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# The Application of Redundant Units and Alternative Designs to Reliability Optimization

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**ABSTRACT**

*Broadly speaking, one of the major criteria for an engineering system implementation is the reliability of that system. The reliability coefficient, owing to the increasing advancement of sciences and technology, intricateness of today's engineering systems and thus, the impact of their unreliable behavior on various areas, on one side, and the tendency for the evaluation of this coefficient in the engineering systems and the need to improve it, on the other side, has gotten of a great significance. In order for making the best or most effective use of these systems, diverse methods are applied to acquire or produce highly reliable goods. Highly capable in optimizing the engineering systems, genetic algorithms are among the most commonly utilized methods. In the present paper, in order to apply these problems, the best or most effectively, a particular model is introduced.*

**Keywords:** Reliability optimization, evolutionary algorithm, alternative design, optimization problem.

**Mathematics Subject Classification:** 9008

**Computing Classification System:** G.1.6

## 1. INTRODUCTION

In its most extended meaning, reliability is a measure of performance of systems. The reliability of a system can be defined as the probability that the system has operated successfully over a specified interval of time under stated conditions (Coit and Liu, 2000). The main goal of reliability engineering is to improve system reliability. In the initial design activity, redundancy allocation is a direct way of enhancing the system reliability (Coit, 2003). As Singh, et. al, (2016) highlighted engineers and technical managers are accountable for the planning, designing, producing and utilization of the most elementary product to the most intricate systems in any modern community. Disturbances at various levels would be resulted from the failure of the products and systems, likely to be regarded as a serious threat to the community and the environment. In consequence, the products and systems are desired by the users, i.e. the people of the society in total, to be reliable, satisfactory and safe. Therefore, the extent to which a system, over its future work life, would be reliable and safe is substantially questioned.

In a broad sense, one of the system implementation criteria is the reliability. Due to more complexity of the systems, the influence of their unreliable behavior in varied areas has gotten more serious. Similarly, the tendency for the evaluation of the reliability coefficient of the systems and the need for the improvement of this coefficient have been remarkably regarded. Consequently, the researchers and managers attempt to utilize the systems' reliability most efficiently employing different methods expanded, to a greater extent, with the advancement in sciences and computer use (such as Mohammadi and Mansouri, 2016; Wang, et al, (2013); and Antonyová, et al. (2013)). One of the optimization methods modeled on nature, while utilizing computer tools is the genetic algorithms. Currently, this method is applied by the engineers and researchers to a greater degree. Rao (2013) utilized ML methods of estimation to calculate the reliability and used Monte Carlo simulation to compare asymptotically its estimator to evaluate the multicomponent stress-strength reliability. He used a set of real data to demonstrate the procedure. Using neural network, Gong, et. al. (2013) indicated a newly combined prediction model and demonstrated the decrease and increase of prediction risk and accuracy respectively. Wang, et.al. (2013) highlighted that Hilbert-Huang Transform, time series modeling procedure and grey theory can be applied for degradation prediction and reliability evaluation. This paper made evident the influential aspect of these models in reliability determination. Montazer Haghighi and Shayib (2010) utilized the Maximum Likelihood Estimator and studied the sample size influence and parameters ratio values to estimated reliability. Although there were many researches about reliability, few of them focused on its optimization through alternative designs and it is necessary to do deep research on its different aspect. Therefore, the objective of the present work paper is to investigate the use of redundant units and alternative designs in reliability optimization, a neglected area to the best of researchers' knowledge.

To accomplish this aim and to respond to a recent call for research to do, this paper employs genetic algorithms to optimize a reliability problem whose objective function and restrictions are fuzzy. It examines the reliability structures. Next, the general trend and used genetic algorithms operators are mentioned. Eventually, we attain the optimum solution of the objective function. Additionally, via multiple failure models, the optimization model of reliability is investigated.

## 2. OPTIMIZATION OF RELIABILITY

Suppose that the system includes $N$ subsystem. Related to every subsystem there are various options of design alternatives that can be applied by the system designer to satisfy the requirements of reliability. We consider the following example:

$$\max \quad R(m,\alpha) = \prod_{i=1}^{14}\left(1-\left(1-R_i(\alpha_i)\right)^{m_i}\right) \qquad (4.1)$$

$$s.t. \quad C(m.\alpha) = \sum_{i=1}^{14} C_i\,(\alpha_i) \times m_i \leq 130 \qquad (4.2)$$

$$W(m.\alpha) = \sum_{i=1}^{14} W_i\,(\alpha_i) \times m_i \leq 170 \qquad (4.3)$$

$$1 \leq m_i \leq u_i \qquad (4,4)$$

$$1 \leq \alpha_i \leq \beta_i \qquad (4,5)$$

$$m_i \geq 0 \;\; and \;\; \alpha_i \geq 0 \;\; are\ integer. \;\; i = 1.2.\cdots.N \qquad (4.6)$$

In which $\alpha_i$ is the sign for the design alternatives accessible to the $i$th subsystem, $m_i$ shows the equal units used in redundancy for the $i$th subsystem, $u_i$ is the upper bound of redundancy for the $i$th subsystem, $\beta_i$ is the upper bound of alternative design for the $i$th subsystem. The problem that raise here is how to decide which alternative to choose and then to determine how many unnecessary units are needed to be used in order to reach the highest reliability while the overall system cost and weight are kept in the allowable amounts.

Determine which alternative to select and then decide how many redundant units to use in order to achieve the greatest reliability while keeping the total system cost and weight within the allowable amounts. Data are shown in Table 1.1.

## 3. GENETIC ALGORITHM AND NUMERICAL EXAMPLE

Genetic algorithms are a branch of social optimization techniques in which biological systems and their principles are used. Although these algorithms provide simple models of biological processes in normal circumstances, in practice have shown a lot of ability and efficiency. The basic idea of providing genetic algorithms is using limited population of elements, each of which precisely determines a point of the search space; where algorithm of population is defined by chromosomes. After this stage, chromosomes population (people) enters the evolutionary process in nature. At first, randomly a population of chromosomes is generated then recombination, mutation, and selection operators appear, then depending on the need other operators of evolution are applied on this chromosome and a new generation of chromosomes can be achieved and then for the new generation optimality criteria are checked. If optimality criteria are met for this new generation, algorithm stops and the best chromosome is chosen as the optimum solution for transport model. Otherwise, different genetic operators are applied on chromosomes within the population, new generations are generated, and until meeting the optimality criteria, the process continues. Genetic algorithm is superior to other methods of optimization algorithms and because of this tend to use them is increasing. Since the beginning of the development of genetic algorithms, the experts tried to make these algorithms simpler and more understandable, and to provide models of genetic algorithms that have less operation and great capabilities in the fields of optimization and the outputs of these algorithms consider the best answer to the problem.

**Representation of Chromosome and Numerical Example,** A gene is defined as an ordered couple of design alternatives $\alpha_{ki}$ and redundant units $m_{ki}$ shown as follows:

$$\boldsymbol{v_{ki}} = (\boldsymbol{\alpha_{ki}}.\boldsymbol{m_{ki}})$$

**Table 1.1: Redundant Units and Alternative Design**

| subsystem | Design Alternative | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | 1 | | | 2 | | | 3 | | | 4 | | |
| i | R | $c_i$ | $w_i$ | R | $c_i$ | $w_i$ | R | $c_i$ | $w_i$ | R | $c_i$ | $w_i$ |
| 1 | 0.90 | 1 | 3 | 0.93 | 1 | 4 | 0.91 | 2 | 2 | 0.95 | 2 | 5 |
| 2 | 0.95 | 2 | 8 | 0.94 | 1 | 10 | 0.93 | 1 | 9 | * | * | * |
| 3 | 0.85 | 2 | 7 | 0.90 | 3 | 5 | 0.87 | 1 | 6 | 0.92 | 4 | 4 |
| 4 | 0.83 | 3 | 5 | 0.87 | 4 | 6 | 0.85 | 5 | 4 | * | * | * |
| 5 | 0.94 | 2 | 4 | 0.93 | 2 | 3 | 0.95 | 3 | 5 | * | * | * |
| 6 | 0.99 | 3 | 5 | 0.98 | 3 | 4 | 0.97 | 2 | 5 | 0.96 | 2 | 4 |
| 7 | 0.91 | 4 | 7 | 0.92 | 4 | 8 | 0.94 | 5 | 9 | * | * | * |
| 8 | 0.81 | 3 | 4 | 0.90 | 5 | 7 | 0.91 | 6 | 6 | * | * | * |
| 9 | 0.97 | 2 | 8 | 0.99 | 3 | 9 | 0.96 | 4 | 7 | 0.91 | 3 | 8 |
| 10 | 0.83 | 4 | 6 | 0.85 | 4 | 5 | 0.90 | 5 | 6 | * | * | * |
| 11 | 0.94 | 3 | 5 | 0.95 | 4 | 6 | 0.96 | 5 | 6 | * | * | * |
| 12 | 0.79 | 2 | 4 | 0.82 | 3 | 5 | 0.85 | 4 | 6 | 0.90 | 5 | 7 |
| 13 | 0.98 | 2 | 5 | 0.99 | 3 | 5 | 0.97 | 2 | 6 | * | * | * |
| 14 | 0.90 | 4 | 6 | 0.92 | 4 | 7 | 0.95 | 5 | 6 | 0.99 | 6 | 9 |

$\boldsymbol{\beta_i = 4\ \ for\ \ i = 1.3.6.9.12.14.\ \ \ \beta_i = 3\ \ for\ \ i = 2.4.5.7.8.10.11.13.\ \ and\ \ u_i = 5\ \forall\ i.}$

Where the subscript $k$ is the index of chromosome the gene belongs to and the subscript $i$ show the subsystem $i$. A chromosome is an ordered list of such genes as follows:

$$v_k = [v_{k1}\ \ v_{k2}\ \ \cdots\ \ v_{k14}]$$

The representation can be revised as follows:

$$v_k = [(\alpha_{k1}.m_{k1})\ \ (\alpha_{k2}.m_{k2})\ \ \cdots\ \ (\alpha_{k14}.m_{k14})]$$

**Initial Population.** The initial population of chromosomes is generated randomly as follows:

**Procedure: Initialization**
**begin**
**for** k=1 **to** $pop_{size}$ **do**
  **for** i=1 **to** 14 do
    $\alpha_{ki}$= random(1, $\boldsymbol{\beta_i}$);
    $m_{ki}$= random(1, $\boldsymbol{u_i}$);
  **end**;
$v_k = [(\alpha_{k1}.m_{k1})\ \ (\alpha_{k2}.m_{k2})\ \ \cdots\ \ (\alpha_{k14}.m_{k14})]$
**end**
**end.**

Where random(1,num) means to return a random integer within the range $[1.num]$. Let $pop_{size} = 4$; the randomly generated chromosomes are given as follows:

$$v_1 = [\ (2.1)\ \ (1.1)\ \ (4.1)\ \ (1.4)\ \ (3.4)\ \ (4.2)\ \ (2.1)\ \ (1.3)\ \ (3.2)\ \ (2.2)\ \ (2.1)\ \ (1.2)\ \ (3.4)\ \ (3.4)\ ]$$
$$v_2 = [\ (2.2)\ \ (1.3)\ \ (2.3)\ \ (1.1)\ \ (2.5)\ \ (2.1)\ \ (1.1)\ \ (3.1)\ \ (4.1)\ \ (3.2)\ \ (1.2)\ \ (1.2)\ \ (1.2)\ \ (1.1)\ ]$$
$$v_3 = [\ (1.3)\ \ (2.3)\ \ (3.1)\ \ (3.2)\ \ (2.3)\ \ (1.1)\ \ (2.1)\ \ (3.1)\ \ (4.1)\ \ (3.4)\ \ (1.2)\ \ (1.2)\ \ (3.4)\ \ (2.1)\ ]$$
$$v_4 = [\ (3.2)\ \ (3.1)\ \ (3.3)\ \ (2.2)\ \ (2.3)\ \ (4.2)\ \ (3.2)\ \ (3.3)\ \ (3.3)\ \ (3.1)\ \ (2.1)\ \ (1.3)\ \ (3.2)\ \ (1.2)\ ]$$

### 3.1. Chromosome Evaluation

If the genetic algorithms are employed to a large-scale integer optimization problem, there will be a serious problem of illegal production of chromosomes which disrupt the system constrains themselves. This major challenge is the outcome of chromosomes produced not only by the genetic operations but also by random initial operations. For the prevention of the undesirable impact of these illegal chromosomes on the system constrains, as a shortcut, we can meet them out a severe punishment which can cause the evolutionary process be erased from illegal chromosomes. The existence of the punishment creates an ineffective and narrowed search space and causes difficulties finding better candidates for the global optimum with any selection mechanism. As a result, the distance of the illegal chromosomes from the possible origin is defined as a special measure function to solve this problem. By the interaction of a constant and severe punishment with each illegal chromosome the refusal of illegal chromosomes from the evolutionary procedure and the genetic search selection of the optimum at the boundary is possible. If the fitness evolution encompasses the information of the degree of infeasibility of the solutions, the optimum from both the feasible and infeasible area will be approached by the genetic search which is broader as is shown the bellow figure as the measure of infeasibility degree for an illegal chromosome.

$$d_{kt} = \begin{cases} 0 & G_t(m.\alpha) \le b_t \\ \dfrac{(G_t(m.\alpha) - b_t)}{b_t} & otherwise \end{cases}$$

Where the index $k$ shows the $k$th chromosome and $t$ shows the $t$th limitation and $T$ is total number of limitations. Then the fitness function defined as follows:

$$eval(v_k) = R(m.\alpha)\left(1 - \frac{1}{T}\sum_{t=1}^{T} d_{kt}\right)$$

The primary chromosomes fitness values are

$$eval(v_1) = 0.656578. \quad eval(v_2) = 0.516839. \quad eval(v_3) = 0.561119. \quad eval(v_4) = 0.573369$$

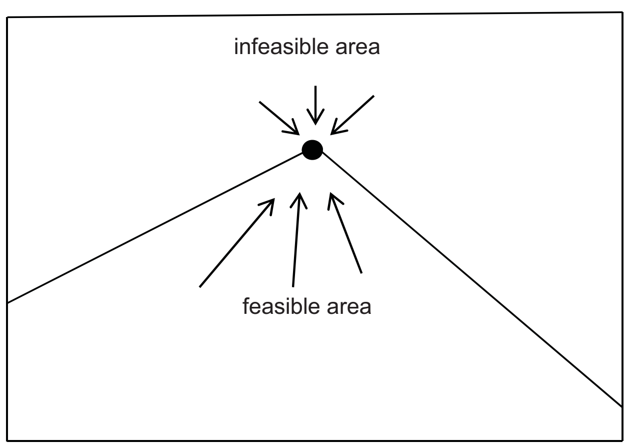


Figure 4.3. Genetic search direction

### 3.2. Recombination

We use the same recombination which is the best alteration for the problem; it has been recognized to be greater to traditional recombination plans for the combinatorial problem. The first thing that a uniform recombination creates is a random crossover mask and then exchanges associated genes among patents according to the mask. A crossover mask is simply a binary string with the same size of chromosome. The equality of each bit in the mask establishes, for each corresponding bit in an offspring from which parent it will receive that bit. This is shown in the following figure.

A random mask:

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| mask | 0 | 0 | 1 | 0 | 0 | 1 | 1 | 0 |

Parents under mask:

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| $v_i$ | $v_{i1}$ | $v_{i2}$ | $v_{i3}$ | $v_{i4}$ | $v_{i5}$ | $v_{i6}$ | $v_{i7}$ | $v_{i8}$ |
| $v_k$ | $v_{k1}$ | $v_{k2}$ | $v_{k3}$ | $v_{k4}$ | $v_{k5}$ | $v_{k6}$ | $v_{k7}$ | $v_{k8}$ |

Exchange relative genes:

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| $o_i$ | $v_{i1}$ | $v_{i2}$ | $v_{k3}$ | $v_{i4}$ | $v_{i5}$ | $v_{k6}$ | $v_{k7}$ | $v_{i8}$ |
| $o_k$ | $v_{k1}$ | $v_{k2}$ | $v_{i3}$ | $v_{k4}$ | $v_{k5}$ | $v_{i6}$ | $v_{i7}$ | $v_{k8}$ |

**Figure 4.4. demonstration of the uniform recombination operation.**

Let the probability of recombination be $p_c = 0.35$ and let the sequence of random numbers be

0.381120 0.141165 0.263125 0.684523

If a random number $r$ is less than $p_c$, select relative chromosome for recombination. It demonstrates that the chromosomes $v_2$ and $v_3$ were chosen for recombination like parents. The randomly produced positions are 4, 7, and 13; then we have

$$v_2 = [\,(2.2)\;(1.3)\;(2.3)\;(1.1)\;(2.5)\;(2.1)\;(1.1)\;(3.1)\;(4.1)\;(3.2)\;(1.2)\;(1.2)\;(1.2)\;(1.1)\,]$$
$$v_3 = [\,(1.3)\;(2.3)\;(3.1)\;(3.2)\;(2.3)\;(1.1)\;(2.1)\;(3.1)\;(4.1)\;(3.4)\;(1.2)\;(1.2)\;(3.4)\;(2.1)\,]$$

The consequent offspring by replacing relative genes of their parents will be

$$o_2 = [\,(2.2)\;(1.3)\;(2.3)\;(\mathbf{3.2})\;(2.5)\;(2.1)\;(\mathbf{2.1})\;(3.1)\;(4.1)\;(3.2)\;(1.2)\;(1.2)\;(\mathbf{3.4})\;(1.1)\,]$$
$$o_3 = [\,(1.3)\;(2.3)\;(3.1)\;(\mathbf{1.1})\;(2.3)\;(1.1)\;(\mathbf{1.1})\;(3.1)\;(4.1)\;(3.4)\;(1.2)\;(1.2)\;(\mathbf{1.2})\;(2.1)\,]$$

The fitness values of the offspring are

$$eval(o_1) = 0.462970 \qquad eval(o_2) = 0.651314$$

### 5.3. Mutation

The performance of random perturbation is Mutation. For a chosen gene $v_{ki} = (\alpha_{ki}.m_{ki})$, $\alpha_{ki}$ will be exchanged by a random integer within $[1.\beta_i]$ and $m_{ki}$ will be exchanged by a random integer within$[1.u_i]$.

Assume the probability of mutation be $p_m = 0 \cdot 1$; it means that, by standard, $14 \times 4 \times 0 \cdot 1 = 5 \cdot 6$ bits were created, and three of them were smaller than 0.1; the bit number, the list below demonstrates relative chromosomes, and the bit position within each chromosome:

| Bit Number | chromosome | position | Random Number |
|---|---|---|---|
| 22 | 2 | 8 | 0.065482 |
| 31 | 3 | 3 | 0.024689 |
| 43 | 4 | 1 | 0.096512 |

That is to say, the chromosomes $v_2 . v_3$ and $v_4$ we selected for mutation. The consequent offspring will be:

$$o_2 = [\,(2.2)\ (1.3)\ (2.3)\ (1.1)\ (2.5)\ (2.1)\ (1.1)\ (3.1)\ (4.1)\ (3.2)\ (1.2)\ (1.2)\ (1.2)\ (1.1)\,]$$
$$o_3 = [\,(1.3)\ (2.3)\ (1.5)\ (3.2)\ (2.3)\ (1.1)\ (2.1)\ (3.1)\ (4.1)\ (3.4)\ (1.2)\ (1.2)\ (3.4)\ (2.1)\,]$$
$$o_4 = [\,(3.4)\ (3.1)\ (4.3)\ (3.1)\ (1.1)\ (2.3)\ (3.2)\ (2.1)\ (3.3)\ (2.2)\ (3.5)\ (1.1)\ (3.1)\ (1.2)\,]$$

The fitness values of offspring are

$$eval(o_2) = 0 \cdot 567215. \quad eval(o_3) = 0 \cdot 613955. \quad eval(o_4) = 0 \cdot 493827$$

### 3.4. Selection

The selection strategy is adopted for the deterministic selection; e.g. we arrange all parents and offspring in descending sort and chose the first $pop\_size$ chromosomes as the new population.

$$v_1' = [\,(3.2)\ (3.1)\ (3.3)\ (2.2)\ (2.3)\ (4.2)\ (3.2)\ (3.3)\ (3.3)\ (3.1)\ (2.1)\ (1.3)\ (3.2)\ (1.2)\,]\ (v_4)$$
$$v_2' = [\,(2.2)\ (1.3)\ (2.3)\ (1.1)\ (2.5)\ (2.1)\ (1.1)\ (3.1)\ (4.1)\ (3.2)\ (1.2)\ (1.2)\ (1.2)\ (1.1)\,]\ (o_2)$$
$$v_3' = [\,(2.1)\ (1.1)\ (4.1)\ (1.4)\ (3.4)\ (4.2)\ (2.1)\ (1.3)\ (3.2)\ (2.2)\ (2.1)\ (1.2)\ (3.4)\ (3.4)\,]\ (v_1)$$
$$v_4' = [\,(1.3)\ (2.3)\ (1.5)\ (3.2)\ (2.3)\ (1.1)\ (2.1)\ (3.1)\ (4.1)\ (3.4)\ (1.2)\ (1.2)\ (3.4)\ (2.1)\,]\ (o_3)$$

The fitness values of new generation are

$$eval(v_1') = 0 \cdot 753369. \quad eval(v_2') = 0 \cdot 567215. \quad eval(v_3') = 0 \cdot 656578. \quad eval(v_4') = 0 \cdot 613955$$

From a random run, the best chromosome at the 452rd generation is

$$v^* = [\,(3.3)\ (1.2)\ (4.3)\ (3.3)\ (2.3)\ (2.2)\ (1.2)\ (1.4)\ (3.2)\ (2.3)\ (1.2)\ (1.4)\ (2.2)\ (3.2)\,]$$

The reliability of system is 0.980124. The statistical data gathered over 30 runs are as the following:

| **Total Runs** | **30** |
|---|---|
| **Frequency for getting optima** | 0.170 |
| **Earliest generation for getting optima** | 161 |
| **Latest generation for getting optima** | 975 |
| **Average generation for getting optima** | 423.8 |

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