IMPLEMENTATION OF EVOLUTIONARY ALGORITHM AND FUZZY SETS FOR RELIABILITY OPTIMIZATION OF ENGINEERING SYSTEMS

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ABSTRACT

This article uses an evolutionary algorithm to solve the series parallel redundancy optimization problem which is in a fuzzy framework. Reliability optimization provides a means to help the reliability engineer achieve such a goal. Most methods of reliability optimization assume that systems have redundancy components in series and/or parallel systems and that alternative designs are available. Optimization concentrates on optimal allocation of redundancy components and optimal selection of alternative designs to meet system requirements. A fuzzy simulation based evolutionary algorithm is then employed to solve these kinds of fuzzy programming with fuzzy Goal and fuzzy constraints. Finally, numerical examples are also given.

Keywords: evolutionary algorithm, reliability analysis, reliability optimization, genetic algorithm, Fuzzy simulation

As systems have grown more complex, the consequences of their unreliable behavior have become severe in terms of cost, effort, lives, and so on, and the interest in assessing system reliability and the need for improving the reliability of products and systems have become very important. The reliability of a system can be defined as the probability that the system has operated successfully over a specified interval of time under stated conditions. In the past few decades, the field of reliability has grown sufficiently large to include separate specialized subtopics, such as reliability analysis, failure modelling, reliability optimization, reliability growth and its modelling, reliability

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and optimal selection of alternative designs to meet system requirements. In the past two decades, numerous reliability optimization techniques have been proposed. Generally, these techniques can be classified as linear programming, dynamic programming, integer programming, and geometric programming, Heuristic method, Lagrange multiplier, and

MODES

Using redundant components is well accepted as a technique to improve the reliability of a system. Usually this problem can be formulated as a nonlinear integer

Notation and Nomenclature

k- Out - of - n: F the system is failed if and only if at least k out of its n elements are

k - Out - of - n: G the systems good if and only if at least k out of its n elements are good

Class O failure modes: Those of which subsystem i is 1 - out - of - mi : F

Class A failure modes: Those of which subsystem i is 1 - out - of - mi : G

Si : total number of failure modes in subsystem i

qi : probability of failure mode u for each element in subsystem i

ui : upper bound of m i element in subsystem i

gti (mi): subsystem i requires this amount of resource t when it contains mi elements

Failure probability in subsystem i (m i elements) for failure mode u of :

\( Q^o_u(m_i), Q^A_u(m_i) \) the class of O or A failure modes

Failure probability in subsystem i (m i elements) subject to class of O :

\( Q^o(m_i), Q^A(m_i) \)

Qi (mi): unreliability in subsystem i (m i elements) R(m): reliability of the system when the element allocation is m

The following assumptions are made for the problem

All elements in subsystem i are s – independent with respect to failure mode u, for each i, u combination taken by itself

The system is 1 - out - of - N: F with respect to its subsystems

The failure modes are a partitioning (mutually exclusive and exhaustive) of the failure event. Thus failure probabilities for failure modes combine to obtain total failure probability

Within a subsystem, the elements are 1 - out – of - mi : G for some failure modes while at the same time are 1 - out - of - mi : F for the other failure modes
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Failures occur when the entire subsystem is subjected to the failure condition. All elements in subsystem i can be failed by only one of the s_i modes at any given time, and

The failure probability $Q^o_{u}(m_i)$ in subsystem i for failure mode u of class O failure modes is

$$Q^o_{u}(m_i) = 1 - (1 - q_{iu})^{m_i} \quad u = 1,2,\ldots, h_i$$

The failure probability $Q^o (m_i)$ in subsystem i subject to the class O failure modes is

$$Q^o (m_i) = \sum_{u=1}^{h_i} Q^o_{u}(m_i)$$

The failure probability $Q^a_{u}(m_i)$ in subsystem i for failure modes u of class A is

$$Q^a_{u}(m_i) = (q_{iu})^{m_i} \quad u = h_i + 1, h_i + 2, \ldots, s_i$$

The failure probability $Q^a (m_i)$ in subsystem i subject to the class A failure modes is

$$Q^a (m_i) = \sum_{u=h_i+1}^{s_i} Q^a_{u}(m_i)$$

The unreliability $Q_i (m_i)$ of subsystem i is obtained by adding the failure probabilities for class O failure modes to those for class A failure modes; that is

$$Q_i (m_i) = Q^o (m_i) + Q^a (m_i)$$

Then the reliability optimization of redundant system with several failure modes can be formulated as follows

$$\text{Max} \quad R(m) = \prod_{i=1}^{N} (1 - Q_i (m_i)) \quad (4)$$

$$\text{St} \quad g_t(m_i) = \sum_{i=1}^{N} g_t (m_i) \leq b_t \quad t = 1,2,\ldots,T \quad (5)$$

$$1 \leq m_i \leq u_i \quad i = 1,2,\ldots,N \quad (6)$$

The optimization problem is to determine the element allocation which maximizes the nonlinear system reliability (4) subject to
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3. RELIABILITY OPTIMIZATION WITH FUZZY GOAL AND FUZZY CONSTRAINTS

3.1. Problem Formulation

The reliability optimization of redundant system with fuzzy goal and fuzzy constraints can be formulated as follows [5]:

\[ \text{Max } R(m) = \prod_{i=1}^{N} (1 - Q_i(m_i)) \succ g_0 \quad (7) \]

\[ \text{S.t } G_t(m) = \sum_{i=1}^{N} g_{it}(m_i) < t, \quad t=1,2,\ldots,T \quad (8) \]

\[ 1 \leq m_i \leq u_i, \quad i=1,2,\ldots,N \quad (9) \]

The symbols \( p \) and \( f \) represent fuzzy inequality, \( g_0 \) represents the desired goal of the reliability \( R(m) \) given by decision maker and the other variable have been defined in the previous section. Let \( 0 \leq Z \) be the worst tolerable value of system reliability. The membership function for objective (7) is defined as follows [7].

\[ \mu_t(m) = \begin{cases} 
1 & G_t(m) < b_t \\
1 - \frac{G_t(m) - b_t}{\delta_t} & b_t \leq G_t(m) \leq b_t + \delta_t \\
0 & G_t(m) < b_t + \delta_t 
\end{cases} \quad (11) \]

Let \( Z_0 \) be the tolerable overuse of the \( t \) - The membership function \( \mu_0(m) \) for

\[ \mu_0(m) = \begin{cases} 
1 & R(m) > g_0 \\
\frac{R(m) - Z_0}{g_0 - Z_0} & Z_0 \leq R(m) \leq g_0 \\
0 & R(m) < Z_0 
\end{cases} \quad (10) \]

Fuzzy reliability optimization problem can transform from (7) to (9) in to equivalent crisp
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Max \[ f(m) = \sum_{t=0}^{T} w_t \mu_t(m) \] (12)

S.t. \[ R(m) - (g_0 - \bar{z}_0) \mu_0(m) \geq \bar{z}_0 \] (13)
\[ (\mu_t(m) - 1)\delta_t + G_t(m) \leq b_t, t = 1, 2, \ldots, T \] (15)

Where \( w_t, t = 0, 1, \ldots, T \) are the weights of membership function \( \mu \) given by the decision maker with

The fuzzy reliability optimization problem is to determine the best number of redundancy units for each subsystem to maximize the grades of system designers' satisfaction for both the achievement of objective of reliability and the best utilization of

Genetic algorithm is stochastic search techniques based on the mechanism of natural selection and natural genetics. Genetic algorithms, deferring from conventional

The strength of each chromosome is the objective function (fitness) that must be optimized. The new generation is formed by three basic operations: reproduction, crossover and mutation from last generation according to their fitness value which provide them with higher probabilities of being selected finally, the population stabilizes, because no better individual can be found. When algorithm converges, and most of individuals in the population are almost identical, it represents a suboptimal solution. A genetic algorithm has three parameters: the population size, crossover rate and mutation rate, these parameter are important to determine the performance of the algorithm. The

To apply GA to solve a specific problem, one has to define the solution representation and the coding of the control variables. Here chromosome is defined as an ordered list of the number of redundant units’ \( m_{x_i} \) shown as follows: \( V_x = [m_{x_1} m_{x_2} \ldots m_{x_6}] \)

The initialized procedure will select the initial population within the range of the control variable with a random number generator. The user can specify the population number in

When evaluating chromosome, each legal one is assigned the value of objective function
Crossover is one of the main distinguishing features of GA that make them different from other algorithms. Its main aim is to recombine blocks on different individual to make a new one. Discrete recombination form is used here as crossover operator with \( P_c = 0.74 \) as crossover probability \[9\].

\[
\text{fitness}(V_k) = \begin{cases} 
  f(m) & R(m) \geq Z_0, \ G_t(m) \leq b_t + \delta_t, \\
  1 \leq m_i \leq u_i & \forall i, t \\
  0 & \text{Otherwise}
\end{cases} \tag{16}
\]

Mutation is performed as random perturbation in the population to avoid premature convergence to local optimum. For a selected chromosome's gene \( nki \), it will be replaced

\[
O_1 = \lambda_1 P_1 + (1 - \lambda_1) P_2 \\
O_2 = \lambda_2 P_2 + (1 - \lambda_2) P_1 \\
\lambda_1, \lambda_2 \in \{0, 1\}
\]

The general selection operator is roulettewheel, also called stochastic sampling with replacement. This is a stochastic algorithm and involves the following technique. The individuals are mapped to contiguous segments of a line, such that each individual's segment is equal in size to its fitness. A random number is generated and the individual whose segment spans the random number is selected. The process is repeated until the desired number of individuals is obtained called mating population. This technique is analogous to a roulette wheel with each slice proportional in size to the fitness. But here rank of each individuals use instead of fitness. In ranking selection, the population is sorted according to the objective values. The fitness assigned to each individual depends

Consider \( N \) the number of individual in this population (least fit individual has \( p = 1 \), the

\[
\text{Rank} \ (P) = 2 - sp + 2 + (sp -1)(p-1)/(n-1)
\]

In elitism one or more of the best individuals is selected and transfer directly to next
The following example, modified by substitution last parameter with defined numbers to maximize the nonlinear reliability objective subject to fuzzy nonlinear constraints:

$$R(m) = \prod_{i=1}^{6} \left[ 1 - \left( 1 - q_{ii} \right)^{m_i+1} \right] - \sum_{i=1}^{4} \left( q_{ii} \right)^{m_i+1} \geq 0.93$$

(19)

$$G_1(m) = (m_1)^2 + (m_2)^2 + (m_3)^2 + (m_4 + 2)^2 + (m_5)^2 + (m_6)^2 \leq 51$$

$$G_2(m) = 20 \sum_{i=1}^{6} \left( m_i + \exp(-m_i) \right) \geq 260$$

$$G_3(m) = 20 \sum_{i=1}^{6} \left( m_i \exp(-m_i/4) \right) \leq 140$$

$$1 \leq m_i \leq 3 \text{ positive integer: } i = 1,2,\ldots,6$$

(20)

Where $m = [m_1\ldots m_6]$. The subsystems are subject to four failure modes ($s_i=4$) with one O failure ($h_i=1$) and three A failure, for subsystems $i = 1,\ldots, 6$. For each subsystem the failure probabilities are shown in Table (1).

$$g_0 = 0.93, z_0 = 0.86, \delta_1 = 14.0, \delta_2 = 8.0, \delta_3 = 8.0, w_0 = 0.85, w_1 = 0.05 \text{ and } w_2 = 0.05$$

respectively. The above problem can be transformed into the following equivalent crisp

$$f(m) = 0.85\mu_0(m) + 0.05\mu_1(m) + 0.05\mu_2(m) + 0.05\mu_3(m)$$

$$R(m) - (g_0 - z_0)\mu_0(m) \geq z_0$$

$$(\mu_1(m) - 1)\delta_1 + (m_1)^2 + (m_2)^2 + (m_3)^2 + (m_4 + 2)^2 + (m_5)^2 + (m_6)^2 \leq 51$$

$$(1-\mu_2(m))\delta_2 + 20 \sum_{i=1}^{6} \left( m_i + \exp(-m_i) \right) \geq 260$$

$$(1-\mu_3(m))\delta_3 + 20 \sum_{i=1}^{6} \left( m_i + \exp(-m_i/4) \right) \geq 140$$

$$m_i \geq 0 \text{ integer, } i = 1,2,\ldots,6$$
Reliability membership is shown as follows which depicted in figure 1 and in figure 2 membership function for First constraint is illustrated which shows legal tolerance for the first constraint.

Figure 1: Reliability membership function

Second and third membership function is depicted simultaneously in figure 3 & 4 which.
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By using mentioned algorithm and operators search process performed to reach solution the best solution from a random run of genetic algorithm with 100 generation is

\[ V^* = [3, 2, 3, 3, 2, 2] \]

<table>
<thead>
<tr>
<th>Subsystem</th>
<th>Failure modes</th>
<th>Failure Probabilities</th>
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<td>1</td>
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<tr>
<td></td>
<td>A</td>
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<td></td>
<td>A</td>
<td>0.18</td>
</tr>
<tr>
<td>2</td>
<td>O</td>
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<tr>
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<td>5</td>
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<tr>
<td></td>
<td>A</td>
<td>0.10</td>
</tr>
</tbody>
</table>

The values of corresponding membership function are:

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Genetic and Evolutionary Algorithm for use with matlab toolbox http://www.geatbx.com