



# Different nano-particles volume fraction and Hartmann number effects on flow and heat transfer of water-silver nanofluid under the variable heat flux

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## ABSTRACT

Nanofluid flow and heat transfer composed of water-silver nanoparticles is investigated numerically inside a microchannel. Finite volume approach (FVM) is applied and the effects of gravity are ignored. The whole length of Microchannel is considered in three sections as  $l_1=l_3=0.151$  and  $l_2=0.71$ . The linear variable heat flux affects the microchannel wall in the length of  $l_2$  while a magnetic field with strength of  $B_0$  is considered over the whole domain of it. The influences of different values of Hartmann number ( $Ha=0, 10, 20$ ), volume fraction of the nanoparticles ( $\phi=0, 0.02, 0.04$ ) and Reynolds number ( $Re=10, 50, 200$ ) on the hydrodynamic and thermal properties of flow are reported. The investigation of slip velocity variations under the effects of a magnetic field are presented for the first time (to the best knowledge of author) while the non-dimensional slip coefficient are selected as  $B=0.01, 0.05, 0.1$  at different states.

## 1. Introduction

Due to low thermal conductivity of simple pure fluids like water, many works have been reported in order to increase that thermal conductivity which led to generate a new composite called nanofluid which was a mixture of suspended nanoparticles in a liquid base fluid [1–6]. Several correlations have also been suggested to predict the nanofluid thermophysical properties; among them, Yu and Choi [7] have presented different models for its thermal conductivity. In the following, a large number of papers concerned nanofluid flow in enclosures and tubes can be addressed in this way [8–11].

The investigation of natural convection inside an enclosure filled with different types of nanofluid as like Cu-water,  $Al_2O_3$ -water and  $TiO_2$ -water was presented by Oztop and Abu-Nada [12] and the most value for Nusselt number was achieved for Cu-water. Moreover Jou and Tzeng [13] and Santra et al. [14] have studied other aspects of the nanofluid in rectangular cavities for example supposed it as the non-Newtonian liquid. Particle base methods like lattice Boltzmann approach can also be applied for the simulation of Cu-water nanofluid natural convection in an inclined lid driven cavity [15]. Many other works might be referred which concerned various types of nanofluid at different applications [16–20].

Simulation of flow and heat transfer through the microchannels has attracted several researchers recently. Due to small size and higher efficiency, that subject has being widely studied especially in the new industrials of MEMS and NEMS [21–24]. Tullius et al. [25] investi-

gated the cooling process of the microchannel walls with various types of fluids and provided a desirable function about. Another interesting work involved nanofluid flow inside a microchannel was reported by Raisi et al. [26] who studied the effects of nanoparticles volume fraction and slip coefficient. Show the effects of gravity on a micro flow were also presented by Karimipour et al. [27] by lattice Boltzmann method. They proposed a range for Knudsen number to include the buoyance forces. More other studies can be addressed concerned the micro flows in this way [28–35].

It is possible to have the transverse magnetic field in several applications of devices; which strongly affects the hydrodynamic and thermal properties of domain. Back [36] investigated the flow and heat transfer between two parallel plates subjected to the electrical conductor. Show the influence of a magnetic field on the nanofluid flow through the microchannel was reported by Aminossadati et al. [37]. Simulations of MHD (magneto hydrodynamic) flows at different conditions are still continuing [38–45]. Also, several other new works can be referred concerned nanofluid flow which implies its novelty by now [50–55]; so that it is seen there is lack of researches concerned the magnetic field effect on nanofluid slip velocity which is presented here.

## 2. Problem statement

Nanofluid flow and heat transfer composed of water-silver nanoparticles is investigated numerically inside a microchannel. Fig. 1 illustrates the physical geometry of the microchannel with its aspect

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Nomenclature		$u_S$	Slip velocity
$B$	Non-dimensional slip coefficient ( $=\beta/h$ )	$U_S$	Slip velocity in non-dimensional form
$B_0$	The power of the magnetic field	$x,y$	Cartesian coordinates, m
$Ha$	Hartmann number ( $=B_0h\sqrt{\sigma_{nf}/\mu_{nf}}$ )	$X,Y$	Non-dimensional Cartesian coordinates
$Nu_x$	Local Nusselt number	<i>Greek symbols</i>	
$Nu_m$	Averaged Nusselt number	$\phi$	Volume fraction of nano particles
$q''_x$	Heat flux, W/m <sup>2</sup>	$\beta$	Slip coefficient, m
$T_c$	The temperature of the cold inlet nanofluid flow, K	$\theta$	Temperature in non-dimensional form
$u,v$	velocity components, m/s	$\sigma$	Electrical conductivity, S/m
$U,V$	Non-dimensional velocity components		
$u_c$	The velocity of inlet nanofluid flow, m/s		

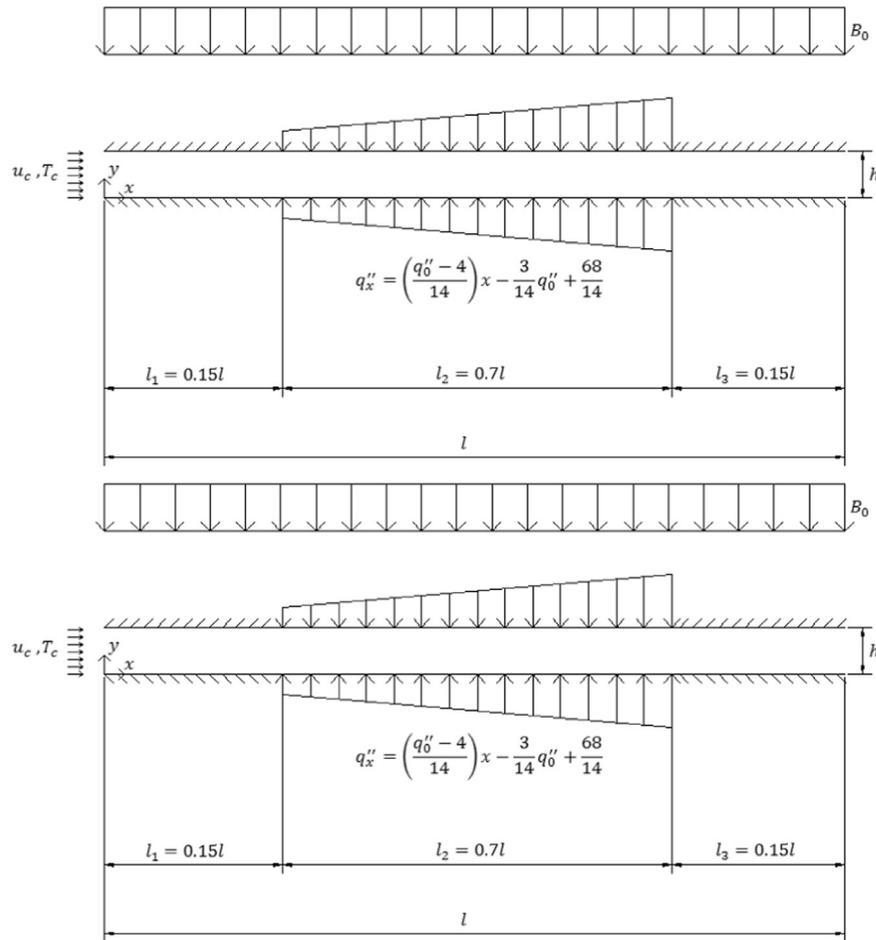


Fig. 1. The schematic physical domain of the microchannel.

Table 1  
Thermo-physical properties of water (base fluid) and silver at 25 °C.

	$\mu$	$\rho$	$C_p$	$K$
Pure water	$0.8908 \times 10^{-3}$	997.1	4179	0.613
Ag nanoparticles		10,500	235	429

Table 2  
Grid study at  $\phi=0.04$ ,  $Ha=10$  and  $B=0.1$ .

Grids		400×20	600×30	800×40	1000×50
Re=10	$Nu_m$	4.060652	4.112199	4.140568	4.158858
	$\theta_{out,(Y=0.5)}$	0.747418	0.747433	0.74744	0.747443
	$U_{out,(Y=0.5)}$	1.1014	1.10263	1.10306	1.10326
Re=50	$Nu_m$	9.241925	9.415255	9.507361	9.565655
	$\theta_{out,(Y=0.5)}$	0.119025	0.118954	0.118921	0.118906
	$U_{out,(Y=0.5)}$	1.10138	1.10263	1.10306	1.10326

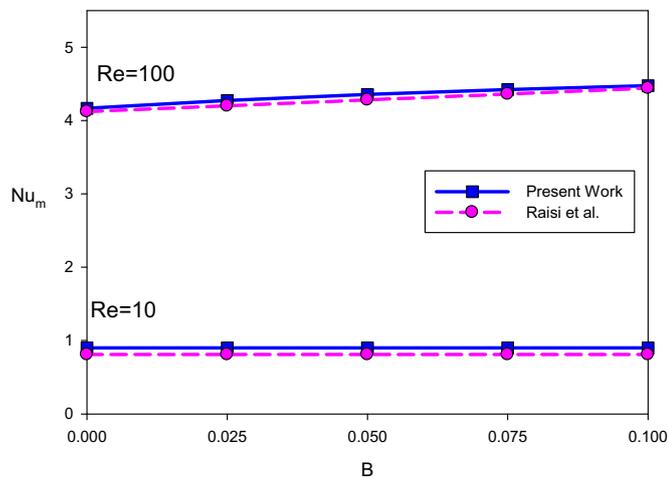


Fig. 2. Averaged Nusselt number in comparison with those of Raisi et al. [26] for  $\phi=0.03$ .

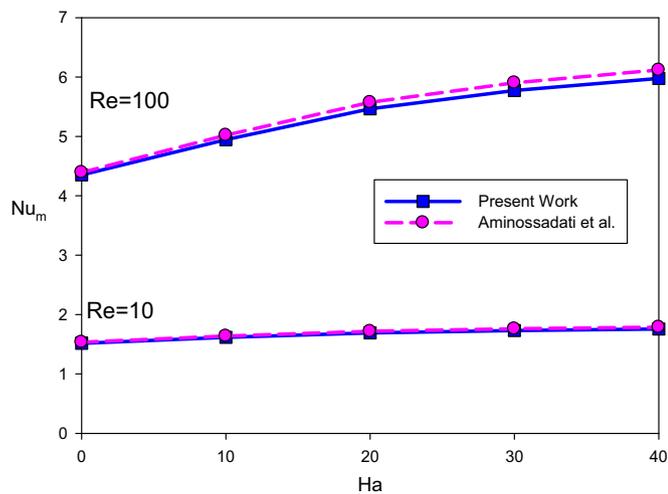


Fig. 3. Averaged Nusselt number in comparison with those of Aminossadati et al. [37] for  $\phi=0.02$ .

ratio of  $\frac{l}{h}=20$ . The whole length of Microchannel is considered in three sections as  $l_1=l_3=0.151$  and  $l_2=0.71$ . The linear variable heat flux affects the microchannel wall in the length of  $l_2$  while a magnetic field with strength of  $B_0$  is considered over the whole domain of it. The inlet cold Newtonian, incompressible nanofluid is supposed at  $T_c=293$  °k. Moreover it is assumed that the silver spherical nanoparticles are at the same size with the diameter equal to  $d_p=10$  nm.

Table 1 shows the thermo-physical properties of Ag nanoparticles and the water as the base fluid which implies that the Prandtl number of water would be 6.2. The influences of different values of Hartmann number ( $Ha=0, 10, 20$ ), volume fraction of the nanoparticles ( $\phi=0, 0.02, 0.04$ ) and Reynolds number ( $Re=10, 50, 200$ ) on the hydrodynamic and thermal properties of flow are reported. The investigation of slip velocity variations under the effects of a magnetic field are presented for the first time (to the best knowledge of author) while the non-dimensional slip coefficient are selected as  $B=0.01, 0.05, 0.1$  at different states.

### 3. Numerical procedure and governing equations

The well-known Navier–Stokes equations are considered and solved numerically to simulate the supposed problem. However; the non-dimensional form of them can be achieved by the variables used in Eq. (1) for a 2-D domain under the effects of a magnetic field as follows:

$$\begin{aligned}
 X &= \frac{x}{h}, Y \\
 &= \frac{y}{h}, H \\
 &= \frac{h}{h} \\
 &= 1, L \\
 &= \frac{l}{h} \\
 &= 20, U \\
 &= \frac{u}{u_c}, V \\
 &= \frac{v}{u_c}, P \\
 &= \frac{p}{\rho_{nf} u_c^2} \theta \\
 &= \frac{T - T_c}{\Delta T}, \Delta T \\
 &= \frac{q_0''}{k_{eff}}, Re \\
 &= \frac{\rho_{nf} u_c h}{\mu_{nf}}, Pr \\
 &= \frac{\nu_{nf}}{\alpha_{nf}}, Ha \\
 &= B_0 h \sqrt{\sigma_{nf} / \mu_{nf}}
 \end{aligned} \tag{1}$$

$$\frac{\partial U}{\partial X} + \frac{\partial V}{\partial Y} = 0 \tag{2}$$

$$U \frac{\partial U}{\partial X} + V \frac{\partial U}{\partial Y} = -\frac{\partial P}{\partial X} + \frac{1}{Re} \left( \frac{\partial^2 U}{\partial X^2} + \frac{\partial^2 U}{\partial Y^2} \right) - \frac{Ha^2}{Re} U \tag{3}$$

$$U \frac{\partial V}{\partial X} + V \frac{\partial V}{\partial Y} = -\frac{\partial P}{\partial Y} + \frac{1}{Re} \left( \frac{\partial^2 V}{\partial X^2} + \frac{\partial^2 V}{\partial Y^2} \right) \tag{4}$$

$$U \frac{\partial \theta}{\partial X} + V \frac{\partial \theta}{\partial Y} = -\frac{\partial P}{\partial Y} + \frac{1}{RePr} \left( \frac{\partial^2 \theta}{\partial X^2} + \frac{\partial^2 \theta}{\partial Y^2} \right) \tag{5}$$

It should be mentioned that nanofluid's properties are applied to determine the amounts of  $Re, Pr$  and  $Ha$  to have better agreements with the real physical problems. Moreover, the strength of magnetic field and electrical conductivity coefficient are shown by  $B_0$  and  $\sigma_{nf}$ .

The nanofluid density is written as:

$$\rho_{nf} = \phi \rho_s + (1 - \phi) \rho_f \tag{6}$$

$\phi$  shows the volume fraction of nano particles; while  $n, f$  and  $s$  are related to the nanofluid, base fluid and solid nanoparticles, respectively.

The thermal diffusivity of nanofluid can be achieved as:

$$\alpha_{nf} = \frac{k_{eff}}{(\rho C_p)_{nf}} \tag{7}$$

Xuan and Li [46] formula is used to determine the nanofluid heat capacity:

$$(\rho C_p)_{nf} = (1 - \phi)(\rho C_p)_f + \phi(\rho C_p)_s \tag{8}$$

And finally the nanofluid viscosity is represented by Brinkman model as follows [47]:

$$\mu_{nf} = \frac{\mu_f}{(1 - \phi)^{2.5}} \tag{9}$$

The model of Chon et al. [48] is used to calculate the nanofluid thermal conductivity. The resent model shows suitable accuracy and also the influences of Brownian motions and nanoparticles diameter can be considered in it.

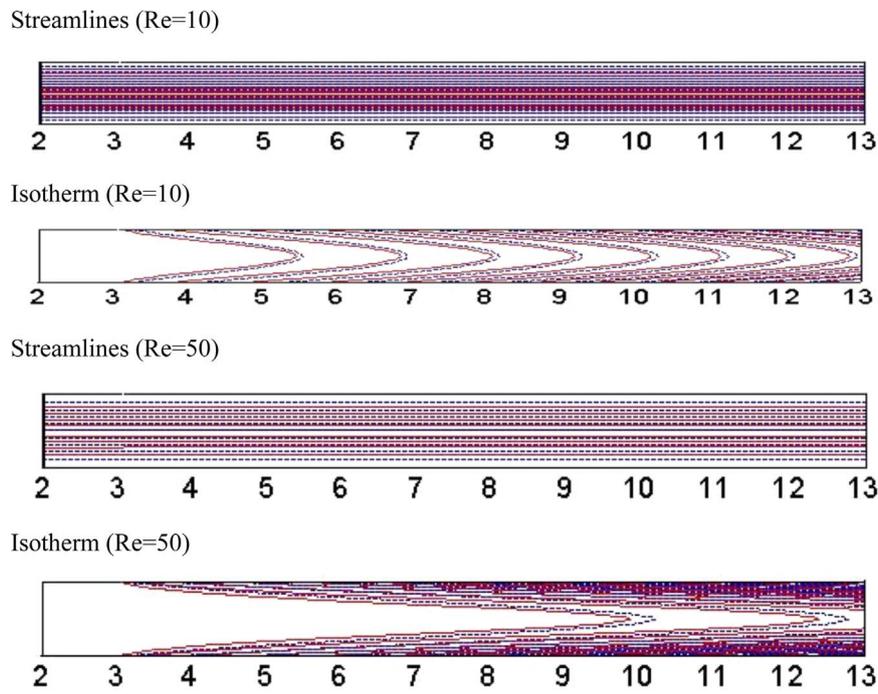


Fig. 4. Stream-lines and isotherm-contours at  $\phi=0.04$  and  $B=0.01$  for  $Ha=0$  (solid lines) and  $Ha=20$  (dash lines).

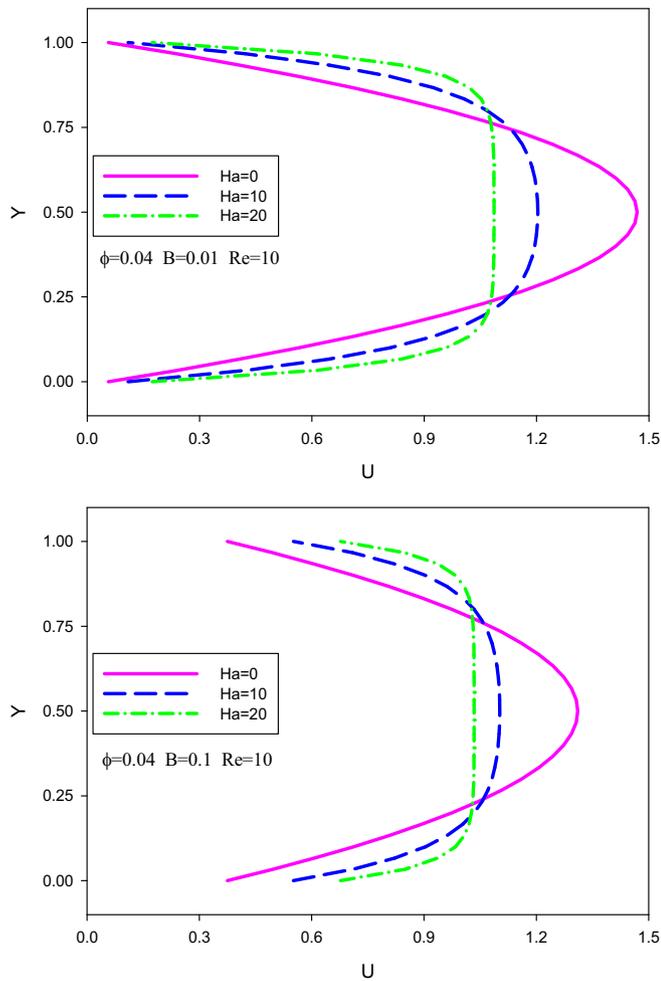


Fig. 5. U profiles on vertical centerline at  $\phi=0.04$ ,  $Re=10$ ,  $B=0.01$  and  $B=0.1$ .

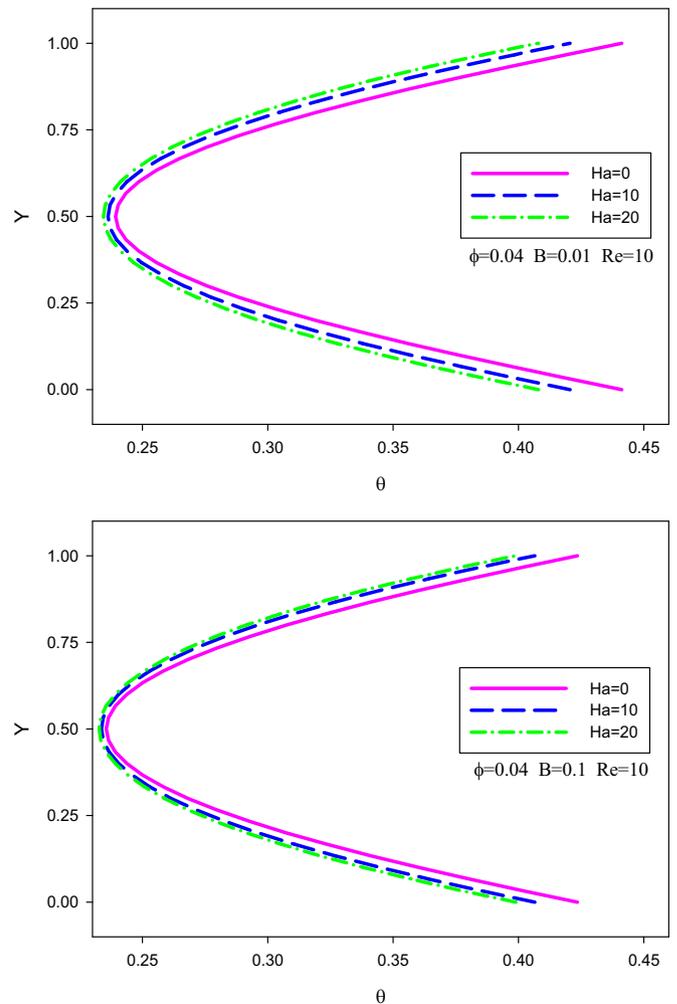


Fig. 6.  $\theta$  profiles on vertical centerline at  $\phi=0.04$ ,  $Re=10$ ,  $B=0.01$  and  $B=0.1$ .

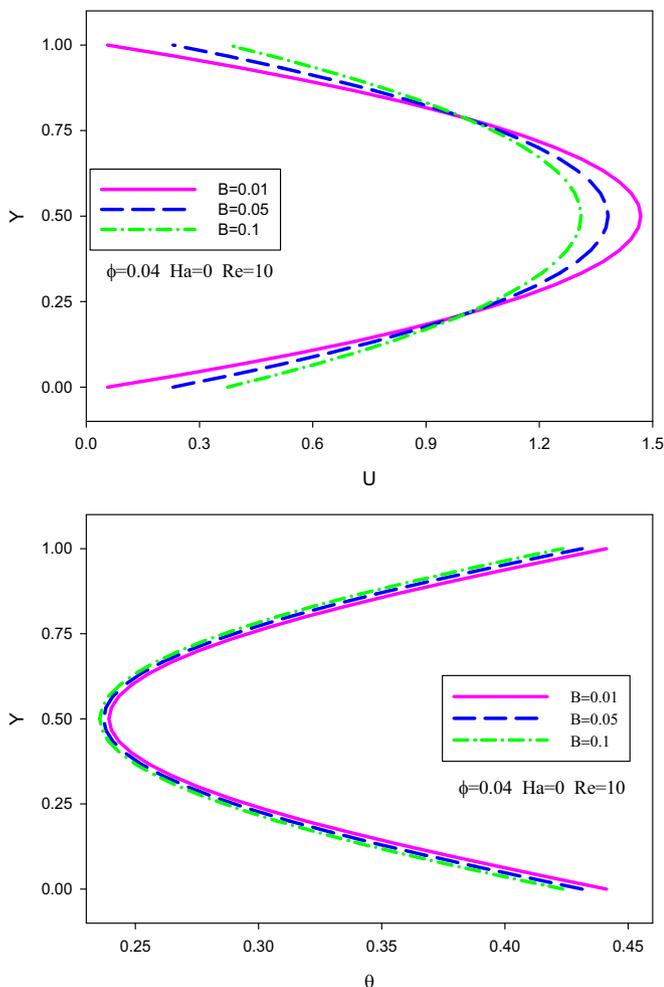


Fig. 7.  $U$  and  $\theta$  profiles on vertical centerline at  $\phi=0.04$ ,  $Ha=0$  and  $Re=10$ .

$$\frac{k_{nf}}{k_f} = 1 + 64.7 + \phi^{0.7460} + \left(\frac{d_f}{d_p}\right)^{0.3690} \left(\frac{k_s}{k_f}\right)^{0.7476} \left(\frac{\mu}{\rho_f \alpha_f}\right)^{0.9955} \left(\frac{\rho_f B_c T}{3\pi\mu^2 L_{BF}}\right)^{1.2321} \quad (10)$$

In Eq. (10),  $B_c=1.3807 \times 10^{-23}$  and  $L_{BF}$  represent the Boltzmann coefficient and the mean free path of water molecules. It should be mentioned that diameter of water molecules are considered at  $d_f=0.17$  nm while the amount of  $\mu$  is determined as below at recent equation:

$$\mu = A10^{\frac{B}{T-C}}, C=140(K), B=247(K), A=2.414 \times 10^{-5} (Pas) \quad (11)$$

As it is said before, the slip velocity boundary condition ( $u_s$ ) is taken in to account at present study. To do this, the slip coefficient is shown by [49]; hence  $u_s$  can be achieved along the microchannel walls as follows:

$$u_s = -\beta \left(\frac{\partial u}{\partial y}\right)_{y=h} \quad u_s = \beta \left(\frac{\partial u}{\partial y}\right)_{y=0} \quad (12)$$

The non-dimensional type of slip coefficient is introduced by  $B=\frac{\beta}{h}$ . As a result, all boundary conditions of the microchannel are shown in non-dimensional form as:

$$\begin{aligned} U &= 1, V \\ &= 0 \quad \text{and} \quad \theta \\ &= 0 \quad \text{for} \quad X \\ &= 0 \quad \text{and} \quad 0 \\ &\leq Y \\ &\leq 1V \\ &= 0 \quad \text{and} \quad \frac{\partial U}{\partial X} \\ &= \frac{\partial \theta}{\partial X} \\ &= 0 \quad \text{for} \quad X \\ &= 20 \quad \text{and} \quad 0 \\ &\leq Y \\ &\leq 1V \\ &= 0, U_S \\ &= B \frac{\partial U}{\partial Y} \quad \text{and} \quad \frac{\partial \theta}{\partial Y} \\ &= 0 \quad \text{or} \quad \frac{\partial \theta}{\partial Y} \\ &= -\frac{1}{14} \left[ \left(1 - \frac{4}{q_0''}\right) (Xh) + \frac{68}{q_0''} - 3 \right] \text{for } Y \\ &= 0 \quad \text{and} \quad 0 \\ &\leq X \\ &\leq 20V \\ &= 0, U_S \\ &= -B \frac{\partial U}{\partial Y} \quad \text{and} \quad \frac{\partial \theta}{\partial Y} \\ &= 0 \quad \text{or} \quad \frac{\partial \theta}{\partial Y} \\ &= \frac{1}{14} \left[ \left(1 - \frac{4}{q_0''}\right) (Xh) + \frac{68}{q_0''} - 3 \right] \text{for } Y \\ &= 1 \quad \text{and} \quad 0 \\ &\leq X \\ &\leq 20 \end{aligned} \quad (13)$$

In which  $U_S$  illustrates the non-dimensional form of slip velocity. Now, the Eqs. (2)–(5) are able to be solved numerically by finite volume method according to the SIMPLE algorithm.

Eqs. (14) and (15) are used for the local Nusselt number in dimensionless and dimensional forms, respectively:

$$(Nu_x)_{y=0,h} = \frac{q_0'' h}{(T_{S,x} - T_c) k_{nf}} \quad (14)$$

$$(Nu_x)_{y=0,1} = \frac{1}{\theta_{S,x}} \quad (15)$$

Finally the averaged Nusselt number is determined as follows:

$$Nu_m = \frac{1}{L_2} \int_{L_1}^{L_1+L_2} Nu_X dX \quad \left( L_1 = \frac{l_1}{h} \quad \text{and} \quad L_2 = \frac{l_2}{h} \right) \quad (16)$$

#### 4. Grid independency and validations

Grid independency study is presented at different states in Table 2 for the amounts of the averaged Nusselt number and the outlet dimensionless velocity and temperature at  $Y=0.5$ ,  $Re=50$  and  $Re=10$ . As a result, the grid nodes of  $800 \times 40$  are found suitable for the next computations. Moreover and for the validation; in Figs. 2 and 3 the values of Nusselt number along the microchannel walls are compared

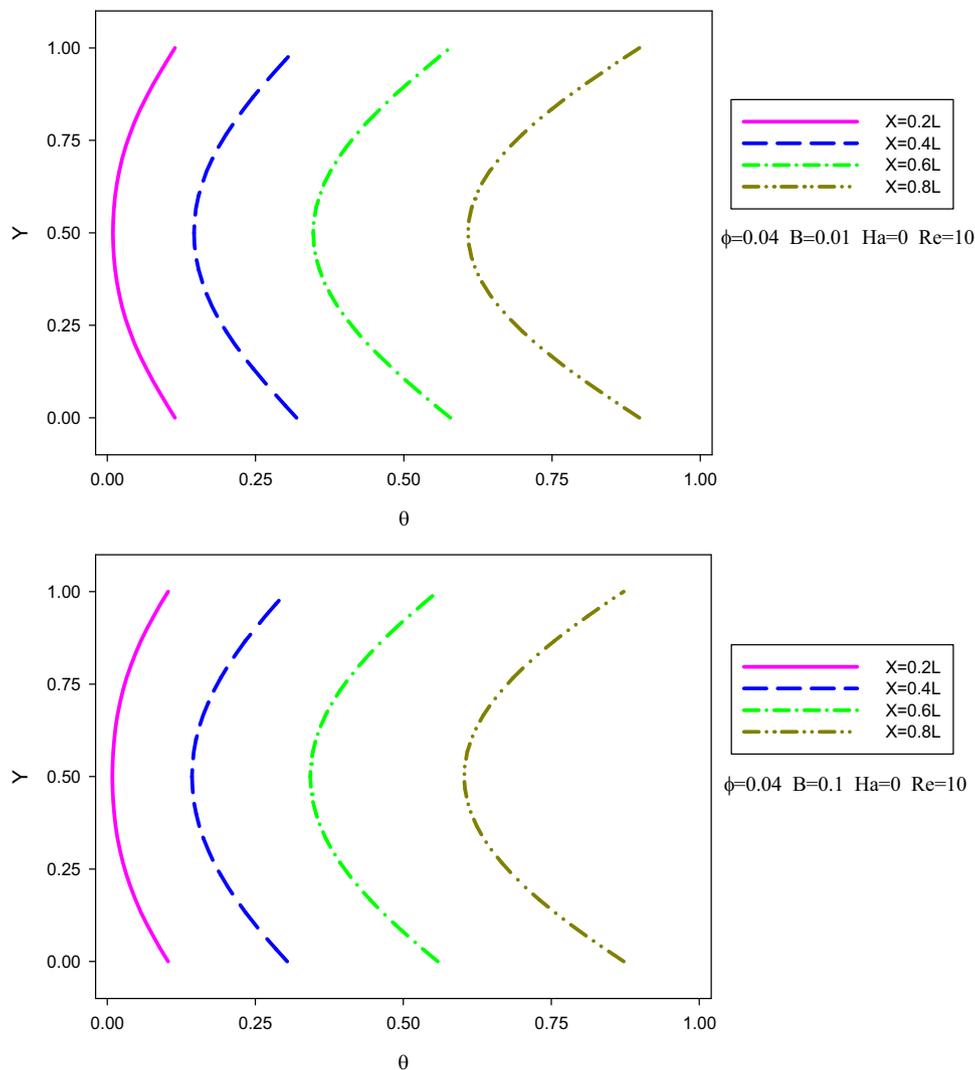


Fig. 8.  $\theta$  profiles along the various cross-sections for  $\phi=0.04$ ,  $Ha=0$  and  $Re=10$ .

with those of Raisi et al. [26] and Aminossadati et al. [37] respectively; and the suitable agreements are seen in these figures.

### 5. Results

The influences of various amounts of nanoparticles volume fractions, Hartmann number, slip coefficient and Reynolds number on the hydrodynamic and thermal properties of the water-Ag nanofluid inside a microchannel are simulated while the whole domain is under the uniform magnetic field. The following results are achieved.

#### 5.1. Influences of Re and Ha

Fig. 4 illustrates the streamlines and isotherms for  $Ha=0, 20$  and  $Re=10, 50$  at  $\phi=0.04$  and  $B=0.01$  which implies to the straight and parallel lines. Different values of Hartmann and Reynolds number don't significantly change these parallel streamlines. Although higher Re corresponds to higher entrance length. It is also seen that the isotherms intensity adjacent to the walls increases at larger amounts of Reynolds number.

In the following and in Fig. 5, the horizontal profiles of  $U$  on the microchannel vertical centerline are presented for  $Ha=0, 10, 20$  and  $B=0.01, 0.1$  at  $\phi=0.04$  and  $Re=10$ . The Lorentz force is generated in opposite direction of flow due to the existence of the magnetic field. Moreover  $U_{max}$  decreases by more Hartmann number which leads to

higher values of  $U$  near the walls. This fact makes the change in the figure of fully developed velocity profiles from parabolic to the flat shape.

Dimensionless temperature profiles of  $\theta$  on the vertical centerline at  $Ha=0, 10, 20$  and  $B=0.01, 0.1$  for  $\phi=0.04$  and  $Re=10$  are presented through the Fig. 6. It is observed that cold nanofluid enters from the left side and then after passing the insulated section, it begins to be warmed under the effects of the heat flux along the length of  $L_2$ . That means the temperature of the nanofluid close the walls would be larger than the central areas. The less effects of Hartmann number on  $\theta$  profiles can also be traced in this figure.

#### 5.2. Influences of B and volume fraction

Fig. 7 presents  $U$  and  $\theta$  profiles of nanofluid on the microchannel vertical centerline at  $B=0.01, B=0.05$  and  $B=0.1$  for  $Ha=0, Re=10$  and  $\phi=0.04$ . The amounts of  $U_s$  along the walls at  $B=0.01, B=0.05, B=0.1$  are achieved as 0.056, 0.23, 0.374. Moreover the value of  $U_{max}$  decreases with  $B$  to satisfy the continuity equation. Although,  $\theta$  profiles are not changed significantly by  $B$ . At present work, the temperature jump is ignored; As a result the slip coefficient only affects the hydrodynamic properties like slip velocity along the walls; so the temperature profiles would not significantly be affected by  $B$ . In Fig. 8, the profiles of  $\theta$  at various cross-sections are observed at  $B=0.1, B=0.01$  and for  $\phi=0.4, Re=10, Ha=0$ . Due to imposed heat flux over the

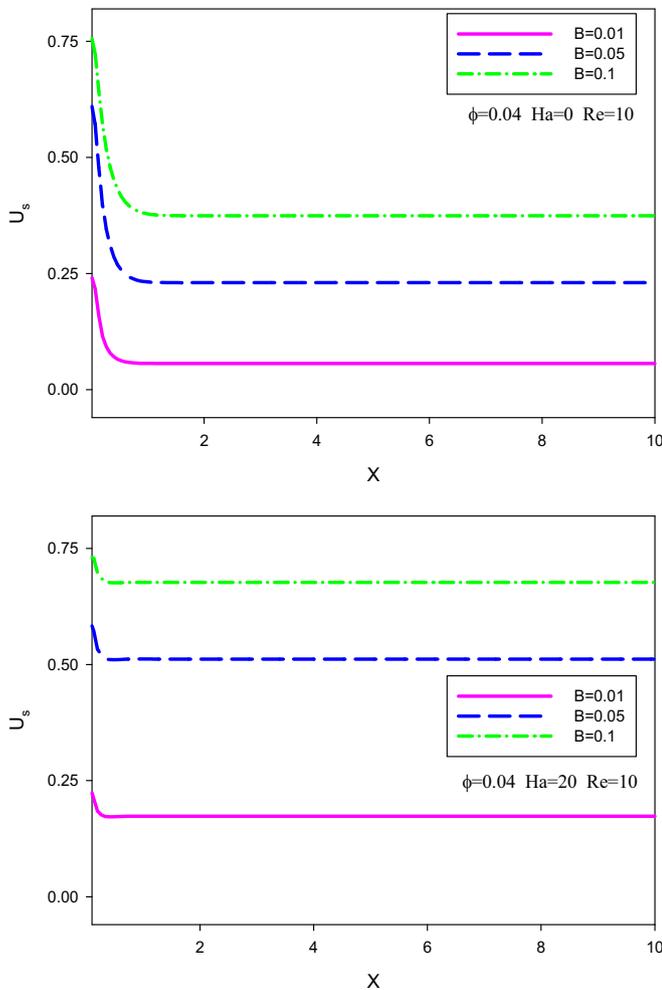


Fig. 9.  $U_s$  on the wall of microchannel for ng the microchannel for  $\phi=0.04$ ,  $Re=10$ ,  $Ha=0$  and  $Ha=20$ .

microchannel walls, the temperature of nanofluid increases with  $X$ . Change the curve figure of temperature profiles from the flat shape at  $X=0.2 L$  to the parabolic one at  $X=0.8 L$  can be observed clearly in this figure; that event has not been reported in previous articles.

Fig. 9 illustrates the  $U_s$  along the microchannel walls at  $B=0.01$ ,  $0.05$ ,  $0.1$  and for  $\phi=0.04$ ,  $Re=10$ . Slip velocity has maximum value at inlet and it starts to decrease with  $X$ , so that it tends to the constant amount. More  $U_s$  is achieved by more slip coefficient; although this event is happened more severely at larger Hartmann number.

Local Nusselt number on the walls at various amounts of Hartmann number and slip coefficient for  $\phi=0.04$  and  $Re=50$  are presented through the Fig. 10. Obviously, Nusselt number equals to zero through the adiabatic sections and then its maximum amount can be found at  $X=3$ . More  $X$  corresponds to less  $Nu_x$  until  $X=17$  where is the start point of second insulated area.

At last, the values of  $Nu_m$  on the length of  $L_2$  are shown in Fig. 11 for various amounts of Reynolds number, Hartmann number, slip coefficient and nanoparticles volume fraction. At the state of  $Re=10$ , the averaged Nusselt number decreases while volume fraction changes from  $\phi=0.02$  to  $\phi=0.04$ . This fact is seen for the first time at present work. However higher Reynolds number corresponds to change in this manner so that at  $Re=200$ , more Nusselt number is observed for larger amount of volume fraction. It should be mentioned that the values of nanofluid Reynolds number through the microchannel are usually small; hence applying nanofluid with more volume fraction to increase Nusselt number would not be so desirable action. Although this deficiency might be improved by the influences of the slip coefficient

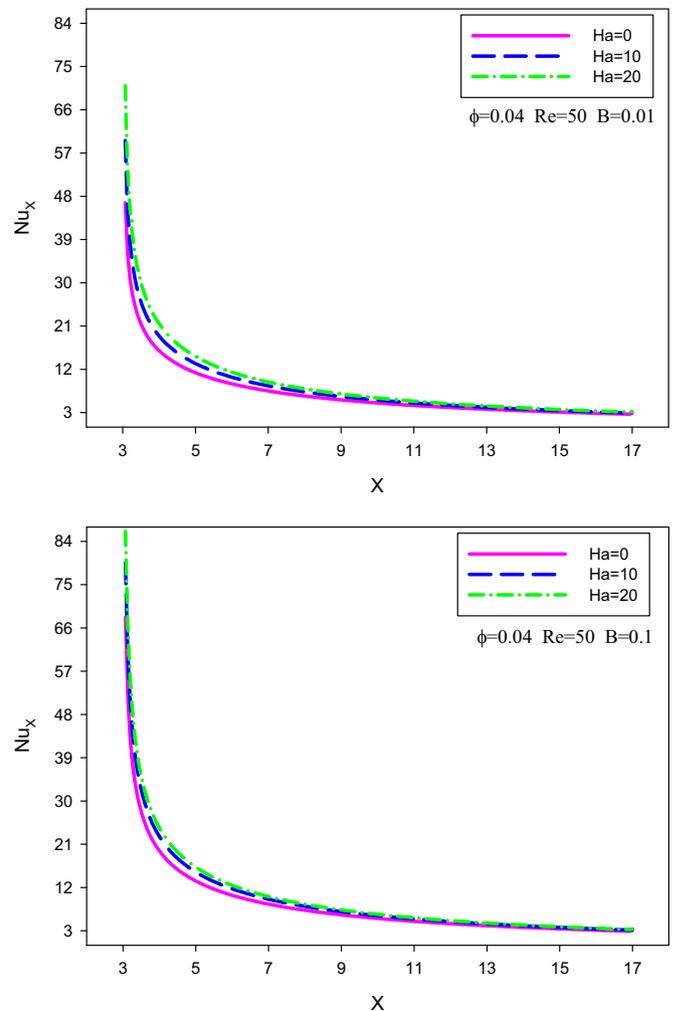


Fig. 10.  $Nu_x$  on the wall of microchannel for  $\phi=0.04$ ,  $Re=50$ ,  $B=0.01$  and  $B=0.1$ .

and the stronger magnetic field.

## 6. Conclusion

Nanofluid flow and heat transfer composed of water-silver nanoparticles was investigated numerically inside a microchannel. Finite volume approach (FVM) was applied and the effects of gravity were ignored. The whole length of Microchannel was considered in three sections as  $l_1=l_3=0.151$  and  $l_2=0.71$ . The linear variable heat flux affected the microchannel wall in the length of  $l_2$  while a magnetic field with strength of  $B_0$  was considered over the whole domain of it. The influences of different values of Hartmann number ( $Ha=0, 10, 20$ ), volume fraction of the nanoparticles ( $\phi=0, 0.02, 0.04$ ) and Reynolds number ( $Re=10, 50, 200$ ) on the hydrodynamic and thermal properties of flow were reported. The investigation of slip velocity variations under the effects of a magnetic field were presented for the first time (to the best knowledge of author) while the non-dimensional slip coefficients were selected as  $B=0.01, 0.05, 0.1$  at different states.

The Lorentz force was generated due to the magnetic field existence in the opposite direction of flow. This fact corresponded to less the maximum value of horizontal velocity and more velocity adjacent to the walls. Larger slip coefficient led to more  $U_s$  especially at higher amounts of Hartmann number. The recent event was reported for the first time at present article and has not been seen in previous ones; hence the detour from continuous regime might be observed sooner at higher strength of the magnetic field.

Increasing  $\phi$  from  $\phi=0.02$  to  $\phi=0.04$  at  $Re=10$  generated lower

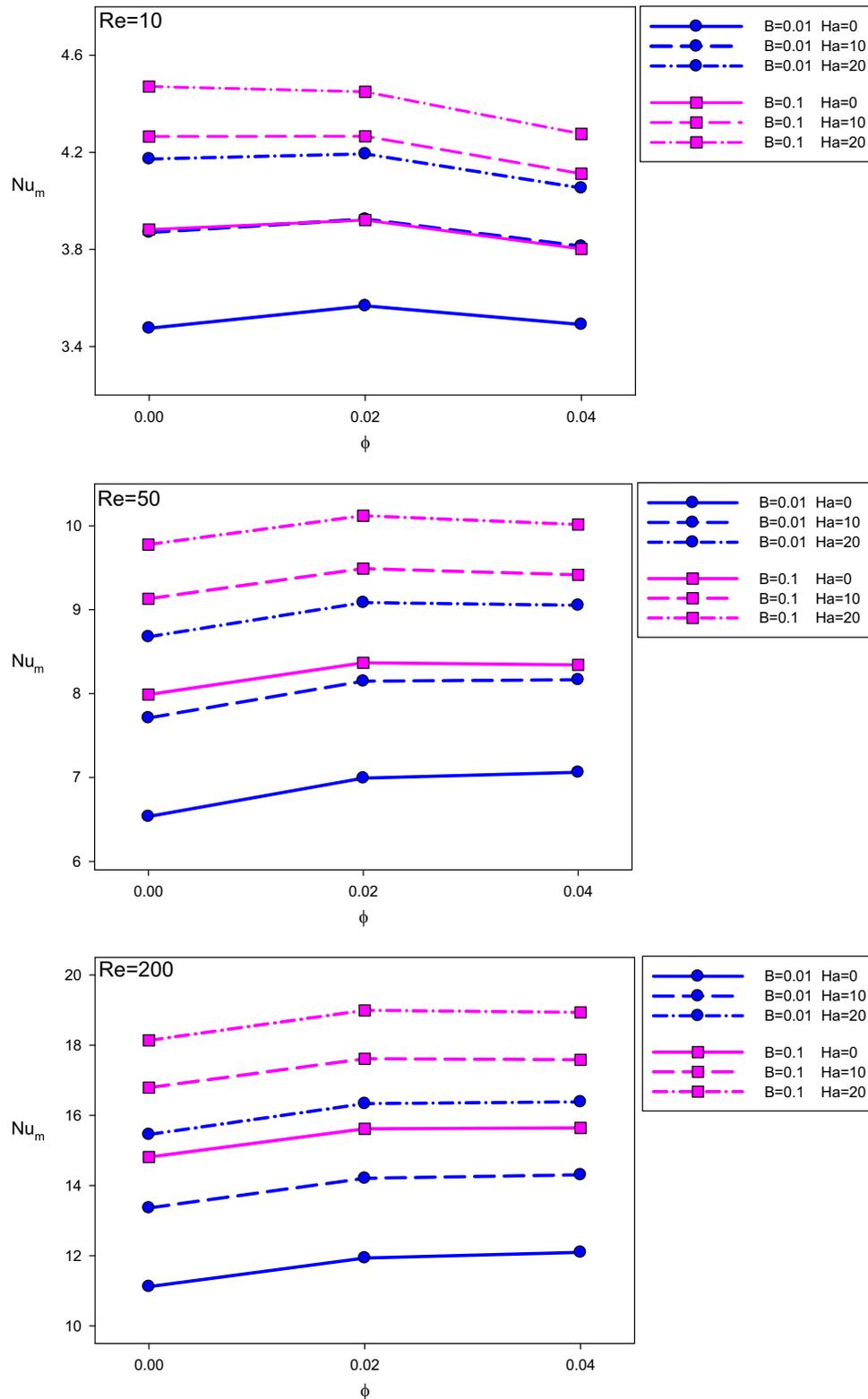


Fig. 11. Averaged Nusselt number on the wall of microchannel for various amounts of  $\phi$ , Ha, Re and B.

$Nu_m$ ; however at  $Re=200$  higher Nusselt number was observed at more volume fraction. It should be mentioned that the Reynolds number of nanofluids inside the microchannels usually had the small amounts; hence using more volume fraction to increase  $Nu$  would not be so desirable at micro and nano flows. Although, this fact might be improved by the magnetic field and slip coefficient influences.

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