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## International Communications in Heat and Mass Transfer

journal homepage: [www.elsevier.com/locate/ichmt](http://www.elsevier.com/locate/ichmt)Friction and heat transfer coefficient in micro and nano channels filled with porous media for wide range of Knudsen number<sup>☆</sup>H. Shokouhmand<sup>a</sup>, A.H. Meghdadi Isfahani<sup>a,\*</sup>, E. Shirani<sup>b</sup><sup>a</sup> School of Mechanical Engineering, College of Engineering, University of Tehran, Tehran, Iran<sup>b</sup> Department of Mechanical Engineering, Isfahan University of Technology, Isfahan, Iran

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## ABSTRACT

Invoking the velocity slip and temperature jump, numerical simulation of Darcy–Brinkman–Forchheimer flow model and forced convection in a circular micro/nano channel filled with porous media are presented. Relating the viscosity to the local Knudsen number, Kn, a generalized diffusion coefficient is obtained in such a way that it can model wide range of Kn regimes of flow. The effect of Kn and Darcy coefficient on velocity and temperature distribution is described. It is shown that despite of the fact that in most of previous researches it is assumed that Kn is constant along the channel, the variations of Kn due to the pressure variations, have considerable effects on heat transfer and temperature distribution across the channel cross section.

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## 1. Introduction

Flow and heat transfer through the porous medium have become one of the most popular research subjects because of its wide applications. With recent advances in micro and nano technology mechanical devices such as micro/nano channels, nozzles and pumps can be fabricated. In micro/nano devices, porous media can be used for micro filtration, fractionation, catalysis and microbiology related applications [1]. For example, micro packed beds or sintered metal fibers can be used in micro structured reactors for catalytic reaction [2,3] and the filter medium prepared from commercially available glass fibers of 70–140 μm thickness is suitable for applications in bio filtration systems [4]. Also charged porous media structures have been employed in micro devices to magnify the pumping [5,6], mixing [7,8] and separating [9] effects.

Also by reducing the dimensions of a channel, heat transfer area per unit volume increases, thus the overall heat transfer coefficient per unit volume increases. Use of porous media in the channel can verify this phenomenon. The micro porous heat exchangers have better heat transfer performance than the micro channel heat exchangers [10].

Flow in micro devices is different from macro devices. The flow rate in micro ducts is higher than predicted from theories based on continuum flow model. This is because of the fact that the Navier–Stokes equations based on the continuum flow model breaks down when the mean free path of the molecules is comparable to the

characteristic length of the flow domain. This phenomenon is called the “rarefaction” (Knudsen flow) [11]. As the channel size decreases the Knudsen number, defined as the ratio of the molecular mean free path ( $\lambda$ ) to the characteristic length of system, increases and the flow regime in the channel changes from continuum, to slip flow, transition and finally free molecular regime.

The Knudsen number is a measure of rarefaction of gases encountered in flows through very small size channels, and also is a measure of the degree of validity of the continuum mode. In large Kn, the mean free path of the molecules is comparable with the characteristic length of the flow, therefore the intermolecular collisions are reduced significantly and the collisions of the molecules with the walls become important. Because of the rarefaction, the density of the gas along the channel varies and the velocity of the gas near the solid walls is no longer equal to the wall velocity.

The existence of slip velocity at the wall was first predicted by Maxwell [12]. Because of the slip, the flow rate in micro and nano devices is higher than predicted from no slip boundary conditions. The temperature jump is a classic physical phenomenon which was observed and studied by Smoluchowski [13] and Knudsen [14]. Depending on the value of Knudsen number, four flow regimes exist:  $Kn < 10^{-3}$  for the continuum flow,  $10^{-3} < Kn < 10^{-1}$  for the slip flow,  $10^{-1} < Kn < 10$  for the transition flow and  $Kn > 10$  for the free molecular flow [15].

Only a few papers can be mentioned for analytical studies on micro channels with cross sections other than circular micro tubes or flow between parallel plate micro channels occupied with porous media [16–23]. All of these papers are limited to the slip regime using Darcy–Forchheimer momentum equation. Also in these papers Kn is assumed constant along the channel which is a crude assumption

<sup>☆</sup> Communicated by W.J. Minkowycz.

\* Corresponding author.

E-mail address: [amir\\_meghdadi@yahoo.com](mailto:amir_meghdadi@yahoo.com) (A.H. Meghdadi Isfahani).

**Nomenclature**

C	Forchheimer coefficient
C <sub>f</sub>	Skin friction coefficient
Da	Darcy coefficient ( $K/\varepsilon r_0^2$ )
h	Convection coefficient
K	Intrinsic permeability of the porous medium
Kn	Knudsen number $\lambda/r_0$
k	Thermal conductivity
k <sub>m</sub>	Overall thermal conductivity $((1-\varepsilon)k_s + \varepsilon k_f)$
Nu	Nusselt number
p	pressure
Pr	Prandtl number ( $\mu C_p/k_m$ )
r	Radial coordinate
r <sub>0</sub>	Channel radius
Re <sub>D</sub> <sup>*</sup>	Modified Reynolds number ( $2\rho_f u_0 r_0/\mu\varepsilon$ )
R	Non-dimensional radial coordinate ( $r/r_0$ )
T	Fluid temperature
T <sub>m</sub>	Mixing cup temperature
T <sub>w</sub>	Wall temperature
T <sub>i</sub>	Inlet temperature
U	Non-dimensional axial velocity ( $u/u_0$ )
u	Axial velocity
u <sub>0</sub>	Reference axial velocity $\varepsilon r_0^2/\mu_0(-dp/dz)$
z	Axial coordinate
Z	Non-dimensional axial coordinate ( $z/r_0$ )

*Greek symbols*

$\lambda$	Mean free path
$\sigma_v$	Tangential momentum accommodation coefficient
$\sigma_T$	Thermal accommodation coefficient
$\gamma$	Specific heat ratio
$\Gamma$	Nondimensional Forchheimer coefficient
$\varepsilon$	Porosity
$\mu$	Generalized diffusion coefficient
$\mu_0$	Dynamic viscosity
$\theta$	Non-dimensional temperature $((T-T_i)/(T_w-T_i))$
$\theta_m$	Non-dimensional mixing cup temperature $((T_m-T_i)/(T_w-T_i))$
$\tau_w$	Shear stress at the wall $(-\mu(\partial u/\partial r)_w)$

*Subscripts*

f	Fluid
m	Mean value
s	Solid
w	Wall

since Kn is a strong function of pressure, and the pressure varies significantly along a micro channel.

In this paper a unified flow model was employed for transition and free molecular regimes that predicts velocity profile in the entire Knudsen regimes in channels and ducts. We will adjust the model for flow in a circular micro/nano channel filled with porous media.

**2. Unified model**

From the Direct Simulation Monte Carlo, DSMC, results and solutions of the linearized Boltzmann equation [24], it is evident that the velocity profiles of flow in a channel in the transition and free molecular regimes remain approximately parabolic. But the velocity profile obtained from continuum-based relations does not predict the flow rate properly [15].

This is because of the fact that the dynamic viscosity which is related to the diffusion of momentum due to the intermolecular collisions must be modified to consider the diffusion of momentum due to the intermolecular collisions and the collision of molecules with the walls. As mentioned in the previous section, in the transition regime, because of the rarefaction, intermolecular collisions and molecule-wall collisions have the same order and in the free molecular regime the molecule-wall collisions is the dominant phenomenon. Polard and Present [25] proposed Eq. 1 for generalized diffusion coefficient as a function of Kn:

$$\mu(Kn) = \mu_0 \left[ \frac{1}{1 + \alpha Kn} \right] \tag{1}$$

where  $\mu_0$  is the dynamic viscosity of the gas at a specified temperature and  $\mu$  is the generalized diffusion coefficient.  $\alpha$  is a parameter that varies from zero (at Kn = 0) to a constant value,  $\alpha_0$ , (as Kn  $\rightarrow \infty$ ) and can be expressed by Eq. 2 [15].

$$\alpha = \alpha_0 \frac{2}{\pi} \tan^{-1} (\alpha_1 Kn^B) \tag{2}$$

$\alpha_0$  is determined to obtain the desired free molecular flow rate as Kn  $\rightarrow \infty$ .  $\alpha_0 = 4$  and  $\beta = 0.4$  are empirical constants.

**3. Governing equations**

Steady laminar forced convection in a circular micro/nano channel filled with porous media is considered. Here the flow is assumed hydrodynamically fully developed and thermally developing. Applying the unified model, the momentum and energy equations with first order velocity slip and temperature jump boundary conditions are applicable for all Kn regimes.

Eq. 3 is the axial momentum equation [26]:

$$1 + \frac{\partial^2 U}{\partial R^2} + \frac{1}{R} \frac{\partial U}{\partial R} - \frac{1}{Da} U - \Gamma U^2 = 0 \tag{3}$$

By substituting Eq. 1 into Eq. 1, Eq. 4 is obtained:

$$1 + \frac{1}{1 + \alpha Kn} \frac{\partial^2 U}{\partial R^2} + \frac{1}{1 + \alpha Kn} \frac{1}{R} \frac{\partial U}{\partial R} - \frac{1}{Da} \frac{1}{1 + \alpha Kn} U - \Gamma U^2 = 0 \tag{4}$$

where term 1 is the non-dimensional pressure gradient along the channel. The energy equation (Eq. 5) remains unchanged:

$$U \frac{\partial \theta}{\partial Z} = \frac{1}{Pr Re_D^*} \left[ \frac{1}{R} \frac{\partial \theta}{\partial R} + \frac{\partial^2 \theta}{\partial R^2} \right] \tag{5}$$

Eqs. 4 and 5 are solved subject to the slip (Eq. 6) [27] and temperature jump (Eq. 7) [28] boundary conditions:

$$U(Z,1) = \frac{-2 - \sigma_v}{\sigma_v} Kn \left( \frac{\partial U}{\partial R} \right)_w \tag{6}$$

$$\theta(Z,1) = 1 - \frac{2 - \sigma_T}{\sigma_T} \left[ \frac{2\gamma}{\gamma + 1} \right] \frac{Kn}{Pr} \frac{\partial \theta}{\partial R} (Z,1) \tag{7}$$

Eqs. 8–10 express the boundary conditions at the entrance and center of the channel.

$$\frac{\partial U}{\partial R} (Z,0) = \frac{\partial \theta}{\partial R} (Z,0) = 0 \tag{8}$$

$$\theta(Z,R) = 0 \tag{9}$$

$$U(Z,R) = 1 \tag{10}$$

where  $\sigma_V$  and  $\sigma_T$  are the tangential momentum and thermal accommodation coefficients respectively which for most practical surfaces are assumed to be 0.7 [20].

The friction factor and Nusselt number are defined as Eqs. 11 and 12 respectively [29].

$$C_f = \frac{\tau_w}{\frac{1}{2}\rho U_0^2} = \frac{\frac{\mu}{\varepsilon} \frac{\partial U}{\partial r} \Big|_w}{\frac{1}{2}\rho f \frac{U_0^2}{\varepsilon^2}} = \frac{4}{Re_D} \frac{1}{(1 + \alpha Kn)^2} \left| \frac{dU}{dR} \right|_w \quad (11)$$

$$Nu = \frac{hD}{K_m} = \frac{2}{1 - \theta_m} \left( \frac{\partial \theta}{\partial R} \right)_w \quad (12)$$

where  $\theta_m$  is non-dimensional mixing cup temperature defined as

$$\theta_m = \frac{\int_0^1 U \theta R dR}{\int_0^1 U R dR}$$

#### 4. Results and discussion

The Eqs. 4 and 5 were solved numerically using the finite difference method. The momentum equation, Eq. 4, was discretized explicitly and was solved by iteration method, while the energy equation, Eq. 5, was discretized implicitly and the resulted system of algebraic equations was solved by Thomas algorithm [30]. To verify the results, it is noticeable that Eq. 4 with negligible microscopic inertial term ( $\Gamma=0$ ) has the analytical solution in the form:

$$U = \frac{Da(1 + \alpha Kn)}{\frac{\alpha_v - 2}{\sigma_v} \frac{Kn}{\sqrt{Da}} I_1 \left( \frac{1}{\sqrt{Da}} \right) - I_0 \left( \frac{1}{\sqrt{Da}} \right)} I_0 \left( \frac{1}{\sqrt{Da}} R \right) + Da(1 + \alpha Kn) \quad (13)$$

where  $I_0$  and  $I_1$  are the zeroth and first order Bessel functions, respectively. In Fig. 1 the exact and numerical solutions for velocity profile are shown when  $\alpha=0$ ,  $Da=1$ ,  $\Gamma=0$  and  $Kn=0.1$ . There is excellent agreement between the two solutions.

Fig. 2 shows the radial distribution of the axial velocity for different Knudsen numbers in the slip and transitional regimes when  $Da=1$ ,  $\Gamma=0.1$ .

It can be seen from the figure that by increasing Knudsen number, the velocity and velocity slip increase and at large Kn, the velocity profile is flatter.

Fig. 3 shows the variation of  $C_f Re_D^*$  with  $Da$  at different Knudsen numbers for the case that  $\Gamma=0.1$ . It can be seen that as  $Da$  increases, firstly,  $C_f Re_D^*$  increases and then it becomes constant. It is because of

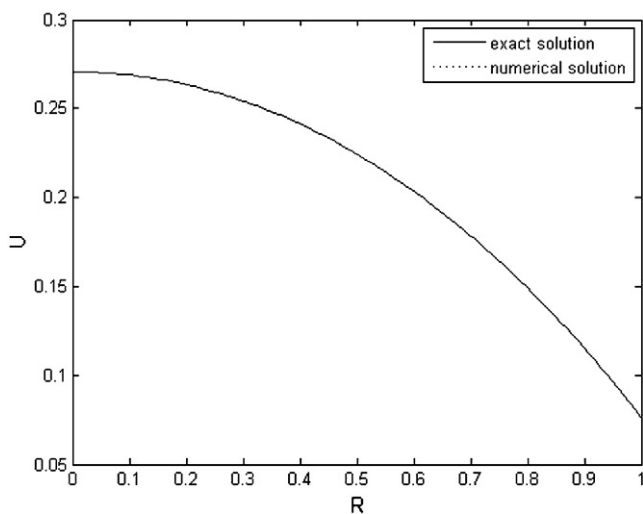


Fig. 1. Velocity profile as a function of radius.

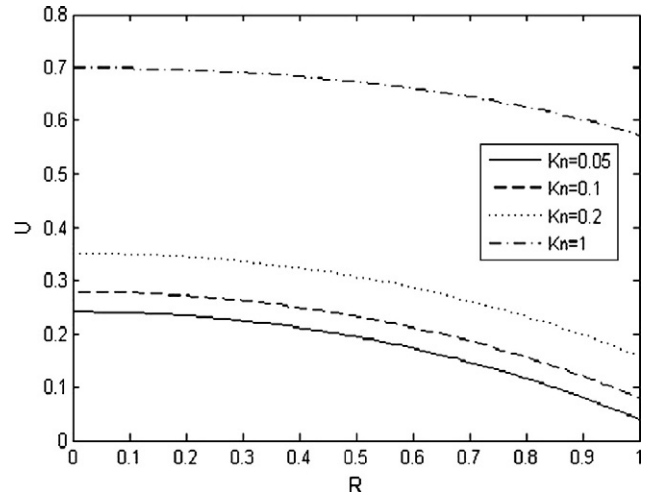


Fig. 2. Radial distribution of the axial velocity.

the fact that as  $Da$  increases, the porosity,  $\varepsilon$ , decreases and the flow in circular channel occupied with porous media approaches the flow in an ordinary pipe, for which  $C_f Re_D^*$  is constant. According to the figure, the value of  $C_f Re_D^*$  even for large Kn is a function of Kn. It does not support the result of Haddad et al. [19] because of the fact that in their work  $\mu$  is assumed constant but in the present work  $\mu$  is a function of Kn.

In Fig. 4,  $C_f Re_D^*$  versus Kn is shown for  $Da=1$ ,  $\Gamma=0.1$  and  $10^{-3} < Kn < 10$ . The figure shows that as the Kn increases,  $C_f Re_D^*$  decreases. As it was mentioned previously, any increase in Kn would result in an increase in the flow velocity and  $Re_D$ . On the other hand,  $C_f$  decreases due to the decrease in the velocity gradient near the wall. Thus  $C_f Re_D^*$  decreases with Kn. This means that the reduction in  $C_f$  is more significant, especially when Kn is in the range of  $10^{-2}$  to 3, i.e. in the transition regime.

Fig. 5 shows the effect of Kn variations on the non-dimensional temperature profile in the micro/nano channel at a certain axial location,  $Z=1$ . Because of the decrease of the pressure along the channel from  $P_i$  at  $Z=0$  to  $P_o$  at outlet, Kn, which is a function of pressure is not constant. In the figure the dashed line is for the case when Kn is assumed constant and equal to 0.3 and the solid line is the curve of the case that Kn varies along the channel according to the equation  $Kn = Kn_o \frac{P_o}{P}$  where the subscript o indicates the output conditions, where  $Z=1$ . In the present case  $Kn_o=0.3$ . It can be seen from the figure that despite the Kn in two cases is the same at  $Z=1$ ,

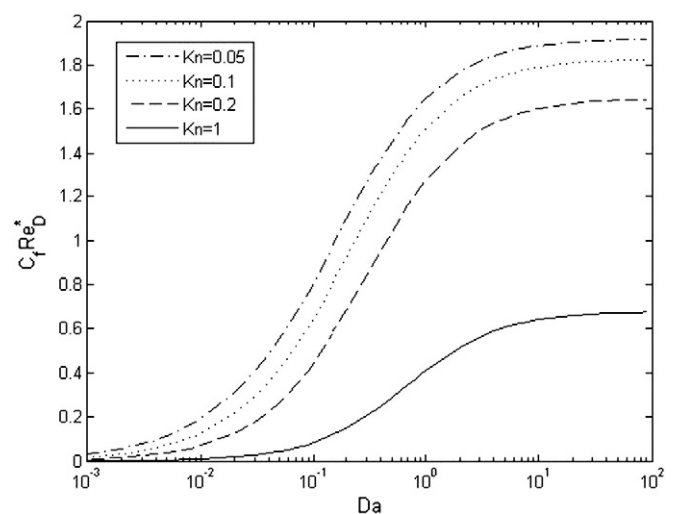


Fig. 3. Variation of  $C_f Re_D^*$  versus  $Da$ .

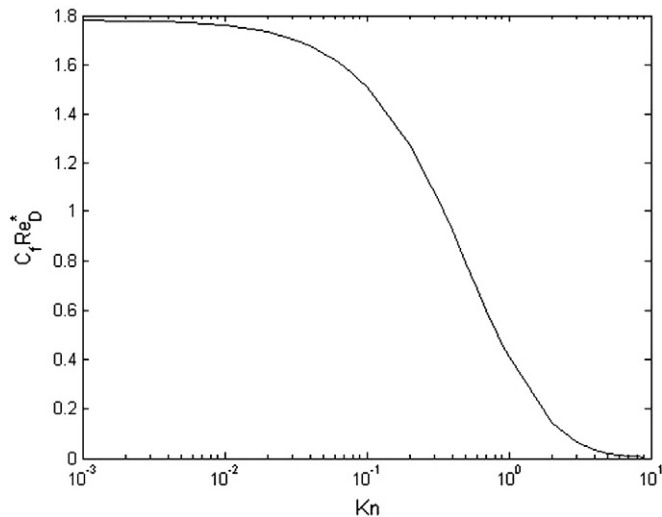


Fig. 4.  $C_f Re_D^*$  as a function of Kn.

the temperature profiles are different. This characteristic can be explained as follow: as the pressure decreases along the channel, the Knudsen number increases. Increasing Kn leads to increase the temperature jump at the wall. As can be seen from the figure, the temperature jump for the case where Kn is constant, is higher. Because Kn in the previous sections of the channel in the case where Kn is constant, is higher than the case where Kn is variable, therefore temperature jump is higher and the fluid does not feel the real temperature of the wall and therefore the temperature is lower.

Fig. 6 shows the Nusselt number along the channel length for the two cases. Despite of the fact that Kn in two cases is the same at  $Z = 1$ , Nu is not. The figure indicates that Nu in the case of constant is less than the other case, initially but it is higher downstream. Increasing Kn has two opposite effects on Nusselt number. As mentioned in the Fig. 5, the first effect is that it leads to increased temperature jump. Therefore heat flux from the wall to the fluid is lower so Nu decreases. The second effect is that it leads to increase in slip velocity at the wall, therefore larger amount of fluid is moved near the wall. As a result larger amount of heat is absorbed from the wall. On the other hand because the temperature of the fluid near the wall is higher than

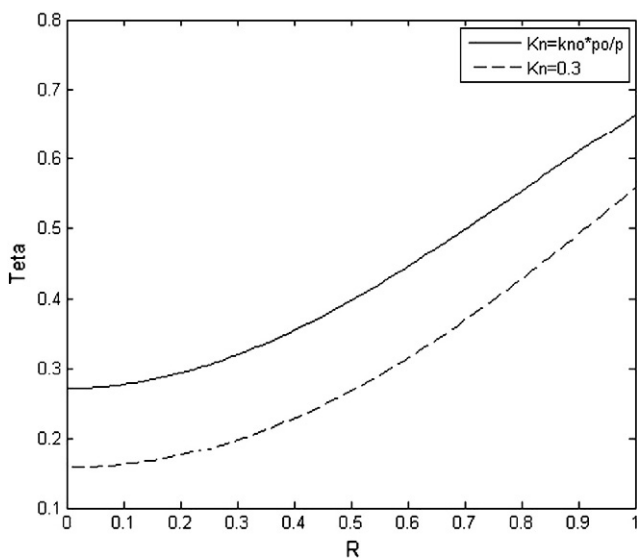


Fig. 5. Temperature distribution for two different cases.

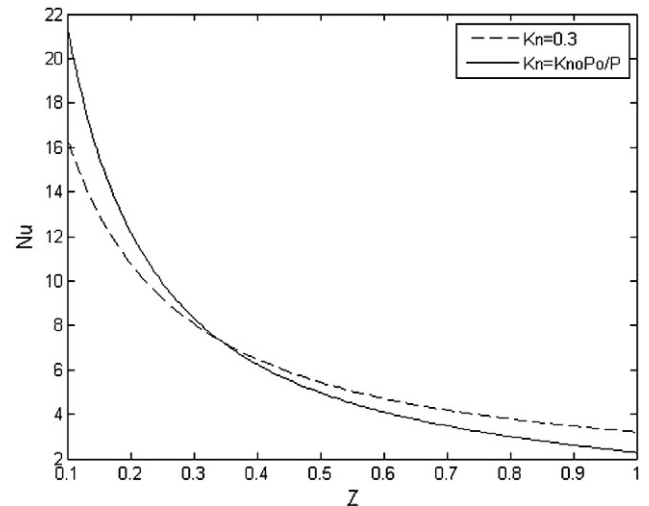


Fig. 6. Nusselt number along the channel for two different cases.

its temperature in center, the increase in velocity near the wall increases the magnitude of  $\int_0^1 UR\theta dR$ , therefore  $\theta_m$  and Nu increase.

For the actual case, when Kn is variable and  $Kn_o = 0.3$ . Therefore temperature jump and Nu for the case where Kn is constant is higher than the case that Kn is variable. But for the case that Kn is constant the amount of flow near the wall is higher, and approaching the outlet, it can dominate the Nu decrease due to the temperature jump. Therefore Nu number in the case that Kn is constant is higher.

### 5. Conclusion

Applying the Darcy–Brinkman–Forchheimer flow model and slip boundary condition, finite difference solution for fully developed velocity distribution in a micro/nano channel of circular cross section, filled with porous media was presented. Convection heat transfer and pressure drop performance of the system, reflected in Nu and  $C_f Re$  were analyzed. Effect of Kn and Da on the system performance is studied. Considering the dependency of viscosity on Kn, a unified flow model for all flow regimes with different Kn was obtained. It was observed that increasing Kn and decreasing Da result in decreasing skin friction. Also, it was found that the variation of Kn along the channel has considerable effects on Nu and temperature distribution across the channel cross section.

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