A useful case study to develop lattice Boltzmann method performance: Gravity effects on slip velocity and temperature profiles of an air flow inside a microchannel under a constant heat flux boundary condition

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Mixed convection heat transfer of air in a 2-D microchannel is investigated numerically by using lattice Boltzmann method. The effects of buoyancy forces on slip velocity and temperature profiles are presented while the microchannel side walls are under a constant heat flux boundary condition. Three states are considered as no gravity, \(Gr = 100\) and \(Gr = 500\). At each state, the value of Knudsen number is chosen as \(Kn = 0.005\), \(Kn = 0.01\) and \(Kn = 0.02\) respectively; while Reynolds number and Prandtl number are kept fixed at \(Re = 1\) and \(Pr = 0.7\). Density-momentum and internal energy distribution functions are used in order to simulate the hydrodynamic and thermal domains in LBM approach. Develop the ability of LBM to simulate the constant heat flux boundary condition along the microchannel walls in the presence of slip velocity and buoyancy forces is proved for the first time at present work. The new and interesting results are achieved such as generating a rotational cell through the fluid flow due to buoyancy forces which leads to see the negative slip velocity at these areas.

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1. Introduction

Many works have been reported concerned Micro and Nano Electro Mechanical Systems which could be applied in microdevices such as micropumps, microchannels and so on. The microflow regimes might be different from the macroflow ones. Therefore a dimensionless number of Knudsen (\(Kn\)) was introduced to classify the flows [1–3]. The continuum flow regime can be observed at the state of \(Kn < 0.001\), which means the classic Navier-Stokes equations are being true and can be applied for the flow simulations. Moreover the slip flow regime, transient regime and free molecular regime would be occurred at \(0.001 < Kn < 0.1\), \(0.1 < Kn < 10\) and \(Kn > 10\), respectively [4–9]. At these regimes the particle based methods like lattice Boltzmann method (LBM), Molecular dynamic (MD) and direct simulation of Monte Carlo ( DSMC) must be applied and the well-known Navier-Stokes equations would not be responded. However Navier-Stokes equations together with slip velocity and temperate jump boundary conditions are able to be used for the slip flows. These mentioned boundary conditions imply the fluid molecule placed on the constant or moving wall, would not have the same velocity and the same temperature with those of wall [10–16].

Among all particle based methods, LBM involves the less computation costs and also can be used for the all flow regimes. Besides, it works with less complex math equations; hence many articles can be addressed using LBM. It can be mentioned that lattice Boltzmann method is a relatively new simulation approach consisted collision and streaming stages between fictive particles located on the lattice nodes. Suitable deal with complex boundaries, using parallel algorithms and easy to use for the multiphase flows are some other advantages of LBM compared with classic CFD approaches. All these benefits are available only after choosing an appropriate collision operator; so that LBM according to BGK model is considered as the alternative approach in this way which shows suitable accuracy and convergence [17–22].

Different models of LBM were developed in order to increase LBM performance such as for the simulation of thermal domains. Among them, double population approach used density-momentum distribution function for the hydrodynamic field and internal energy distribution function for the thermal field showed a better manner [23–31]. Simulation of 2-D isothermal pressure driven flow inside a microchannel by LBM was reported by Lim et al. [32]. More works of Niu and Shu and Chew [33–35] can be
referred corresponded lattice Boltzmann method based on new
definition of relaxation times in term of Kn and by using diffuse
scattering boundary condition model for the slip velocity. Several
other studies on thermal lattice Boltzmann method (TLBM)
through a microchannel considering viscous heat dissipation term
were presented by Tian et al. [36,37]; also other works of LBM were
reported at different conditions and for various type of working
fluids such as nanofluid [38–48].

It is seen that the influences of buoyancy forces are neglected in
the most previous researches which means lack of accuracy in
results. Moreover heat flux boundary condition along the
microchannel walls, in the presence of gravity, has not been
reported by LBM (To the best author’s knowledge). Hence present
article aims to investigate this subject to develop the LBM perfor-
mance through such physical problems.

2. Problem statement

Mixed convection heat transfer of air in a 2-D microchannel is
investigated numerically by using lattice Boltzmann method
(LBM) involved BGK model. Microchannel length is ten times
longer than its width (AR = L/H = 10) and its horizontal walls are
under a constant heat flux of \( q^0 \). The cold air at \( T_i \) enters to the
microchannel from the left side with a constant velocity of \( u_i \),
(see Fig. 1).

Three states are considered as no gravity, \( Gr = 100 \) and \( Gr = 500 \).
The first one implies to ignore the gravity acceleration while the
two other ones represent to invigorate the buoyancy forces. At
each state, the value of Knudsen number is chosen as \( Kn = 0.005, Kn = 0.01 \) and \( Kn = 0.02 \) respectively; while Reynolds number and
Prandtl number are kept fixed at \( Re = 0.02 \) and \( Pr = 0.7 \). It is well
known that the characteristic length is chosen as the hydraulic
length \( D_h \) in a channel flow, which means \( D_h = 2H \) have a small
amount in a micro flow inside a microchannel due to the fact of
low value of microchannel width (H). Hence it’s corresponded Rey-
molds number \( (Re = \rho u_i D_h/\mu) \) will have a low value at the level of
one or even less than one. Density-momentum \( (f) \) and internal
energy distribution functions \( (g) \) are used in order to simulate
the hydrodynamic and thermal domains in LBM approach.

3. Present work novelty

The effect of gravity was ignored in the most published papers
concerned the microflows. However its effect may be important
and considerable on the flow domain at a specified rage of Knudsen
number; while there is no article concerned microchannel in the
presence of constant heat flux especially applying LBM. Some diffi-
culties in this way can be classified as: 1-How to consider the gravity
effects in slip velocity model of LBM based on density-
momentum distribution function, 2-Develop a new model or use
a suitable existence model to simulate the heat flux along the
microchannel walls in the presence of both slip velocity and buoy-
ancy forces.

Hence develop the ability of LBM to simulate the constant heat
flux boundary condition along the microchannel walls in the pres-
ence of slip velocity and buoyancy forces is proved for the first time
at present work. Moreover the new and interesting results are
achieved which they would be presented in the following sections.

4. Formulation

4.1. Lattice Boltzmann method

Boltzmann equation according to the density-momentum and
internal energy distribution functions:

\[
\partial_t f + (\mathbf{c} \cdot \nabla) f = \Omega(f)
\]

(1)

\[
g = 0.5(\mathbf{c} - \mathbf{u})^2 f
\]

(2)

which lead to the thermal lattice Boltzmann equation:

\[
\partial_t g + (\mathbf{c} \cdot \nabla) g = \Omega(g)
\]

(3)

In BGK model, the collision term can be written as:

\[
\Omega(f) = \frac{f^e - f}{\tau_f}
\]

(4)

\[
\Omega(g) = \frac{g^e - g}{\tau_g} - fZ = 0.5(\mathbf{c} - \mathbf{u})^2 \Omega(f) - fZ
\]

(5)

The heat dissipation term of “f Z” is:

\[
fZ = f(c - u) \cdot \partial_u \mathbf{u} + (c \cdot \nabla) u
\]

(6)

\( \tau_f \) and \( \tau_g \) are introduced as the hydrodynamic and thermal relaxation
times.

New distribution functions of \( f_e \) and \( g_e \) are used as:

\[
\tilde{f}_i = f_i + \frac{dt}{2\tau_f} (f_i - f_i^e)
\]

(7)
\[ g_i = \frac{dt}{2 \tau_g} (g_i - g_i^t) + \frac{dt}{2} g_i Z_i \]  \\
(8)

Maxwell-Boltzmann equilibrium distribution functions are shown as \( f_i^e \) and \( g_i^e \) for momentum and internal energy. Subscript \( i \) represents the lattice link number of \( D_2Q_9 \) shown in Fig. 2 [17].

\[ Z_i = (c_i - u_i) D_i u_i \text{ and } D_i = \partial_i + c_i \nabla \]  \\
(9)

\( c_i = (\cos \frac{1}{2} \pi \frac{i}{3}, \sin \frac{1}{2} \pi \frac{i}{3}) \) \( c_i \) \( i = 1, 2, 3, 4 \)

\( c_i = \sqrt{2} (\cos \left[ (\frac{i-5}{9}) \pi + \frac{\pi}{4} \right], \sin \left[ (\frac{i-5}{9}) \pi + \frac{\pi}{4} \right]) \) \( c_i \) \( i = 5, 6, 7, 8 \)

\( c_0 = (0, 0) \)

Now the collision and streaming stages of LBM are presented as:

\[ f_i(x + c_i dt, t + dt) - f_i(x, t) = - \frac{dt}{\tau_f + 0.5 dt} [f_i - f_i^e] \]  \\
(11)

\[ g_i(x + c_i dt, t + dt) - g_i(x, t) = - \frac{dt}{\tau_g + 0.5 dt} [g_i - g_i^e] \]  \\
(12)

\[ f_i^e = \omega_i \rho \left[ 1 + \frac{3c_i \cdot u}{c^2} + \frac{9(c_i \cdot u)^2}{2c^4} - \frac{3(u^2 + p^2)}{2c^2} \right] \]  \\
(13)

\[ g_i^e = -\omega_i \rho e \left[ \frac{\rho e}{c^2} \right] \]  \\
(14)

\[ g_i^{e, 1,2,3,4} = \omega_i \rho e \left[ 1 + 1.5 + 1.5 \frac{c_i \cdot u}{c^2} + 4.5 \left( \frac{c_i \cdot u}{c^2} \right)^2 - 1.5 \frac{u^2 + p^2}{c^2} \right] \]

\[ g_i^{e, 5,6,7,8} = \omega_i \rho e \left[ 3 + 6.6 \frac{c_i \cdot u}{c^2} + 4.5 \left( \frac{c_i \cdot u}{c^2} \right)^2 - 1.5 \frac{u^2 + p^2}{c^2} \right] \]

where \( c_i^2 = 1 \), \( e = \rho RT \) and \( \omega_0 = 4/9 \), \( \omega_i = 1/9 \) for \( i = 1, 2, 3, 4 \) and \( \omega_i = 1/36 \) for \( i = 5, 6, 7, 8 \) in a 2-D simulation. Hence the macroscopic parameters are achieved:

\[ \rho = \sum \tilde{f}_i \]  \\
(15)

\[ \rho u = \sum \tilde{c}_i \tilde{f}_i \]  \\
(15)

\[ \rho e = \sum \tilde{g}_i - \frac{\theta}{4} \sum f Z_i \]

According to the kinetic theory of the microflows, it is reminded that:

\[ Kn = \sqrt{\frac{\pi k Ma}{2 Re}} \]  \\
(16)

\[ \tau_f = \sqrt{\frac{6}{\pi k}} D_h Kn \]  \\
(17)

\[ \tau_g = \frac{\tau_f}{Pr} \]  \\
(18)

Grid independency study at the state of no-gravity, \( Re = 1, Kn = 0.01 \) for three different lattice nodes.

<table>
<thead>
<tr>
<th>Nodes</th>
<th>( U_i )</th>
<th>( Nux_{\text{outlet}} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>350 x 35</td>
<td>0.0779</td>
<td>11.1020</td>
</tr>
<tr>
<td>400 x 40</td>
<td>0.0782</td>
<td>11.1023</td>
</tr>
<tr>
<td>450 x 45</td>
<td>0.0783</td>
<td>11.1024</td>
</tr>
</tbody>
</table>

Fig. 3. Validation of present work versus those of Hooman & Ejlali [6].

Lines: present work; Symbols: those of Kavehpour et al. [7].

Fig. 4. Comparison of present work results with those of Kavehpour et al. [7].

Lines: present work; Symbols: those of Kavehpour et al.
4.2. Boundary conditions

Non-equilibrium bounce back model is applied to determine the unknown values of hydrodynamic distribution functions for the inlet and outlet sections:

\[ \bar{f_1} = \bar{f_3} + \frac{1}{2} \rho_{in} u_{in} \]
\[ \bar{f_5} = \bar{f_7} + \frac{1}{2} (\bar{f_4} - \bar{f_2}) + \frac{1}{2} \rho_{in} u_{in} \]
\[ \bar{f_8} = \bar{f_6} - \frac{1}{2} (\bar{f_4} - \bar{f_2}) + \frac{1}{2} \rho_{in} u_{in} \]  \hfill (19)

\[ \bar{f_3} = \bar{f_1} - \frac{1}{2} \rho_{out} u_{out} \]
\[ \bar{f_7} = \bar{f_5} - \frac{1}{2} (\bar{f_4} - \bar{f_2}) - \frac{1}{2} \rho_{out} u_{out} - \frac{1}{2} \rho_{out} u_{out} \]
\[ \bar{f_6} = \bar{f_8} + \frac{1}{2} (\bar{f_4} - \bar{f_2}) + \frac{1}{2} \rho_{out} u_{out} \]  \hfill (20)

And for the unknown thermal distribution functions at these areas:

\[ \bar{g_5} = \frac{6 \rho_{in} \sum (f_2 \rho_{out} u_{out} + f_1 \rho_{in} u_{in})}{2 \cdot 3 \rho_{in} + 3 \rho_{out}} \times [3.0 + 6 u_{in} + 3.0 u_{in}^2] \]
\[ \bar{g_1} = \frac{6 \rho_{in} \sum (f_2 \rho_{out} u_{out} + f_1 \rho_{in} u_{in})}{2 \cdot 3 \rho_{in} + 3 \rho_{out}} \times [1.5 + 1.5 u_{in} + 3.0 u_{in}^2] \]
\[ \bar{g_6} = \frac{6 \rho_{in} \sum (f_2 \rho_{out} u_{out} + f_1 \rho_{in} u_{in})}{2 \cdot 3 \rho_{in} + 3 \rho_{out}} \times [3.0 + 6 u_{in} + 3.0 u_{in}^2] \]  \hfill (21)

Moreover the slip velocity along the lower wall can be determined by using the following equations; although the amount of accommodation factor of “r” should be chosen in a suitable way to achieve accurate results:

\[ \bar{f_1} = \bar{f_4} \]  \hfill (23-1)

\[ \bar{f_6} = r \bar{f_5} + (1 - r) \bar{f_8} \]  \hfill (23-2)

The corresponded equation for the upper wall can be written similarly:

\[ \bar{f_3} = \bar{f_2} \]  \hfill (24-1)

\[ \bar{f_7} = \bar{f_6} + (1 - r) \bar{f_7} \]  \hfill (24-2)

---

**Fig. 5.** Streamlines and isotherms at Kn = 0.02 for the states of no gravity (top), Gr = 100 (middle) and Gr = 500 (bottom).
4.3. Effects of buoyancy forces

The effects of mixed convection heat transfer through a microflow is studied using Boussinesq approximation which leads to have the buoyancy force as $G = \beta g \Delta T$.

The Boltzmann equation involved the external force of “F” is presented as follows:

$$\partial f + (\mathbf{c} \cdot \nabla) f = \frac{f - f^o}{\tau_f} + F$$

(25)

where “F” based on the buoyancy forces can be written as $F = \frac{G c}{k} f^o$; which means,

$$f(x + \mathbf{c} dt, \mathbf{c}, t + dt) - f(x, \mathbf{c}, t) = -\frac{dt}{2\tau_f} \left[ f(x + \mathbf{c} dt, \mathbf{c}, t + dt) - f^o(x + \mathbf{c} dt, \mathbf{c}, t + dt) \right] + \frac{dt}{2} F(x + \mathbf{c} dt, \mathbf{c}, t + dt)$$

$$+ \frac{dt}{2} F(x, \mathbf{c}, t)$$

(26)

$$\tilde{f}(x + \mathbf{c} dt, \mathbf{c}, t + dt) - \tilde{f}(x, \mathbf{c}, t) = -\frac{dt}{\tau_f + 0.5 dt} \left[ f(x, \mathbf{c}, t) - f^o(x, \mathbf{c}, t) \right] + \frac{\tau_f dt}{\tau_f + 0.5 dt}$$

(27)

Applied the expression $\tilde{f}_i = f_i + 0.5 dt / \tau_f (f_i - f^o_i) - 0.5 dt F$. Now discretized form of Eq. (27) is illustrated as:

$$\tilde{f}_i (x + \mathbf{c} dt, \mathbf{c}, t + dt) - \tilde{f}_i (x, \mathbf{c}, t) = -\frac{dt}{\tau_f + 0.5 dt} \left[ \tilde{f}_i - f^o_i \right] + \left( \frac{dt \tau_f}{\tau_f + 0.5 dt} \frac{3G c u}{c^2 f} \right)$$

(28)

Fig. 6. Streamlines and isotherms at Kn = 0.005 for the states of no gravity (top), Gr = 100 (middle) and Gr = 500 (bottom).

Fig. 7. Slip velocity along the microchannel walls.
\begin{equation}
\dot{f}_i = \frac{\tau f_i}{\tau} + 0.5dt \dot{f}_i + \left( \frac{0.5dt \tau f_i}{\tau} + 3G(c_{iy} - \nu) f_i \right)
\end{equation}

And finally the macroscopic parameters, included the gravity effects:

\begin{equation}
\rho = \sum \dot{f}_i \tag{30-1}
\end{equation}

\begin{equation}
u = (1/\rho) \sum \dot{f}_i c_{iy} + \frac{dt}{2} G
\end{equation}

Moreover the hydrodynamic boundary conditions for the inlet involved gravity:

\begin{equation}
\dot{f}_1 + \dot{f}_5 + \dot{f}_8 = \rho_m - \left( \dot{f}_0 + \dot{f}_2 + \dot{f}_3 + \dot{f}_4 + \dot{f}_6 + \dot{f}_7 \right)
\end{equation}

\begin{equation}
\dot{f}_5 - \dot{f}_8 = \rho_m \dot{u}_m + \left( \dot{f}_0 + \dot{f}_2 - \dot{f}_4 - \dot{f}_6 + \dot{f}_7 \right) - \frac{dt}{2} \rho_m G
\end{equation}

where

\begin{equation}
\dot{u}_m = \frac{\dot{f}_0 + \dot{f}_2 + \dot{f}_4 + 2(\dot{f}_3 + \dot{f}_6 + \dot{f}_7)}{1 - \dot{u}_m}
\end{equation}

and

\begin{equation}
\dot{f}_1 - \dot{f}_3 = \dot{f}_3' - \dot{f}_3'' \Rightarrow \dot{f}_1 = \dot{f}_3 - \dot{f}_3' + \dot{f}_3''
\end{equation}

Now, from Eq. (13) together with Eqs. (31-2), (31-3) and Eq. (34):

Fig. 8. Slip velocity along the microchannel upper and lower walls at \( Kn = 0.005 \).
\[ \tilde{f}_1 = \frac{1}{2} \tilde{f}_3 + \frac{2}{3} \tilde{\rho}_m \tilde{u}_m \]  
\[ \tilde{f}_8 = \tilde{f}_6 - \frac{\tilde{f}_6 - \tilde{f}_2}{2} + \frac{1}{6} \tilde{\rho}_m \tilde{u}_m - \frac{1}{2} \tilde{\rho}_m \tilde{v}_m + \frac{dt}{4} \tilde{\rho}_m \tilde{G} \]  
\[ \tilde{f}_5 = \tilde{f}_3 + \frac{\tilde{f}_6 - \tilde{f}_2}{2} + \frac{1}{6} \tilde{\rho}_m \tilde{u}_m + \frac{1}{2} \tilde{\rho}_m \tilde{v}_m - \frac{dt}{4} \tilde{\rho}_m \tilde{G} \]  
\[ \tilde{f}_2 = \tilde{f}_4 - \tilde{f}_2 - \frac{1}{6} \tilde{\rho}_m \tilde{u}_m \tilde{v}_m - \frac{1}{2} \tilde{\rho}_m \tilde{v}_m + \frac{dt}{4} \tilde{\rho}_m \tilde{G} \]  
\[ \tilde{f}_7 = \tilde{f}_5 + \frac{1}{2} \tilde{f}_4 - \frac{\tilde{f}_4 - \tilde{f}_2}{2} - \frac{1}{6} \tilde{\rho}_m \tilde{u}_m \tilde{v}_m - \frac{1}{2} \tilde{\rho}_m \tilde{v}_m + \frac{dt}{4} \tilde{\rho}_m \tilde{G} \]  
\[ \tilde{f}_6 = \tilde{f}_8 + \frac{1}{2} \tilde{f}_4 - \frac{\tilde{f}_4 - \tilde{f}_2}{2} - \frac{1}{6} \tilde{\rho}_m \tilde{u}_m \tilde{v}_m + \frac{1}{2} \tilde{\rho}_m \tilde{v}_m - \frac{dt}{4} \tilde{\rho}_m \tilde{G} \]  

Through a same approach for the outlet hydrodynamic boundary conditions included buoyance forces, we will have:

\[ \tilde{f}_3 = \frac{1}{2} \tilde{f}_1 - \frac{1}{2} \tilde{\rho}_m \tilde{u}_m \]  
\[ \tilde{f}_5 = \frac{1}{2} \tilde{f}_6 - \frac{1}{2} \tilde{\rho}_m \tilde{u}_m \tilde{v}_m - \frac{1}{2} \tilde{\rho}_m \tilde{v}_m + \frac{dt}{4} \tilde{\rho}_m \tilde{G} \]  
\[ \tilde{f}_6 = \frac{1}{2} \tilde{f}_8 + \frac{1}{2} \tilde{\rho}_m \tilde{u}_m \tilde{v}_m + \frac{1}{2} \tilde{\rho}_m \tilde{v}_m - \frac{dt}{4} \tilde{\rho}_m \tilde{G} \]

Slip velocity along walls of the microchannel considering "G" can be demonstrated as follows; while it should be necessary to satisfy a constraint presented in Eq. (37) (For example for the lower wall),

\[ \tilde{f}_2 + \tilde{f}_5 + \tilde{f}_6 = \tilde{f}_4 v_w + \left( \tilde{f}_4 + \tilde{f}_5 + \tilde{f}_6 \right) - \frac{dt}{2} \tilde{\rho}_v \tilde{G} \]  
\[ \tilde{f}_2 = \frac{1}{2} \tilde{f}_4 - \frac{1}{2} \tilde{\rho}_v \tilde{G} \]  
\[ \tilde{f}_5 = \frac{1}{2} \tilde{f}_4 + \left( 1 - r \right) \tilde{f}_6 \]  
\[ \tilde{f}_6 = \frac{1}{2} \tilde{f}_6 + \left( 1 - r \right) \tilde{f}_7 \]

The upper and lower walls of the microchannel are imposed to the constant heat flux. General purpose thermal boundary condition (GPTBC) is applied to simulate the thermal boundary condi-

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**Fig. 9.** Slip velocity along the microchannel upper and lower walls at Kn = 0.02.
tions along the microchannel walls. That model was presented by D’Orazio et al. [24,25] according to the non-equilibrium bounce back boundary condition of He & Zou [27,28]. In GPTBC, the unknown distribution functions are supposed to have the equilibrium functions with a counter slip thermal energy density of \( \rho e' \); Then it’s value is selected in a way to satisfy the constraints.

\[
\tilde{g}_{4,7,8} = \rho(e + e') \left( g_{4,7,8}' / \rho \right) \quad \text{and} \quad \rho e' = 2 \rho e + 1.5 \Delta t \sum \tau_i Z_i - 3K
\]  

(39)

\( K \) shows the sum of six known distribution functions of “\( g \)" at the neighbor points and \( e \) is the imposed thermal energy density. Now the heat flux in the form of Boltzmann equation and based on the GPTBC model can be written as follows; However more details can be found in Refs. [23–25].

\[
q = \left( \sum_i c_i \tilde{g}_i - \rho \rho e \sum_i \frac{dt}{T} \sum_i c_i f_i Z_i \right) \frac{\tau_s}{\tau_s + 0.5dt}
\]

(40)

Eq. (39) is used to determine the unknown distribution functions along these walls. For example for the upper wall:

\[
\sum_i c_i \tilde{g}_i = 0.5dt \sum_i c_i f_i Z_i + \rho e N V_w + \frac{\tau_s + 0.5dt q_y}{\tau_s} q_y
\]

(41)

which leads to present the following model for the constant heat flux boundary condition along the microchannel walls in LBM configuration:

\[
\tilde{g}_4 = \left[ \frac{1}{3} - \frac{1}{2} \frac{V_n}{C} + \frac{1}{2} \frac{V_n}{C} \right] \times \left[ (g_2 + g_3 + g_6) - \frac{dt}{2} \sum \frac{c_i f_i Z_i}{c} \right] - \rho e N V_w + \tau_s + 0.5dt q_y
\]

(42)

\[
\tilde{g}_8 = \left[ \frac{1}{3} - \frac{1}{2} \frac{V_n}{C} + \frac{1}{2} \frac{V_n}{C} \right] \times \left[ (g_2 + g_3 + g_6) - \frac{dt}{2} \sum \frac{c_i f_i Z_i}{c} \right] - \rho e N V_w + \tau_s + 0.5dt q_y
\]

(43)

And finally the local Nusselt number is achieved as follows:

\[
Nu_x = \frac{q_y D_s}{\Delta T K} = \frac{D_s (\partial T / \partial y)_w}{T_w - T_{bulk}}
\]

(44)

5. Grid study and validation

The values of slip velocity and outlet Nusselt number were examined at different lattice numbers such as 350 × 35, 400 × 40 and 450 × 45 by a developed computer code in FORTRAN language to simulate the flow domain. Very little differences between the results corresponded to 400 × 40 and 450 × 45 were observed which could be ignored so that the lattice nodes of 400 × 40 were found suitable for the following computations (see Table 1).

Fig. 3 illustrates the validation of present work versus those of Hoorman & Ejali [6] concerned a flow and heat transfer through a microchannel at various amounts of Kn and the suitable agreements can be observed between them. Moreover the non-dimensional temperature profiles of \( \theta \) are presented in Fig. 4 versus those of Kavehpour et al. [7] at two different vertical cross sections along the microchannel for the state of \( Re = 0.01 \), \( T_w = 10 \), \( T_{inlet} = 1 \) and \( Kn_{inlet} = 0.01 \). This figure also implies the desirable accuracy of using developed code for the next simulations.

6. Results and discussions

Mixed convection heat transfer in a microchannel is investigated by using LBM. Aspect ratio of microchannel equals to 10

![Fig. 10. Slip velocity along the microchannel upper and lower walls at Gr = 500.](image-url)
while its horizontal walls imposed to the constant heat flux of $q_0$. The cold air at $T_i$ enters to the microchannel from the left side with a constant velocity of $u_i$. Three states are considered such as no gravity, $Gr = 100$ and $Gr = 500$. At each one, the value of Knudsen number is chosen as $Kn = 0.005$, $Kn = 0.01$ and $Kn = 0.02$ respectively; while Reynolds number and Prandtl number are kept fixed at $Re = 1$ and $Pr = 0.7$.

Fig. 5 shows the streamlines and isotherms at $Kn = 0.02$ for different values of $Gr$. At the state of no gravity ($Gr = 0$), the inlet flow from the left side would be affected continuously by the imposed heat flux from the upper and lower walls. However streamlines are completely symmetry and smooth along the walls. Higher temperature of air through the microchannel is quite clear; Moreover isotherms are also completely symmetry to the horizontal central line. More $Gr$ corresponds to more influences of buoyancy forces which lead to downward streamlines especially around the entrance regions. The symmetry of isotherms is also becoming less along the length; insofar as $Gr = 500$ while a rotational long cell
dominates the flow at upper half of domain, isotherms would be affected more and show less symmetries. The effect of lower value of $Kn$ on the streamlines and isotherms at various amounts of $Gr$ can be observed in Fig. 6. The same variations are seen except to generate the stronger and larger cell due to higher buoyancy forces.

Fig. 7 illustrates the non-dimensional slip velocity along the microchannel walls in the absence of buoyancy forces. $U_s$ begins from its highest value at the inlet and then decreases along the walls until where it approaches to a constant amount. It is observed that larger $Kn$ leads to larger $U_s$. Fig. 8 implies the influence of $Gr$ on slip velocity at $Kn = 0.005$. A noticeable fluctuation is seen at $Gr = 500$ along the diagrams of lower wall at $0 < X < 1$ where a negative slip velocity is achieved along the upper wall; which means the flow is in opposite direction of $X$ in this area because of the generating the rotational cell.

Moreover Fig. 9 presents the slip velocity at higher amount of Knudsen number as $Kn = 0.02$. Although the positive effect of $Kn$ on $U_s$ can also be seen in this figure but plots are affected less by $Gr$ compared with those of Fig. 8 which means lower amplitude.
of the fluctuation of the lower wall besides less absolute amount of $U_l$ for the upper wall. In the following, the amounts of $U_l$ along the microchannel upper and lower walls at $Gr = 500$ are shown in Fig. 10 for different values of $Kn$. This figure shows well the negative slip velocity existence along the upper wall at entrance region.

The horizontal dimensionless velocity profiles ($U = u/u_i$) along various vertical cross sections of the microchannel are presented in Fig. 11 at $Kn = 0.005$ for different amounts of $Gr$. At the state of no gravity, the profiles are completely parabolic but with the $U_{max}$ a little less than $U = 1.5$ at horizontal central line (at $Y = 0.5$) due to existence the slip velocity adjacent to the walls. More $Gr$ corresponds to more buoyancy forces which push downward the flow with $X$; as $U_{max} = 1.8$ at around $Y = 0.3$ for $Gr = 100$ and also $U_{max} = 4.5$ at around $Y = 0.25$ for $Gr = 500$. Moreover $U_{max}$ in opposite direction of $X$ equals to $-2.3$ in recent state which clearly shows that how important the gravity effects might be. Fig. 12 presents $U$ profiles at different cross sections at $Kn = 0.01$. Less value of $U_{max}$ for no gravity state at $Y = 0.5$ (due to generating higher slip velocity) and also less absolute amounts of $U_{max}$ in the same and opposite directions of $X$ axis (due to generating stronger buoyancy
forces), are some of other results of this figure in comparison with those of Fig. 11 concerned lower amount of Kn.

Dimensionless temperature profiles (θ = T/T_s) along different vertical cross sections of the microchannel at Kn = 0.005 and Kn = 0.02 are shown in Fig. 13 and Fig. 14, respectively. These figures also represent the major effects of gravity acceleration on temperature profiles. More focus on thermal properties of domain, those of Fig. 11 concerned lower amount of Kn and Gr. At the state of no gravity, Nu_x starts from its maximum and then trends to decrease with X so that reaches to a corresponded constant amount along the microchannel wall. It is observed that lower Kn corresponds to higher Nu (see diagrams related to no gravity state of three plots of Fig. 15). This phenomenon can also be seen more clearly in the plots corresponded to Gr = 500 at Kn = 0.005, Kn = 0.01 and Kn = 0.02 compared to one another. Moreover the amplitude and number of fluctuations increase severely with Gr because of the strong cell generated by buoyancy forces.

![Fig. 15. Local Nusselt number along the microchannel lower wall.](image)

7. Conclusion

Mixed convection of the air inside a microchannel was studied numerically by lattice Boltzmann method (TLBM-BGK). So that the following points could be addressed in brief:

1. A new case study concerned LBM performance was developed to consider both effects of natural and force convections in a microflow imposed to the heat flux boundary condition.
2. The suitable models for slip velocity and heat flux boundary conditions in terms of hydrodynamic and thermal distribution functions of LBM, were developed in order to consider the both effects of gravity and velocity inlet.
3. The symmetric forms of streamlines and isotherms were being less at higher Gr while a rotational strong cell dominated the flow at upper half of domain. However these patterns would be affected more by Gr at lower Kn.
4. Larger Kn led to larger U_x; Although the influence of Kn on U_x decreased at higher amounts of Gr. Moreover the negative slip velocity phenomenon due to the buoyance forces, was observed for the first time at present work.
5. At the state of no gravity, the profiles were parabolic but with U_{max} a little less than U = 1.5 at horizontal central line due to existence the slip velocity adjacent to the walls. More Gr pushed downward the flow with X as U_{max}=1.8 at around Y = 0.3 for Gr = 100 and also U_{max}=4.5 at around Y = 0.25 for Gr = 500. Moreover U_{max} in opposite direction of X equalled to –2.3 in recent state which clearly showed that how important the gravity effects might be.
6. Lower Kn corresponded to higher Nu. Moreover the amplitude and number of fluctuations of Nu, increased severely with Gr because of the strong cell generated by buoyancy forces.
7. Eventually it is seen that the influence of gravity would be important at Kn < 0.01; Hence it should be included at mentioned range to simulate a microflow under the constant heat flux.

Conflict of interest

There is no conflict of interest.

References