Investigation of permeability and porosity effects on the slip velocity and convection heat transfer rate of Fe$_3$O$_4$/water nanofluid flow in a microchannel while its lower half filled by a porous medium

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**Abstract**

In this study, we numerically investigated the forced heat-transfer and laminar flow in a two-dimensional microchannel whose lower half was filled with a porous medium. The nanoparticles used were Fe$_3$O$_4$ and a water-based fluid. The nanoparticles were considered in the form of a completely stable suspension in a water-based fluid. The nanofluid flow in this microchannel was modeled employing the Darcy–Forchheimer equation. We also hypothesized that there was a thermal equilibrium between the solid phase and nanofluid for energy transfer. And the walls of the microchannels were assumed at a constant temperature higher than the inlet fluid temperature. Also, the slip boundary condition was assumed along the walls. The effects of Darcy number, porosity and slip coefficients, and Reynolds number on the velocity and temperature profiles, and local Nusselt number were studied in both porous and non-porous regions in this research. In this study, the Darcy number was assumed to be $Da = 0.1$ and $0.01$, Reynolds number $Re = 25$, $50$, and $100$, slip coefficient $B = 0.1$, $0.01$, and $0.001$, the porosity of the porous medium $\varepsilon = 0.5$ and $0.9$, and the volume percentage of the nanoparticles $\varphi = 0\%$, $2\%$, and $4\%$. With the Darcy number decreasing, the local Nusselt number increased in the non-porous region, and decreased in the porous region. And this phenomenon was observed for the first time. The increase in the Reynolds number increased the heat transfer in both regions. For instance, the local Nusselt number increased 4 times with the Reynolds number changing from 25 to 100 under the same conditions. The decreased Darcy number in the porous medium increased the amount of slip velocity near the walls in the non-porous region, and on the other hand, the decreased Darcy number in the porous medium reduced the slip velocity in the porous region. Also, the jump observed in the slip velocity, was due to the presence of the fluid velocity in the microchannel width.

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1. Introduction

Nowadays, heat transfer is faced with two major challenges: on the one hand, cooling equipment with high thermal flux, and on the other hand, the issue of reducing the size of equipment. Air cooling is the most common method of cooling, while this method does not work in transferring high heat fluxes. Therefore, engineers have become interested in liquid cooling methods. Cooling liquids usually have poor heat transfer properties, and they definitely will not be used much in the future to transfer ultra-high heats [1–6]. In the last one hundred years, scientists and engineers have made many efforts to enhance the low thermal conductivity of the liquids through the addition of nano-sized solid particles. Since metal solids have higher thermal conductivity coefficients than the fluids have, it is expected that the suspension of metal solid nanoparticles in the fluid, increases its thermal conductivity as well as other properties [7–23]. Masoud Hosseini et al. [24] examined a dimensionless model is discussed to forecast effective thermal conductivity of nanofluids regarding the dimensionless groups. Their finding non-linear correlation for thermal conductivity which demonstrates a good compatibility between present model and experimental data of Al$_2$O$_3$/H$_2$O nanofluids compared to other models. Abdollahzadeh Jamalabadi et al. [25] studied Thermal loading by radiant heaters is used in building heating and hot structure...
design applications. The effect of thermal and geometrical parameters on entropy generation and the distribution field. Abbasian Arani et al. [26] studied laminar flow and heat transfer of nanofluid water/single-wall carbon nanotubes have been investigated in a novel design of double layered microchannel heat sink (MCHS). Goshayeshi et al. [27] examined effect of γ (gamma) and α (alpha) Fe2O3/Kerosene nanofluids for a closed loop pulsating heat pipe under the magnetic field. The results indicated that both heat transfer coefficient and thermal performance of the pulsating heat pipe are enhanced by the addition of Fe2O3 nanoparticles and. The increased input heat flux rises the heat transfer coefficient of the condenser and the evaporator. This study was to investigate the effect of a porous medium on a non-porous medium as well as heat transfer. But, if gravity is taken into account, it will be possible that the results change with changes in the position of the porous medium. Recently, some studies have been conducted on the effect of gravity on the heat transfer and kinematics of nanofluids, among which we can refer to the study conducted by Karimpour et al. [28]. They studied heat transfer in an oblique cavity at different angles while taking into account the gravitational force, the results show NuS numbers can be achieved at larger values of the inclination angle and nanoparticles volume fraction at free convection domination. Results imply the appropriate ability of LBM to simulate the mixed convection of nanofluid in a shallow inclined cavity. Manca et al. [29] studied the flow of water-aluminum oxide nanofluid in a two-dimensional channel with the uniform flux boundary condition on the external walls. The assumptions of this research included the stable properties of the nanofluid and use of a single-phase model for the nanofluid. They investigated the effects of the presence of ribs with different shapes, and changes in the Reynolds number and volume percentage of the nanoparticles on the heat transfer of the nanoparticles, and showed that increases in the Reynolds number and volume percentage of the nanoparticles will increase the heat transfer of the nanofluid. Mangrulkar et al. [30] investigated the heat transfer of nanofluids with changes in parameters such as: the flow geometry, boundary conditions, and the increased thermal conductivity of the fluid. They also investigated the effects of Reynolds number, mass concentration, and particle size on the heat transfer rate. The results of their work showed that increases in the Reynolds and Prandlt numbers increase the heat transfer coefficient, thus increasing the heat transfer rate. On the other hand, the addition of nanoparticles to the base fluid increases the thermal conductivity in the fluid. Ahmed et al. [31] numerically investigated a water-copper nanofluid in a channel using the finite difference method. The results of their work showed that increases in the Reynolds number and volume percentage of nanoparticles will increase the heat transfer of the nanofluid, while the pressure drop will be negligible. Akbarinia et al. [32] investigated the forced convection of a water-aluminum oxide nanofluid in a two-dimensional rectangular microchannel, and showed an increase in the heat transfer due to the increased volume percentage of the nanoparticles. Their results showed that the Reynolds number in the microchannel, and showed an increase in the heat transfer due to the increased volume percentage of the nanoparticles. Li et al. [33] studied the flow of water-copper nanofluid and pure water in a trapezoidal microchannel. The results showed that the presence of copper nanoparticles in the water-based fluid, would increase the thermal performance. Jung et al. [34] investigated the flow of water-aluminum oxide nanofluid in a rectangular microchannel. In their work, they showed an increase in the heat transfer due to the increase in the Reynolds number and volume percentage of the nanoparticles. They also found out that an increase in the Reynolds number would cause an increase in the Nusselt number. Maiga et al. [35] numerically investigated the heat transfer of a nanofluids inside a pipe in both laminar and turbulent regimes. The result of their work showed that the heat transfer of Al2O3-ethylene glycol nanofluid was higher than that of Al2O3 water nanofluid. Kalteh et al. [36] numerically investigated the forced convective heat transfer of water-copper nanofluid in a laminar regime using a two-phase model. In this work, they investigated the difference of velocity and temperature between the liquid phase and nanoparticles phase, and the results showed that the relative velocity and temperature in the phases were very small and could be ignored. They showed that the volume percentage distribution of the nanoparticles was uniform, and the increase in the heat transfer was higher in the two-phase model than in the homogeneous model. Use of porous material in increasing the heat transfer, is considered as one of the important issues. Nowadays, scientists and researchers have conducted a lot of research on how to use porous media. By investigating the effective parameters in heat transfer enhancement, including: different arrangements to reduce pressure drop due to spaces blocked by porous media, changes in the permeability, and the effect of the porosity of different materials, they have been trying to enhance heat transfer using this method [37–42]. Pavel and Mohamad [43] numerically and experimentally investigated heat transfer in a channel with constant and uniform flux boundary condition on the walls and a net-shaped metal porous material placed in the center of the channel. The study was conducted for both the laminar and turbulent regimes in the range of Reynolds numbers from 40 to 1000, and the effects of porosity, the diameter of the porous material, the conductive heat transfer coefficient, and the Reynolds number on the heat transfer was investigated. Their results showed that in the case that the diameter of the porous material was close to

<table>
<thead>
<tr>
<th>Nomenclature</th>
<th>Y non-dimensional vertical Cartesian coordinate</th>
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<tbody>
<tr>
<td>B</td>
<td>non-dimensional slip coefficient (/h)</td>
</tr>
<tr>
<td>Da</td>
<td>darcy number (=K/H^2)</td>
</tr>
<tr>
<td>d_p</td>
<td>diameter of nanoparticles (= 15 nm)</td>
</tr>
<tr>
<td>F</td>
<td>inertia coefficient</td>
</tr>
<tr>
<td>K</td>
<td>permeability (m^2)</td>
</tr>
<tr>
<td>Pr</td>
<td>Prandtl number (= u_0/σ)</td>
</tr>
<tr>
<td>q”</td>
<td>heat flux, (W/m^2)</td>
</tr>
<tr>
<td>Re</td>
<td>Reynolds number (=u_b/h_0)</td>
</tr>
<tr>
<td>U</td>
<td>non-dimensional horizontal velocity</td>
</tr>
<tr>
<td>U_s</td>
<td>non-dimensional slip velocity</td>
</tr>
<tr>
<td>X</td>
<td>non-dimensional horizontal Cartesian coordinate</td>
</tr>
</tbody>
</table>

Greek symbols
- α: thermal diffusivity (=k/ρc, m^2 s^-1)
- β: slip coefficient (m)
- φ: volume fraction of nanoparticles
- θ: non-dimensional temperature (= (T_1 - T_e)/(T_{H1} - T_{e}))
- λ: convection heat transfer coefficient (W/m^2 K)
the diameter of the channel; i.e., the channel was assumed to be filled with the porous material, the increase in heat transfer was acceptable due to an increase in pressure drop. Nusselt number would be about 5.2 times that in the case of a non-porous channel. Huang et al. [44] numerically and experimentally investigated heat transfer in a channel with copper net-shaped blocks having identical diameters. They considered the constant flux boundary condition for the channel walls. They used three porosity models (0.951, 0.966, and 0.975), and Reynolds numbers in the interval (1000–19000), and numerically investigated the effects of the nets’ radii on the system performance. The results showed that the use of a porous material with the same size as the pipe diameter would substantially increase heat transfer. Sheikholeslami and Ganji [45] studied heat transfer of nanofluid in a porous medium in presence of magnetic field the results show Nusselt number has a direct relationship with Darcy number and Rayleigh number also temperature gradient enhance with rise of Darcy number Rayleigh number and nanofluid mass fraction. Nebbali and Boughadef [46] investigated the heat transfer of non-Newtonian fluids in channels with porous blocks. Their work was performed based on the Brinkman-Forchheimer equation. They used two models in their work. The first model was a channel with a porous block, and the second model consisted of two porous blocks alternately inserted at the top and bottom of the channel. They investigated the effects of Darcy number, Reynolds number, and the thermal conductivity ratio on heat transfer. The results showed that heat transfer would be greater in the single-block mode, where the porous material occupied more space. Jiang and Lu [47] analytically and numerically studied a channel with impermeable walls and a porous medium under a constant heat flux. The results showed that the forced convective heat transfer coefficient for the fluid (air) was four to eight times greater in a channel with a porous medium than in a channel without a porous medium. In recent years, due to advances made in the field of manufacturing parts and components, a new branch has emerged in the manufacture of micro- and nanoscale mechanical systems. Therefore, in order to enhance the design quality, and improve the manufacture of micro- and nanocomponents, we need to correctly and accurately understand the physical properties of fluids on the micro- and nanoscale in microchannels. Recently, more focus has been put on microchannels, which is due to their advantages over large-scale channels. Among these advantages, we can refer to their low costs, small sizes, lower masses, and their capability of longer time reactions. But unfortunately, the pace of progress in theoretical and/or laboratory sciences to study the characteristics of the fluid flow and its heat transfer in microchannels, is lower than the pace of progress in their manufacturing technology. In recent decades, methods of manufacturing equipment in very small dimensions, were accompanied by significant and rapid developments [48–51]. Karimipour [52] investigated the forced heat transfer of water-silver, water-copper, and water-alumina nanofluids in a microchannel with the slip velocity and temperature jump boundary conditions, using the lattice Boltzmann method (LBM). The results showed that the addition of nanoparticles would increase the heat transfer of the nanofluid, as well as increasing the amount of temperature jump at the inlet of the microchannel. Karimipour et al. [53] studied the heat transfer of water-copper nanofluid flow in a microchannel with the slip velocity boundary condition, using the lattice Boltzmann method (LBM). The results showed that the addition of nanoparticles to the base fluid would increase the Nusselt number. Li et al. [54] explored empirically nanofluid including water and aluminium oxide. They studied effective thermal conductivity of nanofluid at static mode. Moreover, they used nanofluids such as aluminium oxide- water, aluminium oxide- ethylene glycol, copper oxide- water, and copper oxide- ethylene glycol. Their findings showed that nanofluid has intrinsically higher thermal conductivity than pure fluids. Similarly, with respect to high thermal conductivity of nanofluid ethylene glycol in which ethylene glycol serves as base fluid, it possesses higher effective thermal conductivity. Alternately, nanofluid which includes copper oxide possesses higher effective thermal conductivity than nanofluid that contains aluminium oxide.. Karimipour et al. [55] studied heat transfer in a microchannel by considering the gravitational force using the lattice Boltzmann method (LBM). The results showed that the gravitational force affected the slip velocity on the lower wall. The slip boundary condition is one of the cases which has attracted the attention of many scientists and researchers. The presence of fluid velocity on the wall has had an impact on the heat transfer rate and temperature of the fluid close to the surface [56–60]. Rahman et al. [61] numerically studied a two-dimensional steady slip flow under a magnetic field. They used two different nanofluids in their work. Their results showed that the velocity of the nanofluid was less than the velocity of the base fluid. Also, the flow of the nanofluid has a thinner hydrodynamic boundary layer and thicker thermal boundary layer than the flow of the base fluid has. By comparing the two types of nanofluids, they found out that the heat transfer rate was higher in the water-copper nanofluid than in the water-aluminium oxide nanofluid. Heat transfer enhancement due to the increased slip coefficient is one of the other results. Since in problems relating to porous media, the position geometry of the porous medium is of particular importance, many researchers have conducted their studies on this issue. These studies can be divided into a few groups in terms of the position of the porous material including: completely filled channels with the porous medium being positioned in the direction of the longitudinal axis of the channel and/or with the porous medium being positioned in a transverse manner in the channel, as well as with the porous medium being positioned in a staggered manner in the channel and/or different geometries; as porous media, being placed on the places where the fluid passes. In this study, we compared the two regions; i.e. the porous region and the empty region in a porous medium, and investigated the effect of these two on each other. Nowadays, microchannels are used in cooling various systems with small dimensions. The suggestion of using a microchannel with a semi-porous medium would be applicable in cooling electronic chips, in which it is necessary to asymmetrically control the temperature of the two sides of a circuit. Convective heat transfer and fluid flow with porous medium occur in power stations of many engineering applications where cooling or heating is required such as cooling turbine blades cooling electronic equipment and combustion systems. One of the ways to increase heat transfer is to employ porous medium with nanofluid.

2. Problem definition

The problem being studied is a two-dimensional microchannel with an aspect ratio of $AR = L/H = 30$, whose lower half is filled with a porous material, and whose upper half has no porous medium as shown in Fig.1. The intended nanofluid contains water and $Fe_2O_3$ nanoparticles. Moreover, the diameter of nanoparticles is $d_p = 15 \text{ nm}$. The cold nanofluid with a temperature of $T_i = 298 \text{ K}$ and constant velocity of $u_i$ enters the microchannel, and exits from its end after cooling the walls of the microchannel which are at a temperature of $T_w = 308 \text{ K}$. The nanofluid flow inside the microchannel has been considered to be laminar, steady, Newtonian, and incompressible. The thermophysical properties of water-$Fe_2O_3$ nanoparticles at a constant temperature of $T = 25 \text{ °C}$ are provided in Table 1. The slip boundary condition has been considered on the walls of the microchannel. The values of Reynolds number for the inlet nanofluid are considered to be $Re = 25$, $Re = 50$, and $Re = 100$. Changes in the slip coefficient values on the walls.
of the microchannel are as follows: B = 0.001, B = 0.01, and B = 0.1. Also, the volume percentage values of the nanoparticles are as follows: φ = 4%, φ = 2%, and φ = 0%. The Darcy numbers have been considered to be Da = 0.01 and Da = 0.1, and the porosity 0.5–0.9. The Reynolds number was considered within this interval in order to show the effect of fluid velocity in heat transfer and thermal development. The Darcy number was considered within this interval, so that the permeability effect is comparable in the porous medium on the nanofluid. Given that the nanofluid was considered in a microchannel, the slip coefficient was considered within the interval 0.1–0.001, so that the effect of the slip boundary condition could have been.

3. Formulation

The finite volume method was used to discretize the governing equations. The SIMPLEx algorithm was used to solve the velocity and pressure fields. Also an organized network was used to solve the governing equations and discretized the solution area [62–72]. The convergence condition in solving the discretized governing equations, is that the parameter values should be less than 10^8. The second-order upwind scheme has been used to discretize the permeability and convection terms in the governing equations.

3.1. Governing equations

Continuity equation

\[
\frac{\partial}{\partial x} (\rho u_i) + \frac{\partial}{\partial y} (\rho v_i) = 0
\]  

(1)

Momentum equations

\[
\frac{\partial}{\partial x} (\rho v_i u_i) + \frac{\partial}{\partial y} (\rho v_i v_i) = \frac{\partial p}{\partial x} + \rho u_i \frac{\partial u_i}{\partial x} + \rho v_i \frac{\partial u_i}{\partial y} - \frac{\mu_i}{K} \frac{\partial u_i}{\partial y} + \frac{\mu_{nf}}{K} \frac{\partial u_i}{\partial y} - \frac{\rho_{nf} F}{\sqrt{K}} u_i v_i
\]  

(2)

\[
\frac{\partial}{\partial x} (\rho v_i v_i) + \frac{\partial}{\partial y} (\rho v_i u_i) = \frac{\partial p}{\partial y} + \rho v_i \frac{\partial v_i}{\partial x} + \rho u_i \frac{\partial v_i}{\partial y} - \frac{\mu_i}{K} \frac{\partial v_i}{\partial x} + \frac{\mu_{nf}}{K} \frac{\partial v_i}{\partial x} - \frac{\rho_{nf} F}{\sqrt{K}} u_i v_i
\]  

(3)

where \( \rho_{nf} \) is considered as the nanofluid density, \( u_i \) the component of instantaneous speed along the axis \( x_i \), the component of instantaneous speed along the axis \( y \), \( \mu_{nf} \) the nanofluid viscosity, \( T_c \) the temperature of the cold nanofluid, and \( F \) Forchheimer’s correction coefficient. \( u_i \) Also, \( K \) is the permeability coefficient of the porous medium.

Energy equation

\[
\frac{\partial}{\partial x} (\rho_{nf} c_{nf} u_i T_c) + \frac{\partial}{\partial y} (\rho_{nf} c_{nf} u_i T_c) = \frac{\partial}{\partial x} (k_e \frac{\partial T_c}{\partial x}) + \frac{\partial}{\partial y} (k_e \frac{\partial T_c}{\partial y})
\]  

(4)

where \( c_{nf} \) is the specific heat capacity of the nanofluid, and \( K_e \) the effective thermal conductivity of the porous medium.

\[
\frac{\partial}{\partial x} (U) + \frac{\partial}{\partial y} (V) = 0
\]  

(5)

Dimensionless momentum equations

\[
\frac{\partial}{\partial x} (\tilde{U} \tilde{U}) + \frac{\partial}{\partial y} (\tilde{V} \tilde{V}) = \frac{\partial \tilde{P}}{\partial x} + \frac{\partial \tilde{P}}{\partial y} + \frac{\partial \tilde{U}}{\partial x} + \frac{\partial \tilde{V}}{\partial y} - \frac{U}{DaRe} - \frac{F}{\sqrt{Da}} \frac{U}{|U|}
\]  

(6)

\[
\frac{\partial}{\partial x} (\tilde{U} \tilde{V}) + \frac{\partial}{\partial y} (\tilde{V} \tilde{U}) = \frac{\partial \tilde{P}}{\partial x} + \frac{\partial \tilde{P}}{\partial y} + \frac{\partial \tilde{V}}{\partial x} + \frac{\partial \tilde{U}}{\partial y} - \frac{V}{DaRe} - \frac{F}{\sqrt{Da}} \frac{V}{|V|}
\]  

(7)

Dimensionless energy equation

\[
\frac{\partial}{\partial x} (\tilde{U} \tilde{T}) + \frac{\partial}{\partial y} (\tilde{V} \tilde{T}) = \frac{1}{Pr} \frac{\partial (\tilde{E})}{\partial x} + \frac{1}{Pr} \frac{\partial (\tilde{E})}{\partial y}
\]  

(8)

where \( Da \) is the Darcy number and \( Pr \) the Peclet number.

In the equations above, the dimensionless numbers are all defined based on the properties of the nanofluid. Also, in Eqs. (1)–(8), the following parameters have been used:

\[
H = \frac{h}{h}, \quad L = \frac{l}{h}, \quad Y = \frac{y}{h}, \quad X = \frac{x}{h}, \quad U = \frac{u}{u}, \quad V = \frac{v}{u}
\]  

(9)

\[
\theta = \frac{T - T_c}{T_H - T_c}
\]

\[
P = \frac{P}{\rho_{nf} u_i h}, \quad Re = \frac{\rho_{nf} u_i h}{\mu_{nf}}, \quad Pr = \frac{\mu_{nf}}{k_{nf}}
\]

(10)
3.2. Darcy’s law

Thermal and hydraulic effects are considered as two important factors when investigating the flow in the porous medium. The first study was conducted by Darcy [74], and Darcy’s law was registered in his own name. Darcy’s law states that the pressure drop per unit length for a flow in a porous medium is proportional to the product of the fluid velocity and its dynamic viscosity. Targui et al. [75] added the inverse proportionality constant to Darcy’s law. This constant has been called permeability, which is a measure of the fluid’s resistance when passing through the porous medium. Darcy’s law is defined as below:

\[ \Delta p = -\mu U / k \]

Table 2

<table>
<thead>
<tr>
<th>Grid points</th>
<th>600 × 25</th>
<th>900 × 30</th>
<th>1800 × 60</th>
</tr>
</thead>
<tbody>
<tr>
<td>U</td>
<td>1.833</td>
<td>1.845</td>
<td>1.845</td>
</tr>
<tr>
<td>h</td>
<td>0.465</td>
<td>0.466</td>
<td>0.466</td>
</tr>
</tbody>
</table>

Fig. 2. Present work’s fully developed velocity profiles with those of Karimipour and Afrand for different values of B [81].

Fig. 3. Present work’s fully developed velocity profiles with those of Teamah et al. for different Da [82].

Fig. 4. a. Present work’s fully developed velocity profiles with those of Radiom et al. for different Da [83]. b. Variation of heat transfer coefficient with Re from present work versus of experimental study [84] for different values of bulk temperature.
\[ \frac{dp}{dx} = \frac{\mu}{K} u \quad (11) \]

where \( \frac{dp}{dx} \) is the pressure gradient, \( u \) the average velocity, \( \mu \) the fluid's dynamic viscosity, and \( K \) the permeability of the porous medium. Darcy's law can only be used when the Reynolds number of the flow is low, and the viscous and compressive forces are dominant. In fact, this relation is only limited to low-velocity flows and saturated media with low porosities. Porosity is one of the parameters of porous media, which is defined as below:

\[ \mathcal{E} = \frac{\text{The void volume within the porous medium}}{\text{The total volume of the porous medium}} \quad (12) \]

\[ \mathcal{E} = \begin{cases} 0 < \varepsilon < 1 & \text{The porous medium} \\ 1 & \text{The fluid} \end{cases} \quad (13) \]
As you know, porosity is defined as the ratio of the empty space of a porous medium to the filled space of a porous medium. In order to compare the effect of this parameter, we considered these two values to evaluate the effect of this parameter on the heat transfer and velocity of the nanofluid. Therefore, in the condition where the porosity is considered 0.9, the nanofluid travels the length of the porous area of the microchannel in less time. Conversely, this time will be longer for a porosity of 0.5. Because in this case, in order to travel the path in the microchannel, the fluid needs to pass through a greater number of holes with smaller dimensions in the porous medium, thus increasing the heat transfer rate. We considered this numerical interval in this study to show and compare the effect of this parameter (porosity) on the nanofluid velocity and heat transfer.

As the Reynolds number increases, the inertial term affects the momentum equation, and a deviation from the Darcy equation will be observed. In all the media studied, the axial pressure drop includes two terms, one of which is a linear function of velocity (viscous), and the other is a quadratic function of velocity (inertial). The latter is Forchheimer’s modification of Darcy’s law. At high velocities, Forchheimer’s coefficient will be a corrector in the Darcy equation, and will correct its deviation. Albeit, at low velocities, this coefficient does not play any role. Flow of fluid through a porous medium hinges on Darcy’s law, where unload rate of fluid flow in the porous media is proportionate to the pressure drop and the viscosity of the fluid over a certain space. Darcy’s law was thereafter established to be bounded in precision and only usable for low velocity flow that is incompressible and isothermal. More research brought forth the Dupuit-Forchheimer extension to Darcy’s law which is characterized in Eq. (14). This addition to Darcy’s law expresses the form drags effect on the flow which is denoted by the inertial resistance coefficient (F). The connection of inertial resistance is non-linear and affects higher velocity flows,
but it varies with respect to porosity, internal structure, and pore size. Flow in porous media are described as being in one of three ranges, which are concluded from the flow Reynolds number. They are laminar, non-linear laminar (the transition between the Darcy regime, where viscous effects dominate, to the Forchheimer regime, where inertial effect dominate) and turbulent rages [73,76,77].

\[
\frac{dp}{dx} = \frac{\mu}{K} u + \frac{F\rho}{\sqrt{K}} u^2
\]

where \(F\) is the coefficient of inertia, and \(\rho\) the fluid density.

3.3. The velocity boundary conditions

At the macroscopic level, the non-slip condition is one of the simplest boundary conditions, which states that a liquid adjacent to the solid surface has the same surface velocity. Recently, researchers have shown that the no-slip boundary condition may not be appropriate for flows in microscale channels, because a microscale flow of a liquid is different from a macroscale flow. In fact, in slip regimes, there is an area in the vicinity of the wall, where the molecules of the fluid are volatile. Therefore, only by considering the slip boundary condition, it will be possible to use the Navier-Stokes equations. To solve the slip velocity, \(u_{\text{fluid}}, u_{\text{wall}}, L_s\) are defined as the slip length, wall velocity, and fluid velocity, respectively [78]:

\[
\Delta u_{\text{wall}} = u_{\text{fluid}}(y \to \text{wall}) - u_{\text{wall}} = L_s \frac{\partial u_{\text{fluid}}(y)}{\partial y}_{\text{wall}}
\]

As a result, the slip velocity of the nanofluid \(u_s\) on the walls, is calculated as follows [79]:
where the parameter $b$ is called the slip coefficient, and the dimensionless form of Eq.(16) is defined as follows:

\[ \frac{u_s}{\beta} = \frac{\partial u}{\partial y} |_{y=0,1} \]  

And the other dimensionless boundary conditions are as follows:

\[ U = 1, V = 0 \quad \text{and} \quad \theta = 0 \quad \text{for} \quad X = 0 \quad \text{and} \quad 0 \leq Y \leq 1 \]

\[ V = 0 \quad \text{and} \quad \frac{\partial U}{\partial X} - \frac{\partial \theta}{\partial X} = 0 \quad \text{for} \quad X = 30 \quad \text{and} \quad 0 \leq Y \leq 1 \]

Eqs. (5)–(8) are numerically solved under the aforesaid boundary conditions, with the aid of the SIMPLE algorithm and using a computer code. The local Nusselt number is as follows:

\[ Nux_{\text{lower wall}} = \frac{K_f}{K_c} \left( \frac{\partial \theta}{\partial Y} \right) |_{Y=0} \]

\[ Nux_{\text{upper wall}} = \frac{K_f}{K_c} \left( \frac{\partial \theta}{\partial Y} \right) |_{Y=1} \]
\[ \dot{\lambda} = \frac{q^*}{T_0 - T_c} \]  

\[ \dot{\lambda} = \frac{q^*}{T_0 - T_c} \]  

Fig. 32. Dimensionless temperature profiles at vertical centerline at \( \varphi = 0.04, \varepsilon = 0.5, \text{Re} = 50, \beta = 0.1 \) for different values of \( Da \).

Fig. 33. Dimensionless temperature profiles at horizontal lines of \( Y = 0.25H \) and \( Y = 0.75H \) in Porous and non-porous medias of microchannel at \( \varphi = 0.04, \varepsilon = 0.5, \text{Re} = 50, \beta = 0.1 \) for different values of \( Da \).

Fig. 34. Dimensionless temperature profiles at horizontal lines of \( Y = 0.25H \) and \( Y = 0.75H \) in Porous and non-porous medias of microchannel at \( \varphi = 0.04, \varepsilon = 0.5, \text{Re} = 50, \beta = 0.1 \) for different values of \( Da \).

Fig. 35. Nu of non-porous along the microchannel wall at different values of \( \text{Re} \) for \( \varphi = 0.04, Da = 0.1, \varepsilon = 0.5 \) at \( \beta = 0.1 \).

\( k \) is the convection heat transfer coefficient \([80]\):

\[ k = \frac{q^*}{T_0 - T_c} \]  

Because the results are clearer, on the chart and since the results showed that the addition of nanoparticles to the base fluid increased the heat transfer rate in the base fluid; thus, use of this coefficient would be a great help in showing this result. Also, in their study on the effect of a magnetic field on the heat transfer of the nanofluid, slip velocity, and temperature jump in a microchannel to express the effect of nanoparticles on heat transfer, Karimipour and Afrand \([81]\) investigated the mean Nusselt parameter. They used the coefficient \( \dot{\lambda} \) to show the results of their study.

4. Results and discussion

4.1. Network Independence and validation

Table 2 shows the numerical values of dimensionless velocity and temperature of the nanofluid in the middle of the microchannel for three models of meshing in different networks including: \( 600 \times 1800, 30 \times 900, \) and \( 25 \times 600 \) for \( \text{Re} = 50, \) in \( \varphi = 4\% \) and \( \beta = 0.001 \). It can be seen that the differences between the networks are small in the results obtained. Therefore, the network \( 30 \times 900 \) is selected to continue the computations.

To validate the work, the developed velocity profiles of a water-copper nanofluid in a microchannel for different values of slip coefficient of velocity are shown in Fig. 2 along with the results obtained by Karimipour and Afrand \([81]\). They conducted a numer-
ical study on the forced convection of the laminar flow of water-copper nanofluid in a microchannel with slip and non-slip boundary conditions. Also in Fig. 3, the velocity profiles for the fluid (air) in a channel for different values of Darcy number have been compared with the results obtained by Teamah [82]. Teamah numerically studied the forced convection of the laminar flow of the fluid (air) in a tube with different arrangements of the porous material. The velocity chart is drawn for two modes; $Da = 0.01$ and $Da = 0.0001$ in Fig. 4a. The obtained results are slightly different from those of Reference. Forced convective heat transfer coefficient at the inlet of a horizontal pipe for water/FMWCNT nanofluid in $\varphi = 0.12\%$ and different bulk temperatures and Reynolds numbers were compared with the work of Amrollahi et al. [84] in Fig. 4b. Amrollahi et al. experimentally measured convective heat transfer coefficient of $0.12\%$ nanofluids to that of base fluid in entrance region of laminar and turbulent flow as a function of Reynolds number in different temperatures. Reynolds number was considered to be in the range of $1592 \leq Re \leq 4778$ and the experimental area was a pipe of $1$ m length and $11.42$ mm diameter. Their results revealed that compared to water, heat transfer coefficient of the nanofluid in laminar and turbulent regimes increases with the rise in nanoparticles concentration.

4.2. The effect of Reynolds number on temperature and velocity contours

Figs. 5–7 related to the temperature contours of the nanofluid at different Reynolds numbers along the microchannel, show that an increase in the Reynolds number causes the nanofluid to have less
time for heat exchange with the walls of the microchannel. Therefore, the nanofluid will have a delay in reaching thermal development. Also, Figs. 8–10 show the velocity contours of the nanofluid at different Reynolds along the microchannel. Due to the presence of the porous material in the lower part of the microchannel, we will be witnessing speed changes in the inlet. But with the advancement of the fluid in the microchannel, these changes decrease, and ultimately, the velocity will become constant in both regions. The increase in the Reynolds number increases the range of these changes along the inlet of the microchannel.

4.3. The effects of permeability and porosity on temperature and velocity contours

Figs. 11 and 12 show the effect of Darcy number on the temperature contours of the nanofluid along the microchannel. It can be seen that with the Darcy number decreasing, the heat transfer between the nanofluid and the lower wall increases in the porous region due to the reduced permeability. Therefore, the asymmetry of temperature distribution along the microchannel is indicative of changes in the Darcy number.

Through comparing Figs. 13 and 14 with Figs. 11 and 12, we can see the effect of porosity on the temperature distribution of the nanofluid along the microchannel. The increased porosity in the porous medium provides more opportunity for the fluid to exchange heat with the wall. Thus, the greater will be the nanofluid temperature in the microchannel inlet.

Figs. 15–18 show the effect of Darcy number and porosity level on the velocity contours of the nanofluid in different Darcy numbers and different porosities along the microchannel. Due to the
presence of the porous material in the lower part of the microchannel, we will be witnessing speed changes in the inlet. But with the advancement of the fluid in the microchannel, these changes decrease, and ultimately, the velocity will become constant in both regions. The decreased Darcy number and porosity number reduces the inlet length, which includes velocity changes.

4.4. The effect of the slip coefficient on the nanofluid velocity

Fig. 19 shows the effect of different values of slip coefficient on the velocity profile along the vertical line in the middle of the microchannel at \( \varphi = 4\% \), \( Re = 50 \), \( Da = 50 \), and \( \varepsilon = 0.9 \). Due to the slip boundary condition on the walls, the nanofluid on the walls moves fast, and with the slip coefficient increasing, the velocity on the walls increases too. The presence of a porous medium in the lower half of the microchannel has prevented the nanofluid from moving uniformly in the width of the microchannel. In fact, the nanofluid volume is greater in the upper half than in the lower half of the microchannel. Thus, the greater will be the slip velocity near the upper wall.

4.5. Effects of permeability and porosity on the velocity and temperature of nanofluid

Due to the fact that the microchannel consists of two areas: open and porous, the nanofluid will act differently at the microchannel inlet when colliding with these two areas. The part of the nanofluid, which passes through the upper space of the microchannel, will travel faster, because the path is open. And the part of the nanofluid, which passes through the lower space, will slow down between the cavities of the porous medium, and also due to the permeability of the porous material. Therefore, the presence of a porous medium has resulted in different volumetric flow rates for the nanofluid in its passing from the upper half and lower half, which is the reason for different velocities in these two regions. Figs. 20 and 21 show the velocity profile along the vertical line in the middle of the microchannel, at \( \varphi = 4\% \) and \( Re = 50 \), for different values of Darcy number and porosity. Due to the slip boundary condition on the walls, the nanofluid on the walls moves fast, and with the slip coefficient increasing, the velocity on the walls increases too. Here, due to the presence of the porous medium in the lower half of the microchannel, the velocity profile will not be parabolic, and the slip velocity will also increase near the wall. The porous medium reduces the passage of fluid through itself. Therefore, the values of nanofluid velocity will be different in the microchannel cross sections for each region (upper and lower), which is why the values of nanofluid velocity are different on the microchannel walls.

Figs. 22–25 show the effects of different values of Darcy number and porosity on the dimensionless velocity profile in a direction perpendicular to the microchannel direction at different cross sections in the inlet at \( \varphi = 4\% \), \( B = 0.1 \), and \( Re = 50 \). Due to the presence of a porous medium in the lower half of the microchannel, part of the nanofluid enters the porous medium, and part of it is directed towards the upper half. Therefore, we will be witnessing the fluid velocity at the microchannel inlet in a direction perpendicular to the microchannel direction. Fig. 26 shows the velocity in a direction perpendicular to the microchannel direction at the microchannel inlet. In this picture, with the Darcy number and porosity decreases, the velocity increases in the direction perpendicular to the microchannel direction, and as the fluid advances in the length of the microchannel, the value of this velocity decreases and reaches zero.

As you know, porosity is the ratio of the empty space with a porous medium to the empty space without a porous medium. Therefore, the smallness of this number represents a higher resistance to the movement of the nanofluid and the deviation of a larger part of the nanofluid towards the upper half of the microchannel. In fact, by analyzing this chart, we will conclude that the smaller the Darcy number, the greater the amount of nanofluid transferred from the porous medium to the empty space, thus witnessing a more severe change in the flow direction along the vertical direction of the microchannel at the microchannel inlet. Finally, as it is clear in the chart, after traveling its way along the microchannel and passing through the inlet, the nanofluid will have a velocity along the microchannel, which will eventually occur with different Darcy numbers and different porosities, but at different points. Figs. 27–30 show the effects of different values of Darcy number and porosity on the dimensionless temperature profile at different cross sections of the microchannel at \( \varphi = 4\% \), \( B = 0.1 \), and \( Re = 50 \). Due to the presence of a porous medium in the lower half of the microchannel, we will be witnessing asymmetry in the temperature profiles. The decreased velocity in the porous medium increases heat transfer. Thus, the nanofluid temperature increases faster in the lower half of the microchannel than in the upper half. Figs. 31 and 32 show the dimensionless temperature profile of the nanofluid in the middle of the microchannel for different values of Darcy number and porosity. By comparing the two pictures, we have shown the effects of porosity and Darcy number on the increased heat transfer in the lower half of the microchannel.

Figs. 33 and 34 show the rate of temperature increase in the nanofluid along the microchannel in the upper half and lower half. As can be seen, the porous medium has significantly increased heat transfer in the lower half. Therefore, reduced Darcy number and porosity increase heat transfer.

4.6. Effects of \( Re \), volume percentage, permeability and porosity on \( Nu \inlet \)

The Nusselt number is a criterion to measure the heat transfer rate in the microchannel. Therefore, the presence of a porous medium in half of the microchannel, has caused a change in velocity in the two areas (porous and non-porous) along the microchannel inlet. Hence, it can be concluded that the reason for this sudden change is the change in the direction of part of the nanofluid due to the resistance of the porous medium. As you know, in a microchannel, an increased Reynolds number will increase the
Nusselt number. And in this study, the presence of a porous medium will increase the nanofluid velocity in the upper (non-porous) half. And a decrease in the Darcy number will result in an increased velocity in the upper half. In addition, the sudden change for a Darcy number of 0.01 at the inlet in the Nusselt chart, shown in Fig. 37, is due to a direction change in the velocity of the nanofluid in the vertical direction of the microchannel. The local Nusselt number has the maximum value at the inlet of the microchannel due to the highest temperature difference between the nanofluid and the microchannel walls, and decreases along the microchannel with an increased X.

Figs. 35 and 36 show the effect of Reynolds numbers on the local Nusselt. The results show that with the Reynolds number increasing, the value of local Nusselt number increases both in the upper and lower half of the microchannel. With the exception that in the upper half, there are greater differences between the values of local Nusselt numbers for different cross-sections and different Reynolds numbers compared with those in the lower half of the microchannel, which is due to a decrease in the velocity because of the presence of a porous medium in the lower half.

Figs. 37 and 38 show the effect of porosity on the local Nusselt graph in both halves of the microchannel. As shown, the reduced Darcy number has increased the local Nusselt number in the upper half of the microchannel. But, in the lower half, due to the presence of a porous medium, the increased Darcy number reduces the local Nusselt number.

One of the goals of this study is to investigate the effect of nanoparticles on heat transfer rate in the base fluid. Figs. 39 and 40 show the increase in the local Nusselt number for each increase in the volume percentage of suspended nanoparticles. Thus, in both the upper and lower half of the microchannel, local Nusselt increases with the increased volume percentage of nanoparticles in the base fluid.

4.7. The effects of the slip coefficient, permeability, and porosity on the slip velocity

As you know, slip velocity has a maximum value at the microchannel inlet, which decreases as the nanofluid advances along the microchannel until reaching a fixed value. In this study, we provided a different condition. Due to the slip boundary condition, the nanofluid will have a velocity on the microchannel walls. Slip velocity is expected to be at its maximum value at the microchannel inlet, and to decrease along its path. But due to the use of a porous medium in half of the microchannel, conditions are different for the nanofluid velocity in the two areas (porous and non-porous). At the microchannel inlet, we will be witnessing a direction change in the velocity of the nanofluid due to the presence of a porous medium. Therefore, the reason for this change is the resistance of the porous medium to the passage of the nanofluid and its influence on the direction of the nanofluid flow at the microchannel inlet. Figs. 41 and 42 show the graph of slip velocity along the microchannel for different values of slip coefficient. As can be seen, in both the upper and lower regions of the microchannel, the slip velocity has maximum value near the wall at the inlet of the microchannel, and within a very short distance from the inlet wall of the microchannel, it decreases with the increased X, and then reaches a constant value.

Figs. 43 and 44 show the graph of slip velocity along the microchannel for different values of Darcy number and porosity. As can be seen, in the upper region of the microchannel, the slip velocity increases with the decreased Darcy number. Due to the presence of a porous medium in the lower half, the nanofluid has a velocity in a direction perpendicular to the microchannel direction, at the microchannel inlet. And this velocity causes a sudden change in the slip velocity at the microchannel inlet. Since the lower half of the microchannel contains a porous medium, the nanofluid velocity will be less in the lower half than in the upper half. Therefore, a reduced Darcy number and porosity will reduce the slip velocity.

5. Conclusion

The forced convection heat transfer of a nanofluid composed of water and Fe$_3$O$_4$ nanoparticles in a two-dimensional microchannel was numerically investigated with the aid of a computer code. The microchannel walls are at a constant temperature greater than the temperature of the inlet nanofluid. Also, the slip boundary condition has been assumed along the walls. The lower region of the microchannel is filled with a porous material, but there is no porous material in its upper half. Our investigations showed that with the increased Reynolds number, the nanofluid velocity increased, thus increasing the local Nusselt number. Also, the reduced permeability would increase the local Nusselt number in the upper half of the microchannel, and would reduce it in the lower half. The presence of a velocity difference at a same cross-section of the microchannel due to the presence of a porous medium in the lower half of the microchannel, led us to be witnessing an increase in the slip velocity in the upper half and a decrease in the slip velocity in the lower half of the microchannel as the Darcy number was decreasing.

Conflict of interest

There is no conflict of interest.

References


