Forecasting the spread of epidemic diseases based on a modified SEIR model using the fuzzy-fractal system

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Abstract: Epidemics like SARS, Ebola, and COVID-19 (the most contagious recent virus) have caused global health and economic crises. Controlling pandemics requires understanding their complexity through mathematical modeling and fuzzy controllers. This study uses a model to forecast outbreaks and a genetically optimized fuzzy controller to manage restrictions like social distancing and quarantine, minimizing infections while accounting for economic impact via GDP growth rate as a cost function. Additionally, cultural factors, represented by fractal dimension, are analyzed, showing significant effects on reducing infections. Results demonstrate that incorporating fractal dimension improves pandemic control.

Keywords: Epidemic diesease, Pandemic Control, Mathematical Model, modified SEIR Model, Fuzzy System, Fractal Dimension

1. Introduction

Various global epidemics, including influenza, SARS, MERS, Ebola, and the recent COVID-19, highlight the need to understand their complexity through mathematical modeling. Differential equation-based models, alongside artificial intelligence (AI) techniques like fuzzy systems, help predict and control pandemics [1-12].The basic SIR model [1] includes susceptible, infected, and recovered individuals. Expanded models like QSIR [2] add quarantine, while SEIR [3] incorporates exposed individuals. The SEIRD model [4] includes deaths, and modified SEIR models [5-7] integrate social distancing, quarantine, and vaccination effects.Controllers such as PID [8], SMC [9], and VSC [10] help manage pandemic spread by adjusting restrictions. AI-based controllers, like fuzzy logic [11,12], simplify control without complex equations. Combined methods, such as the SQEIAR model with optimal control [13], further improve pandemic management.Cultural-social factors, measured via fractal dimension, influence compliance with restrictions [11,12]. Research [11,12] uses fuzzy logic with fractal inputs (e.g., LFDC, LFDD, NLFDC, NLFDD) to tailor controls to different societies. Economic impacts, like GDP growth rate, are also considered to minimize financial losses.

This study combines fractal dimension (cultural indicator) and fuzzy controllers, optimized via genetic algorithms, to determine quarantine and social distancing limits. The paper is structured in five parts: model and tools (Section 2), proposed methods (Section 3), simulations (Section 4), and conclusions (Section 5).

2. Materials and methods

2-1 Modified SEIR Model

In this research, the modified eight-part model of SEIR [5] has been used for the proposed closed-loop model. The advantage of this model is the variety of different parts of the disease and the two limitations of social distance and quarantine, which have been investigated. This model has eight variables (S, E, I_a, I_p, U_a, D_a, U_p, D_p) and ten parameters (β_α, β_p, σ, γ_α, γ_p, α, v_α, v_p, r, u). The values of the parameters are in [5]. The desired mathematical model is described with the set of relationships (1) for various disease classes in a society:

$$\begin{cases} \frac{ds}{dt} = -\frac{u(\beta_a I_a + \beta_p I_p)}{N} S \\ \frac{dE}{dt} = -\frac{u(\beta_a I_a + \beta_p I_p)}{N} S - \sigma E \\ \frac{dI_a}{dt} = \alpha\sigma E - \gamma_a I_a - r v_a I_a \\ \frac{dI_p}{dt} = (1-\alpha)\sigma E - \gamma_p I_p - r v_p I_p \\ \frac{dU_a}{dt} = \gamma_a I_a \\ \frac{dD_a}{dt} = r v_a I_a * \\ \frac{dU_p}{dt} = \gamma_p I_p \\ \frac{dD_p}{dt} = r v_p I_p * \end{cases} \quad (1)$$

Because it is necessary to consider the total number of daily confirmed cases for this research, variable C for this definition is introduced as follows [5]. Then, this variable is used daily in the model.

$$C = D_a + D_p + U_p \quad (2)$$

Social distancing (u) reduces contact between susceptible and infected individuals, involving measures like canceling events, closing public spaces, and avoiding crowds [15]. Initially set to 1 (no intervention), it later varies between [0,1]. Test-quarantine (r) isolates potentially exposed individuals (e.g., travelers or contacts of COVID-19 cases) for 14 days [16], removing them from the infected population (modeled as $rv_a I_a$ and $rv_p I_p$ in equations (6) and (8)). Initially zero, it adjusts over time within [0,1]. The effect of these two interventions in the modified SEIR model is investigated through the u and r parameters, which are supposed to be controlled by the fuzzy controller.

2-2 Fuzzy Logic

Fuzzy systems are rule-based controllers that use intuitive IF-THEN rules with linguistic variables defined by membership functions (0-1 range) rather than complex equations [17,18], consisting of inputs/outputs with Gaussian-shaped membership functions and a knowledge-based rule base. These model-free systems face design challenges in determining optimal linguistic variables, membership function parameters, and rule sets, which this study addresses through genetic algorithm optimization to enhance controller performance while maintaining simplicity and data-driven intelligence.

2-3 Fractal Dimension

Introduced by Benoit Mandelbrot in the 1970s, fractal geometry's key concept of fractal dimension [19] measures nonlinear system complexity, where time series analysis reveals behavior through numerical values - near 1 for flat curves and approaching 2 for oscillating patterns covering more plane area [20]. Among various calculation methods, this study employs the Higuchi algorithm [21] which computes fractal dimension D by analyzing power spectrum distributions following power law principles, beginning with regular-interval time series observations as its foundation:

$$X(1), X(2), X(3), \dots, X(N) \quad (3)$$

From the given time series, a new time series X_k^m is created, which is defined as follows:

$$X_k^m; X(m), X(m+k), X(m+2k), \dots, X\left(m + \frac{N-m}{k} k\right), \quad m = 1,2,\dots,k \quad (4)$$

where both k and m are integers. m and k represent the initial time and interval time, respectively. For time interval equal to k, we obtain k new time series [21].

The length of the curve $X_k^m$is defined as follows:

$$l_m(k) = \left\{\left(\sum_{i=1}^{\left[\frac{N-m}{k}\right]} |X(m+ik) - X(m+(i-1)k)|\right) \frac{N-1}{\left[\frac{N-m}{k}\right]k}\right\} / k \quad (5)$$

In the expression $\frac{N-1}{\left[\frac{N-m}{k}\right]k}$, k represents the normalization factor for the length of the subset time series curve. The length of the curve for the time interval k, that is, L(K), is defined as the average value on the set k of $L_m(k)$. If $L(k) \propto k^{-D}$ then it is a sectional curve (fractal) with dimension D.

The average $L_m(k)$ to find HFD is calculated as follows [22, 23]:

$$HFD = \frac{1}{K} \sum_{M=1}^{K} L_m(k) \quad (6)$$

The fractal dimension serves as a mathematical measure of socio-cultural complexity derived from population statistics, enabling more accurate predictions of societal responses to interventions like social distancing and quarantine.

2-4 Gross domestic product (GDP) rate

Gross Domestic Product (GDP) quantifies the total value of goods/services produced within a country during a specific period (typically annually), measured in local currency requiring conversion for cross-national comparisons [14]. To examine how social distancing measures impact economic growth, reference [14] proposes an ordinary least squares combined model, establishing an analytical framework to evaluate this critical relationship between public health interventions and macroeconomic performance:

$$GDP_{c,t} = \alpha_{c,t} + \beta 1\left(Stringency\ Index_{c,t}\right) + \beta 2\left(Covid19\ Confirmed\ Cases_{c,t}\right) + \sum_{k=1}^{k} \beta_k X_c^k + \varepsilon_{c,t} \quad (7)$$

G represents the GDP rate, and t and c represent the square and a country, respectively. SI indicates the seriousness index. The GDP growth rate is the dependent variable, measured at the seasonal frequency of three months. α is a constant value that is calculated in the third section. The seriousness index means the social distancing policies of the government. Confirmed cases of epidemic disease (e.g. COVID-19) are equal to the total number of new confirmed cases of disease in a quarter of a country. The model includes several control variables represented by X_c^k [14]. Estimating and checking regression coefficients is one of the challenges of using this relationship, which is explained in part 4.

3 Proposed methods presented

This study presents two closed-loop fuzzy controller models for Iran's pandemic data: Model A (Figure 1) uses two fuzzy inputs without socio-cultural considerations, while Model B (Figure 2) incorporates societal cultural effects through fractal dimension analysis. The community model consists of two inputs: social distance (u) and quarantine (r), which need to be controlled by the fuzzy system. The output of the model is daily infected people (Cd). Here, error means as $e(t) = X - Cd(t)$ where X is the target value, which is considered to be 4000 people according to the population of Iran.

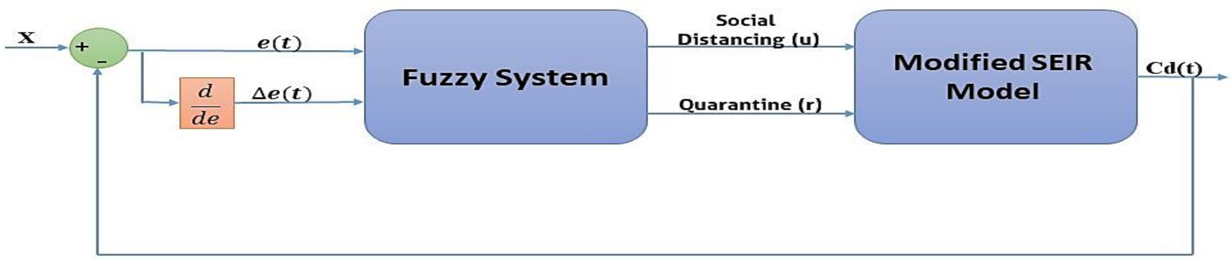

Fig 1- Proposed SEIR model with two-input fuzzy controller

Model A uses a Gaussian-based Mamdani fuzzy controller with two inputs error (e) and error derivative (de) each with three membership functions. Outputs social distance (u) and quarantine (r) have four Gaussian membership functions, tuned via trial and error. The system follows 9 fuzzy rules but excludes socio-cultural effects, which will later be added via a fractal dimension input in Model B.Model B: SEIR proposed model with fuzzy-fractal approach. Model B adds fractal dimension as a third input to incorporate cultural influence, while keeping other inputs/outputs like Model A (see Figure 2). The linear fractal dimension (calculated via the Higuchi algorithm) reflects societal compliance with restrictions—lower graph volatility and a fractal number closer to 1 indicate better pandemic control, reducing the need for strict measures.

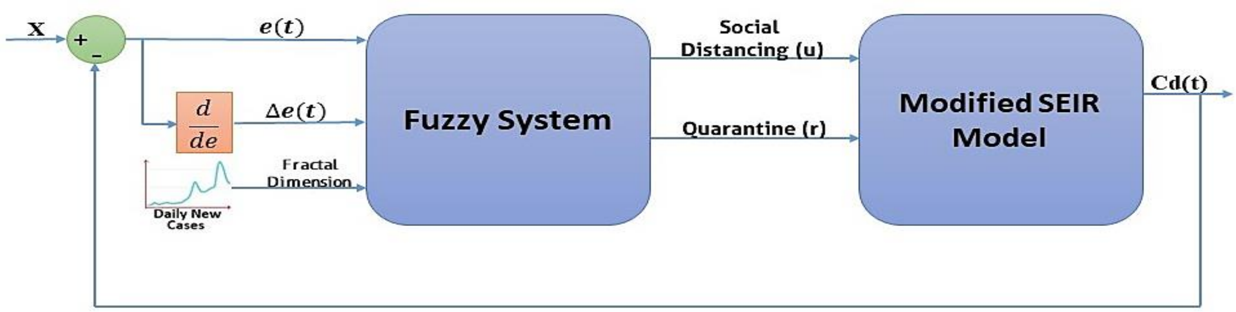

Fig 2- Diagram of the proposed SEIR model with the fuzzy-fractal approach

Model B's controller extends Model A by adding a third input (fractal dimension, range [1,2]) with 2 membership functions**,** modifying the fuzzy rules. With 3 membership functions for error inputs and 2 for fractal dimension**,** the system now uses 18 fuzzy rules and requires 15 parameters for fuzzy set placement, optimized via a genetic algorithm.

3-1 gross domestic product (GDP) rate used in the article

To analyze the relationship between the criteria of social distance and quarantine and economic growth, equation (8) is modified as follows and is used as a part of the objective function in genetic algorithm optimization. We consider the following relation for COVID-19:

$$G = \alpha + \beta 1(SI) + \beta 2(i) \quad (8)$$

The β1 and β2 coefficients were obtained from the correlation matrix in [14]. β1, which is the stringency index coefficient obtained from the first row of the Stringency Index row of the matrix, and β2, which is the coefficient of confirmed infected cases, is obtained from the first row of the Covid19 Confirmed Cases row of the matrix. The coefficients of -0.14 and 0.12 were extracted from the matrix for the stringency index and the cases of COVID-19, respectively. α is defined as a fixed value. The method of obtaining it is described below. By equation 10 and inserting the values of Table 1. From the article [14], the number α with the value α=-44.135 is obtained.

Table 1- The values needed to obtain the constant number α

β_2	β_1	confirmed cases of Covid-19	Stringency Index	GDP rate	Name
0.12	-0.14	429.298	54.789	-0.289	Value

3-2 Optimization with genetic algorithm

The genetic algorithm optimizes the fuzzy controller's structure [20,24,25] using an inverse GDP-based cost function (from Eq. 17) to maximize economic growth while minimizing pandemic impacts. This approach adjusts membership function ranges to maintain optimal GDP levels during restrictions:

$$y = \frac{\frac{i}{N}}{G + 0.00001} \tag{9}$$

The equation incorporates normalized daily infections and GDP rate**. A** genetic algorithm optimizes 15 fuzzy set parameters using a 200-population, 70-generation setup to maximize GDP growth—the study's primary objective.

4 Simulation results

The study first simulates pandemic spread without restrictions (u=1, r=0) using a modified SEIR model in Matlab2016 (Fig.3) for a population of 85 million, with parameters from [5]. Initial conditions reflect the 60th day of outbreak, representing a 2-month delay before restrictions are imposed. Models A and B are then compared.

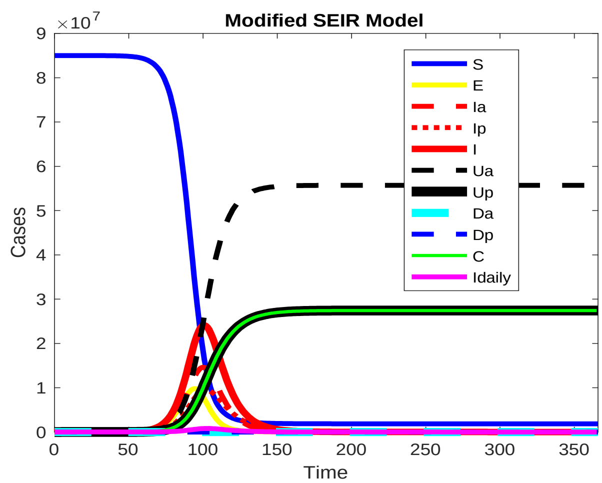

Fig 3- modified SEIR model simulation diagram

4-1 The results of Model A

The study applies a two-input-two-output fuzzy controller to manage COVID-19 in Iran, targeting 4,000 daily cases. Input ranges are [-792,600, 4,000] for error and [-100,000, 30,000] for error derivative, while outputs (social distance and quarantine) use [0,1] intervals with Gaussian membership functions. With R_0=0.51, simulations show infections peaking at 53,334 (day 8) before declining to 2,133 (day 30), with restriction dynamics illustrated in Figure 4c.

4-2 The results of model B

Model B extends Model A by adding a third input (fractal dimension) with range [1,2] and two membership functions (low/high). This modification results in infections peaking at 45,960 by day 9 before declining to 524 by day 30 (Figure 5). The improved R_0=0.1548 demonstrates significantly reduced virus spread, with detailed restriction adjustments shown numerically.

4-3 The results of the model

A genetic algorithm optimized the fuzzy system parameters to maximize GDP growth (using the previously defined objective function). The optimized model shows infections peaking at 44,990 by day 8 and declining to 535 by day 30 (Figure 8). While achieving higher GDP, this comes with trade-offs: social distance restrictions show reduced intensity/fluctuations, quarantine values remain stable, and R_0 increases to 0.2986 (compared to non-optimized Model B). Figures 7-8 demonstrate that incorporating fractal dimension leads to more dynamic adjustments in restrictions, resulting in better infection control despite the slightly higher R_0 value.

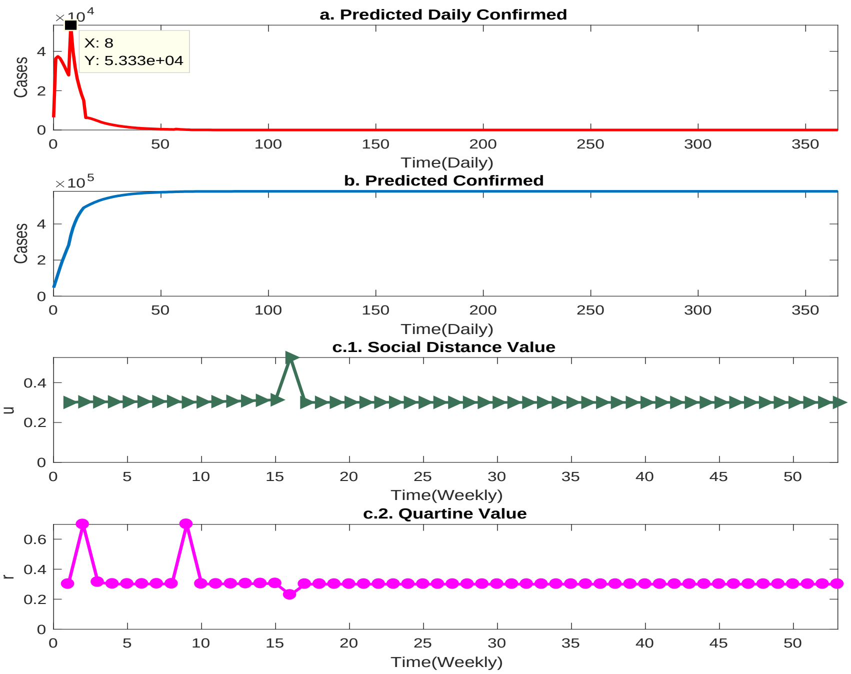

Fig 4- The results of daily approved persons, total approved persons, and the numerical value of u and r interventions with the approach of model A for the country of Iran

The results on cases without controller and two proposed models are compared in Table 2. By comparing the results, it can be seen that the number of infected patients has decreased significantly by applying model B to society. This shows the positive effect of the fractal dimension in reducing the pandemic. Also, by examining the results of the optimized model B, it can be seen that when this model is used, infected people will decrease more than before. Iran's GDP rate is also calculated every week, just like interventions.

Table 2 Comparison of results without controller, with model A and model B

*	Without controller	model A	model B	optimized model B
The peak number of confirmed infected people	More than 27 million people in 120 days	580,000 people in 70 days	43,233people in 50 days	42,970 people in 50 days
Peak number of daily confirmed infected people	800,000 people in 44 days	5,3334 people in 8 days	45,960 people in 8 days	44,990 people in 8 days
The amount of reduction within n days	3722people within 120 days	2133people within 30 days	524people within 30 days	535people within 30 days
social distance	-	between 0.3 and 0.52	between 0.1 and 0.3	between 0.3 and 0.4
Quarantine	-	between 0.22 and 0.7	0.7 between 0.3 and 0.9	between 0.3 and 0.9
R_0	3.76488	0.51	0.1548	0.2986

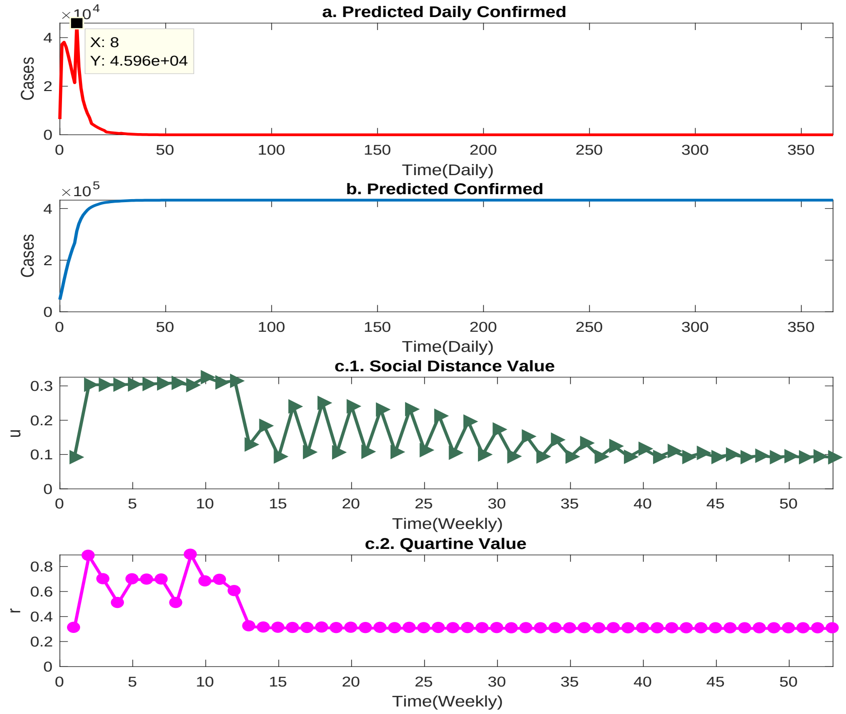

Fig 5- The results of daily approved persons, total approved persons, and the numerical value of u and r interventions with the B model approach for the country of Iran

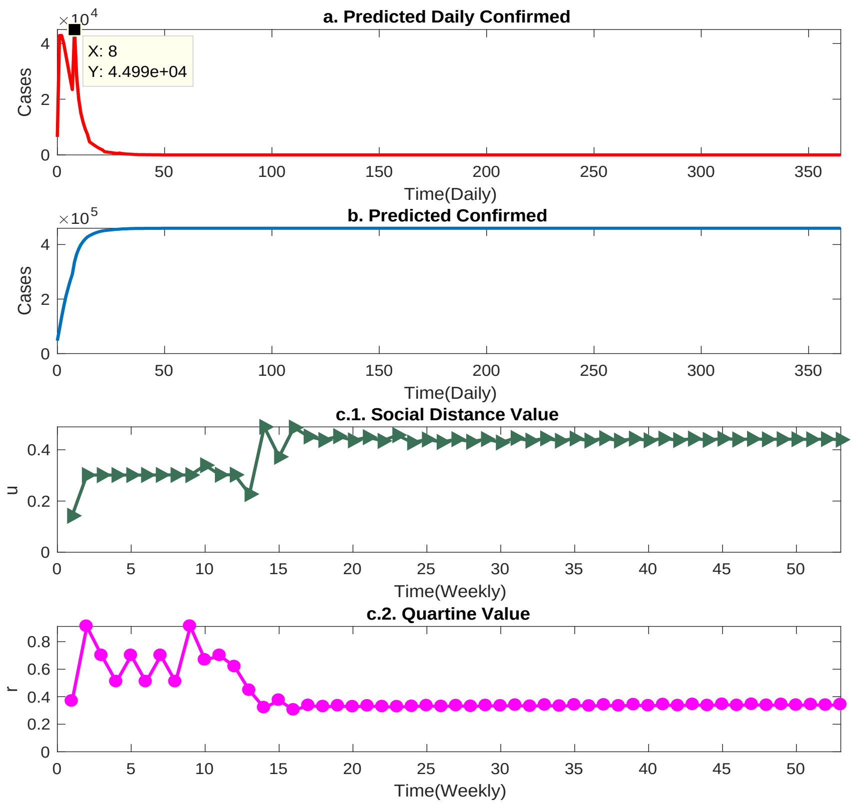

Fig 6- The results of daily confirmed people, total confirmed people, and the numerical value of u and r interventions with the optimization of model B for the country of Iran

5 Conclusion

Global pandemics like COVID-19 require effective prediction and control methods to mitigate health and economic impacts. This study presents a modified SEIR model with fuzzy control, incorporating two key societal factors: fractal dimension (cultural) and GDP growth rate (economic). Two fuzzy systems were developed - a basic two-input version and an enhanced three-input version adding fractal dimension (range [1,2]) to account for cultural influences. Genetic algorithm optimization was applied using a GDP-based objective function to balance economic and health outcomes. Results demonstrated that including fractal dimension led to more dynamic restriction adjustments (social distancing/quarantine), significantly reducing infections while maintaining economic oversight. The optimized fuzzy-fractal model showed particular effectiveness in controlling case numbers.

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