objective function is determined. This process continues until all objective functions are optimized and thus the range of each objective function is determined.
- **Step 4:** The interval between the two optimal values of the subsidiary functions is divided into a predetermined number (q_i) of parts and a table of values for the ε is obtained.

$$\varepsilon_i^k = f_i^{max} - \frac{ri}{qi} * k \qquad k=0, 1, \dots, q_i \tag{22}$$

- **Step 5:** Each time considering each of the ε values, the problem is solved with the primary objective function. In this way, the constraints related to the subsidiary objective functions are transformed into equality constraints using appropriate surplus or deficit variables. By considering 10^{-3} to 10^{-6} for these surplus or deficit variables, the problem is solved and feasible solutions are generated. The new problem is defined as follows:

$$\begin{aligned} &\text{Min } \{f1\ (x) + \delta^* (s_2 + s_3 + \dots + s_p)\} \\ &F_2\ (x) = \varepsilon_2 + s_2 \\ &F_3\ (x) = \varepsilon_3 + s_3 \\ &\dots \\ &F_p\ (x) = \varepsilon_p + s_p \\ &x \in X,\ s_i \in R^+ \end{aligned} \tag{23}$$

- Step Six: Finally, the obtained pareto solutions are reported (38, 40).

4. RESEARCH FINDINGS

In this section, by considering the developed multi-objective robust probabilistic programming optimization model, the evaluation and validation of the model are performed by using the enhanced ε -constrained method. To fully understand the execution process of the research stages, the following steps were taken sequentially: input data presentation, solving the model by using the enhanced ε -constrained method and plotting the pareto diagrams in this study.

4. 1. Inputs of the Robust Probabilistic Programming Mathematical Programming Model and its Analysis

In this section, to evaluate the performance of the final robust probabilistic programming optimization mathematical model, the proposed mathematical model is supplied with real data from a number of gas distribution network projects. Table 1 summarized the inputs of mathematical model. After solving the mathematical model by using the GAMZ software, the results are presented as follows.

It is worth mentioning that since the mathematical model is multi-objective, the mathematical model is first transformed into a single-objective form using the ε -constraint method. Then cuts are applied to the model for implementation.

4. 2. Solving the Enhanced ε -Constrained Method Along with a Numerical Example

Since the mathematical model presented in this study has four primary objective functions and considering the use of

TABLE 1. Inputs of the Mathematical Model

Index	Parameter Description	Uniform Distribution Function
C_{jm}	Fixed costs of construction and gas distribution network of project j in state m	Uniform [5-15]
W_{jim}	The costs of each day of the implementation of the one-way gas distribution network of phase i of project j in state m	Uniform [10-20]
W'_{jim}	Costs resulting from each day of the implementation of the multi-route gas distribution network of phase i of project j in state m	Uniform [1000-2000]

the enhanced ε -constrained method, objective Functions 1 and 2, representing time and cost, have been selected as the main objective function (these two objective functions are quantitative and are merged together and considered as one objective function). The other two objective Functions (3 and 4) are treated as constraints under the names of the first (risk) and second (quality) objective functions and finally, efficient pareto-optimal solutions are recorded.

According to Equation 23, using the enhanced ε -constrained method, optimal pareto solutions are obtained. In this equation, (r_i) represents the domain of objective function i (the length of the objective function's variation range), (ϑ) is a small number between 0.001 and 0.000001 (ϑ indicates relative error or tolerable error) and (S_i) is a non-negative auxiliary variable. It should be noted that as the error value decreases, more time is required to solve the model.

First, the value of (NIS_{fi}) is obtained as the worst value and (PIS_{fi}) as the best value for each objective function. Then, the domain value of objective function i is calculated according to the following equation:

$$r_i = PIS_{fi} - NIS_{fi} \quad (24)$$

is worth mentioning that in this method, the model needs to be solved for all obtained ε values. According to Equation 25, (η) represents the number of grid points obtained.

$$\varepsilon_i^{\eta} = NIS_{fi} + \frac{r_i}{l_i} * \eta \quad (25)$$

Then, using the following equation, the values of ε are obtained.

$$\varepsilon_i^{\eta} = \mathrm{PIS}_{fi} + \frac{r_i}{l_i} * \eta \quad (26)$$

After coding in the GAMZ software and replacing the value of (l_i) with the number 11 and the value of (ϑ) with the number 0.001, the results are calculated and presented based on the values of each of the variables and the obtained ε values, considering the set of pareto optimal solutions as follows.

Considering the 11 ε points in Table 2, the model has found optimal values for all points as solutions in the two objective functions created, as shown in pareto Charts (1 and 2). Additionally, the two objective functions are comparably illustrated in the pareto Chart 3.

Based on the results and information obtained from Charts (1 and 2), it can be inferred that the two objective

TABLE 2. Objective Function Values for Different ε Values

ε	The amount of the first objective function (risk)	The amount of the second objective function (quality)
1558	7659	1833
2947	10379	2982
4336	14275	4336
5725	18415	5728
7114	22585	7118
8503	26740	8503
9892	30914	9895
11281	35084	11285
12670	39374	12672
14059	43619	14062
15451	51376	15451

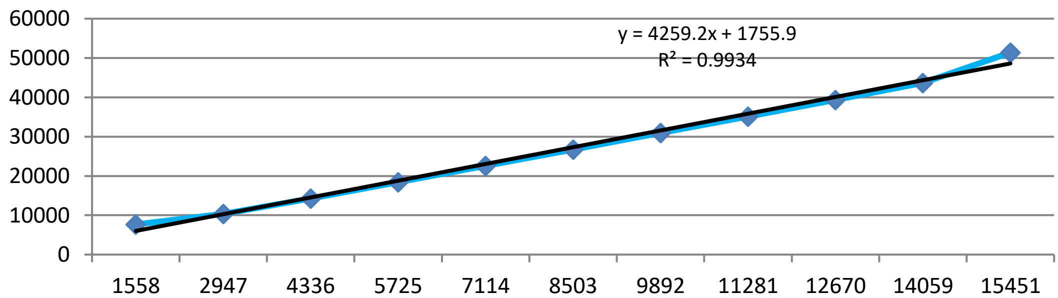

Chart 1. Pareto chart of the first objective function (Risk)

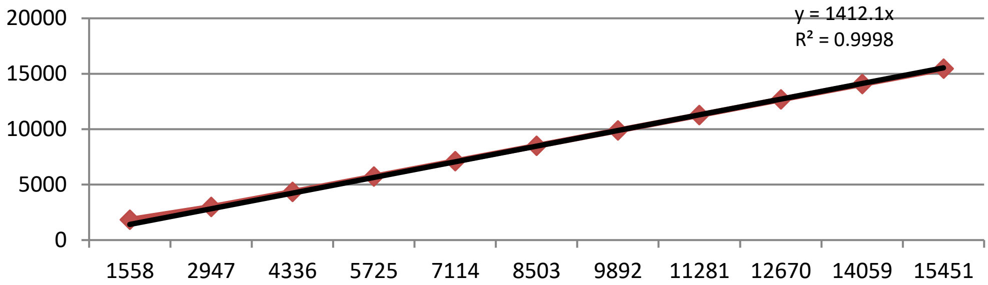

Chart 2. Pareto chart of the second objective function (Quality)

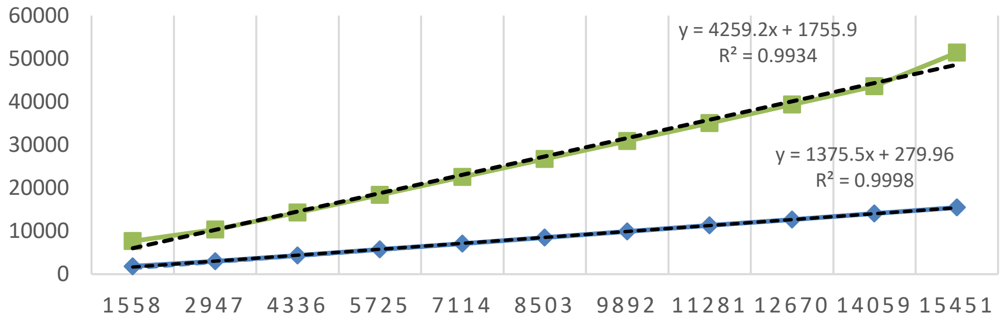

Chart 3. Comparative pareto chart of the two obtained objective functions

functions form a first-degree curve. The closer the value and ratio of error (R^2) to one, the lower the error. Conversely, as (R^2) approaches zero, the deviation error increases. Considering the values of (R^2) in the two pareto charts of the first and second objective functions, it can be stated that the enhanced ε -constrained method in the current research provides 99% confidence.

5. VALIDATION OF THE MATHEMATICAL MODEL

For evaluating the performance and efficiency of the ε -constrained method, five indices are utilized: NPS, CPU time, MID, MS, and SNS. Based on the results presented in Table 3, they indicate the accuracy of the model and the efficiency of the enhanced ε -constrained method.

TABLE 3. Results of Model Validation

Method	NPS		CPU time		MID		MS		SNS	
	value	gap	Value	gap	value	gap	value	gap	value	gap
ε - Constrained	10	0.17	94.261	15.86	38.1	0	06.369174	0	86.296471	0.18

6. CONCLUSION AND FUTURE RESEARCH RECOMMENDATIONS

This research, considering real-world projects and specific inputs in gas distribution network projects implementation and utilizing the expertise of specialists in the gas industry, has achieved the research objectives. In this study, a multi-objective robust probabilistic programming mathematical programming model has been developed and formulated by using an appropriate solution method, namely the enhanced ε -constrained method. By leveraging real data and information from various dimensions in the execution of gas distribution network projects, a case study in the gas industry has been solved. The results of the current research, with the presence of expert stakeholders, have been reviewed and found to have acceptable satisfaction with the implementation of objective functions and problem constraints, making it valuable and worthwhile.

The results of this research enable gas project managers to select the best solution among various options for the gas distribution network projects implementation by considering different values among the four objective functions: time, cost, quality, and risk, to achieve strong optimization. On the other hand, to minimize or maximize the three objective functions of time, cost, and risk and maximize the quality objective function in gas distribution network projects implementation to search for better solution spaces, the enhanced ε -constrained method has been utilized.

The findings of this research indicate that the two objective functions obtained from the enhanced ε -constrained method form a first-degree curve. Considering the consideration of 11 ε points and the (R^2) values in the two Pareto charts of risk and quality objective functions, it can be stated that the enhanced ε -constrained method used in the current research provides 99% confidence and the results indicate the accuracy of the model and the efficiency of the proposed method.

The obtained results can assist project managers in enhancing performance and efficiency in gas distribution network projects implementation through effective and optimized management while adhering to the objectives and constraints of this research.

Regarding research suggestions, it is recommended to consider green supply chain management and other multi-objective fuzzy optimization methods such as artificial neural network (ANN) system and additionally, utilizing like ant colony optimization (ACO), artificial bee colony, ant-based pheromone spaces for generation constructive hyper-heuristics, adaptive polyploid and island metaheuristic algorithms would be beneficial. Comparing the results obtained with the current research in the studied industry would also be advantageous for future investigations.

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Persian Abstract

چکیده

بهینه سازی زمان، هزینه، کیفیت و ریسک چهار فاکتور اصلی در مدیریت پروژه های گازی هستند. با توجه به جایگاه پروژه های گازی در توسعه گاز کشور به عنوان یکی از برترین منابع درآمدزایی و اقتصادی و برتری های منابع مهم انرژی، مدیریت بهینه و هدفمند در اجرای پروژه های شبکه های توزیع گاز امری ضروری محسوب می گردد. هدف از انجام این پژوهش توسعه مدل ریاضی برنامه ریزی امکانی استوار چند هدفه با استفاده از روش حل اپسیلون – محدودیت تقویت شده با رعایت اهداف مسأله کمینه سازی توابع زمان، هزینه و ریسک و بیشنه سازی تابع کیفیت مورد انتظار می باشد. بدین منظور حجم نمونه جامعه آماری این پژوهش ۳۶ نفر خبره و متخصص در حوزه صنعت گاز انتخاب شده است. نتایج این پژوهش به مدیران پروژه های گازی این امکان را می دهد که براساس مدیریت موثر با مقادیر مختلف بین چهار تابع هدف زمان، هزینه، کیفیت و ریسک بهترین جواب از بین چند حالت ممکن انتخاب شود تا به یک بهینه سازی قوی تبدیل گردد. همچنین نتایج نشان می دهد دو تابع هدف حاصل شده از روش حل اپسیلون – محدودیت تقویت شده به منظور جستجوی بهتر فضای حل مسأله در تولید جواب های برتر، ۹۹ درصد اطمینان دارد و نمودارهای پارتو از دو تابع هدف ریسک و کیفیت صحت مدل و کارایی روش پیشنهادی را بیان می کند.