Influence of T-semi attached rib on turbulent flow and heat transfer parameters of a silver-water nanofluid with different volume fractions in a three-dimensional trapezoidal microchannel

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\section*{ABSTRACT}

This study aimed at exploring influence of T-semi attached rib on the turbulent flow and heat transfer parameters of a silver-water nanofluid with different volume fractions in a three-dimensional trapezoidal microchannel. For this purpose, convection heat transfer of the silver-water nanofluid in a ribbed microchannel was numerically studied under a constant heat flux on upper and lower walls as well as isolated side walls. Calculations were done for a range of Reynolds numbers between 10,000 and 16,000, and in four different sorts of serrations with proportion of rib width to hole of serration width (R/W). The results of this research are presented as the coefficient of friction, Nusselt number, heat transfer coefficient and thermal efficiency, four different R/W microchannels. The results of numerical modeling showed that the fluid's convection heat transfer coefficient is increased as the Reynolds number and volume fraction of solid nanoparticle are increased. For R/W=0.5, it was also maximum for all the volume fractions of nanoparticle and different Reynolds numbers in comparison to other similar R/W situations. That's while friction coefficient, pressure drop and pumping power is maximum for serration with R/W=0 compared to other serration ratios which lead to decreased fluid-heat transfer performance.

\section*{1. Introduction}

With the industry development, new methods of heat transfer with high thermal efficiency become essential in research. These methods are helpful in industrial tools and processes, including cooling and heating of thermal resources and production processes in industries, such as transportation, pharmacy, electronic, automotive as well as micro- and nanoelectromechanical systems.

Small-scale cooling systems, e.g. cooling microchannels, have important role in removing extra heat from the machines and devices. Electromechanical devices, complex circuits and laser diodes are heated through the flow crossing. In other words, more function of speed transistors, more current is crossed on them, which leads to extra heat. Heat transfer improvement using new methods has led to significant saving in costs and energy resources as well as environment protection. Some of the methods are the use of suspensions instead of working fluid in heat transfer equipment [1], use of the finned surfaces to raise the heat transfer surfaces [2] and use of the rough surfaces to mix up laminar sublayer in boundary layer of the turbulent flow [3]. After that, researchers also developed secondary flows for better mix of the working fluid in the heat transfer environment [4–6]. One of the abovementioned suggested methods for increasing the heat transfer rate is to increase the fluid conductivity. Several experimental and numerical studies have explored effect of increased fluid thermal conductivity by suspending the ultrafine particles in the fluid, which has been proposed as new strategies in the heat transfer process. In 1891, Maxwell conducted a research using the additive particles in millimeter and micrometer scales [7]. The problem was that the particles precipitated and the flow slowed down. In addition, tubes erosion and drastic pressure drop in flow were other weaknesses of these kinds of fluids [8]. In contrast, micro- and nanoparticles can produce stable suspensions due to the high surface to volume ratio. Thus, the sediments in channels and tubes are decreased. For the first time, Choi suggested adding metal materials of nanometer size into...
**Nomenclature**

A \quad Area, m$^2$

$C_f$ \quad Poiseuille number

$C_p$ \quad Special heat, J/kg K

$D_h$ \quad Hydraulic diameter, m

D \quad Solid particle molecule diameter, nm

f \quad Coefficient of friction

H \quad Microchannel height, nm

h \quad Convective heat transfer coefficient, W/m$^2$ K

k \quad Thermal conductivity, W/m K

L \quad Microchannel length, mm

$L_1$ \quad Microchannel Input length, mm

$L_2$ \quad Microchannel output length, mm

$N_u$ \quad Nusselt number

p \quad Perimeter, m

P \quad Pressure, Pa

PEC \quad Performance evaluation criterion

$P_p$ \quad Pumping power, W

$P_r$ \quad Prandtl number

$q_{//}$ \quad Heat flux, W/m$^2$

R \quad Rib width, mm

Re \quad Reynolds number

T \quad Temperature, K

u, v, w \quad Velocity component in direction of x, y, z m/s

W \quad Rib width, mm

**Greek symbols**

$\rho$ \quad Density, kg/m$^3$

$\mu$ \quad Viscosity, Pa s

$\alpha$ \quad Thermal diffusion, m$^2$/s

$\beta$ \quad Volumetric expansion coefficient, 1/K

$\phi$ \quad Volume fraction

**Subscript**

ave \quad Average

f \quad Fluid

b \quad Basefluid

m \quad Mean

nf \quad Nanofluid

p \quad Solid particles

s \quad Smooth

T \quad Top

w \quad Wall

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Fig. 1. Schematic shape of the channel, serration dimensions and serration types investigated in this research.
usual working fluid, such as water, ethylene glycol and industrial oil [2].

Many numerical studies have been conducted to evaluate the use of nanofluids in the heat transfer equipment. Goodarzi et al. assessed the use of nanofluids in heat exchangers [12]. Also, Duangthongsuks and Wongwises [13] studied the forced convection heat transfer and characteristics of water-Titanium Oxide nanofluid flow with 0.2 vol% in a counter flow double-tube heat exchanger and in the turbulent flow condition. They found out compared to the base fluid, the convection heat transfer coefficient of nanofluid would be increased by 11%. The convection heat transfer coefficient goes higher as the mass rate of base fluid and nanofluid increase and the nanofluid temperature decreases.

Many researchers have monitored the behavior of fluid flow and heat transfer in channels and microchannels in different cross sections. Most of them have focused on the influence of parameters, such as the Reynolds, cross-section shape of the channel, geometrical characteristics, arrangement of serrations and serrations’ angle of attack. Jung et al. [14] investigated forced convection heat transfer of nanofluids in microchannels with water-Aluminum Oxide nanofluid. This study revealed that nanofluid’s convection heat transfer coefficient with 1.8 vol% of nanoparticles was higher than 32% than the water’s convection heat transfer coefficient. Moreover, in microchannels with smaller dimensions, heat transfer coefficients in lower Reynolds numbers were comparable to the heat transfer coefficients of bigger microchannels with higher Reynolds numbers.

Manca et al. [15] conducted a numerical study on the forced convection heat transfer of a nanofluid in a ribbed channel with constant heat flux and concluded that increased Reynolds number and particle’s concentration would increase the average Nusselt number and pressure drop. Han et al. [16] investigated influence of the serration shape, serration’s angle of attack and pitch to height ratio on the heat transfer. It was shown that with similar friction ability, serration with angle of attack with 45° has better heat transfer performance than the serration of 90°. Han and Park [17] studied the heat transfer in rectangular channels with different dimensions and concluded that channels with square section and serrations with angle of attack of 30–45 degrees yield the best heat transfer performance. Jones and Smith [18] studied impact of height and distance between serrations on heat transfer coefficient. The findings showed that distance between serrations is the most important geometrical parameter which should be considered as specific length. Liou and Hwang [19] investigated the heat transfer performance of fully-developed flow in channels with square, semicircle and triangle serration shapes. It was revealed that those serrations had appropriate heat transfer performance. Moreover, areas with lower heat transfer at the back of the square serration were observed. Huh et al. [20] studied effect of serration height on heat transfer in rectangular channels. Higher blockage ratio (channel height to hydraulic diameter) was revealed to improve the heat transfer. Hwang et al. [21] analyzed heat transfer and behavior of flow in rectangular channels with alternative serration arrangement parallel to the flow on one side of main wall. They concluded that acceleration and turbulence intensity of flow have effect on heat transfer coefficient. In an experimental investigation, Wang and Sunden [22] calculated the local heat transfer coefficient in a ribbed square channel and found out that the amount of local heat transfer was strongly dependent on the serration shape. Rau et al. [23] reached to an optimized pitch to height of serration ratio and reported that increase in heat transfer coefficient is not only dependent on the ratio of serration dimensions, but also on the shape of serration. Zhang et al. [24] measured the amount of heat transfer in rectangular channels with normal and grooved serrations. Grooving in the square serrations increased the heat transfer while decreased the pressure drop along the path. Han et al. [25] studied the heat transfer in a square channel for seven different angles of V-shape serration and concluded that the V-shape serration with an angle of 60° resulted in about 4.5 times more heat transfer than the smooth channel. It also exhibited a better performance in comparison to the continuous serration. Shen et al. [26] considered the flow and heat transfer in a microchannel with rough wall with water as the working fluid and reported that natural roughness of surface has significant effect on the heat transfer of microchannel in laminar flow.

Nowadays, due to the comprehensive advance in nanotechnology, some good alternatives to the above-mentioned methods are available. For instance, cooling microchannels with nanofluids can be used instead of the fans. There is evidence in literature that supports the usefulness of nanofluids for cooling [27–29]; however, nanofluids should be investigated further [30–32]. In addition, investigating the thermal performance of nanofluids in tools with micro and nano dimensions should be considered in research studies [33]. The present research attempted to numerically study effect of the semi-attached T-shape serrations on the flow and turbulent heat transfer of the water-Silver nanofluid with different volume fractions of nanoparticle in a three-dimensional trapezoidal microchannel. For validation purposes, the obtained results were also compared with the literature. The results from the present study may lead to design of the new microchannels in solar collectors and electronic devices [33].

2. Statement of problem

In this study, a three-dimensional trapezoidal microchannel with T-shape serrations was analyzed. Due to the T-shape form of the serrations, part of the cooling fluid can pass through the serration. Characteristics of the heat transfer and dynamics of fluid have been studied for four different arrangements of serration with the channel width to serration whole width ratio (R/W) in the following conditions: R/W=0.0, 0.125, 0.250, and 0.500. Range of the Reynolds in this study was $1 \times 10^3 < Re < 6 \times 10^4$ and the Water-Silver nanofluid was used with volume fractions of $\varphi=0.00$, 0.02 and 0.04 as the working fluid. Fig. 1 shows the investigated microchannel in this study.

The main assumptions were that:

### Table 1

<table>
<thead>
<tr>
<th>$\varphi$</th>
<th>$\rho$ (kg/m$^3$)</th>
<th>$C_p$ (J/kgK)</th>
<th>$k$ (W/mK)</th>
<th>$\mu$ (Pa s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0%</td>
<td>997.1</td>
<td>4179</td>
<td>0.613</td>
<td>8.91e–04</td>
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<td>3.7998e+03</td>
<td>0.8849</td>
<td>9.6700e–04</td>
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<td>3.4813e+03</td>
<td>1.1623</td>
<td>0.0011</td>
</tr>
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<td>1.2822e+03</td>
<td>3.2101e+03</td>
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<tr>
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<td>1.3772e+03</td>
<td>2.9762e+03</td>
<td>1.7345</td>
<td>0.0013</td>
</tr>
</tbody>
</table>

### Table 2

Thermophysical characteristics of nanofluid in different volume fractions and solid nanoparticle [35].
– The fluid is Newton type and non-compressible.
– Characteristics of the fluid and solid are assumed unchanged with the change in temperature.
– The flow is turbulent and developed hydrodynamic.
– There is no slipping on the walls.
– The fluid enters into the microchannel in inlet areas with constant velocity.
– Shapes of the nanoparticles are alike and spherical.
– Constant thermal flux is applied to upper and lower walls, and the side walls (trapezoidal routes) are isolated.
– The simulated space is three-dimensional.

3. Fundamental equations ruling on three-dimensional turbulent flow

Fundamental equations ruling on the heat transfer and flow of fluid included the continuity, momentum and the energy conservation equations, which are stated below [15]:

Continuous equation:

$$\frac{\partial}{\partial x_i}(\rho u_i) = 0$$  \hspace{1cm} (1)

Momentum vector equation:

$$\frac{\partial}{\partial x_j}(\rho u_i u_j) = - \frac{\partial P}{\partial x_i} + \frac{\partial}{\partial x_j} \left[ \left( \frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} - \frac{2}{3} \delta_{ij} \frac{\partial u_k}{\partial x_k} \right) \right] + \frac{\partial}{\partial x_i} (-\rho u_i u_j)$$  \hspace{1cm} (2)

Energy equation:

$$\frac{\partial}{\partial x_i} \left( \rho (u_i E_T + P) \right) = \frac{\partial}{\partial x_j} \left[ \left( \lambda + \frac{C p \mu}{Pr} \right) \frac{\partial T}{\partial x_j} + u_j (\tau_{ij})_{ij} \right] = 0$$  \hspace{1cm} (3)

where $E$ is total energy.
\( \tau_{\text{eff}} \) is the deviatoric tensor stress, defined as follows:

\[
\begin{bmatrix}
\partial_{x_{ij}} \left( \rho u_{i} \right) - \frac{2}{3} \partial_{x_{ii}} \rho u_{i} \\
\partial_{x_{ij}} \left( \rho \omega u_{i} \right) - \frac{2}{3} \partial_{x_{ii}} \left( \rho \omega \right)
\end{bmatrix}
\]

The transfer equation for k-ω shear stress transfer model is as shown below [34]:

\[
\begin{align*}
\frac{\partial}{\partial X_{i}} \left( \rho k \right) &= \frac{\partial}{\partial X_{i}} \left( \Gamma_{k} \frac{\partial k}{\partial X_{i}} \right) + \bar{G}_{k} - Y_{k} + S_{k} \\
\frac{\partial}{\partial X_{i}} \left( \rho \omega k \right) &= \frac{\partial}{\partial X_{i}} \left( \Gamma_{\omega} \frac{\partial \omega k}{\partial X_{i}} \right) + \bar{G}_{\omega} - Y_{\omega} + D_{\omega} + S_{\omega}
\end{align*}
\]

In the above equation, \( \bar{G}_{k} \) is the turbulence kinetic energy due to the average velocity gradient and \( G_{\omega} \) represents its productivity from \( \omega \):

\[
\bar{G}_{k} = \text{min} \left( \bar{G}_{k}, 10 \beta_{1} \kappa \omega \right), \quad G_{\omega} = -\rho \omega \mu' \frac{\partial u_{i}}{\partial X_{i}} \quad G_{\omega} = \frac{\alpha_{\omega}}{\omega_{\infty}} \left( \frac{a_{\omega} \left( a_{\omega} + R_{e}/R_{e}^{*} \right)}{1 + R_{e}/R_{e}^{*}} \right)
\]

In the above sentence, \( \nu_{t} \) is the turbulent kinematic viscosity, \( \beta^{*} \) is the constant of model and the amount of \( \alpha \) is calculated as follows:

\[
\begin{align*}
\alpha &= \alpha_{\omega} \left( a_{\omega} \right. + R_{e}/R_{e}^{*} \right) \\
(1 + R_{e}/R_{e}^{*})
\end{align*}
\]

\( R_{e}=2.95 \) and \( \alpha_{\infty} \) is stated as [15]:

\[
\begin{align*}
\alpha_{\infty} &= \beta_{1} \frac{\rho_{\infty}}{\rho_{\omega}} \frac{1}{\alpha_{\omega}} \\
\alpha_{\infty} &= \beta_{2} \frac{\rho_{\infty}}{\rho_{\omega}} \frac{1}{\alpha_{\omega}} \\
\kappa &= 0.41, \quad \beta_{1} = 0.072, \quad \alpha = \alpha_{\omega} = 1.0
\end{align*}
\]

In the above equation, the amount of \( \kappa \) is 0.41 and \( \beta_{1} = 0.072 \). In the high Reynolds, \( \alpha=\infty=1.0 \). In Eqs. (10) and (11), \( \Gamma_{\omega} \) and \( \Gamma_{k} \) are effective diffusions of \( \omega \) and \( k \), which are defined as below:

\[
\begin{align*}
\Gamma_{k} &= \mu + \frac{\rho_{k}}{\tau_{k}} \\
\Gamma_{\omega} &= \mu + \frac{\rho_{\omega}}{\tau_{\omega}}
\end{align*}
\]

\( \alpha_{\omega} \) and \( \alpha_{\omega} \) are stating the turbulent Prandtl in \( \omega \) and \( k \) models and are defined as follows:

\[
\begin{align*}
\alpha_{\omega} &= 1 + 0.41 \left( 1 - \frac{R_{e}}{R_{e}^{*}} \right) \\
\alpha_{\omega} &= 1 + 0.41 \left( 1 - \frac{R_{e}}{R_{e}^{*}} \right)
\end{align*}
\]
\[ \sigma_1 = \frac{1}{F/\sigma_{1,1} + (1 - F)/\sigma_{1,2}} \]

\[ \sigma_2 = \frac{1}{F/\sigma_{1,1} + (1 - F)/\sigma_{1,2}} \]

\[ \mu_t = \frac{\mu^* \rho_k}{\omega} \]

\[ a^* = \frac{a^* (a_{\sigma_0} + R_\lambda / R_\kappa)}{(1 + R_\lambda / R_\kappa)} \]

\[ a^* = a^*_{\omega} = 1.0 \]

\[ F \lambda = k \rho / \omega \mu \quad R_\kappa = 6 \quad a_{\sigma_0} = 6^{1/3} \quad \beta_t = 0.072 \]

\[ F_1 \text{ (blending function equation) equals to:} \]

\[ F_1 = \tan(\Phi^1) \]

\[ \Phi^1 = \min \left[ \max \left( \frac{\sqrt{\frac{\sqrt{\frac{200}{\rho_k}}} {\rho_\omega}} \frac{4k}{\alpha_k \omega^2} \right) \right] \]

\[ D_{\omega}^+ = \max \left[ 2r \left( \frac{1}{\alpha_{\omega}^2} \right) \frac{d \omega}{d \omega} \right] \]

where \( y \) is the distance to the next surface and \( D_{\omega}^+ \) is the positive part of diffusion term in the lateral section. \( Y_k \) and \( Y_{\omega} \) show the losses of \( k \) and \( \omega \) according to the turbulence as follows:

\[ Y_k = \rho \beta_t k \omega \]

\[ Y_{\omega} = \rho \beta_t \omega^2 \]

\[ \beta_t = F \beta_{\omega} + (1 - F) \beta_{\omega} \]

Fig. 4. Poiseuille number according to \( R/W \) in different Reynolds numbers and volume fractions of nanofluids.
\[ D_\rho = 2(1 - F_k)\rho\sigma_{w} \frac{1}{\omega_x} \frac{\partial k}{\partial x_i} \frac{\partial \omega}{\partial x_i} \]  

(20)

Constants of shear stress transport turbulence model are shown in Table 1.

4. Boundary conditions

Boundary conditions are:

In inlet:

\[ V = V_{in} \]  

(21)

\[ T = T_{in} = 293\text{K} \]  

(22)

In the output:

\[ P = P_{out} \]  

(23)

In the microchannel walls, the constant heat flux was applied and the amount of applied flux to these walls was equal to \( q'' = 20,000 \text{ W/m}^2 \).

\[ U = V = W = 0 \]  

(24)

\[ q'' = -k_f \frac{\partial T_f}{\partial n} \]  

(25)

5. Calculation of the nanofluid characteristics

The used nanofluids in this simulation were suspension from silver nanoparticles in the water (as the base fluid) with volume fractions of \( \phi = 0.00, 0.02 \) and 0.04. Thermo-physical properties of the base fluid, nanofluid in different volume fractions and silver powder are presented in Table 2.

In order to calculate the nanofluid density, the following formula was used [36]:

\[ \]
The effective thermal diffusivity of nanofluid was calculated as below [37]:

$$\alpha_{nf} = \frac{k_{nf}}{\rho C_{nf} \beta}$$  \hspace{2cm} (26)

The specific heat capacity of nanofluid was calculated as [38]:

$$\rho C_{nf} \beta = (1 - \phi) \rho C_f \beta + \phi \rho C_p \beta$$  \hspace{2cm} (27)

In a turbulent flow, the below relation was used to determine the effective dynamic viscosity [39]:

$$\mu_{nf} = \mu_f (123\phi^2 + 7.3\phi + 1)$$  \hspace{2cm} (29)

Relation of Patel et al. [40] was used to calculate the coefficient of thermal conductivity for suspensions with spherical particles,

$$k_{nf} = k_f \left[ 1 + \frac{k_f A_p}{k_p A_f} \right]$$  \hspace{2cm} (30)

where the experimental constant "c" is 36,000 and:

$$Pe = \frac{n d_p^2}{\nu}$$  \hspace{2cm} (31)

In the above relations, $d_f$ is the diameter of water molecule and $d_p$ is the diameter of silver nanoparticle molecule. $U_b$ is the velocity of Brownian motion of nanoparticles and is determined as below:

$$U_b = \frac{2k_B T}{m d_p^2}$$  \hspace{2cm} (32)

In the above relation, $k_B = 1.3807 \times 10^{-23} \text{ J/K}$ (Boltzmann constant) [41].

Fig. 6. Performance evaluation criterion according to the R/W in different Reynolds numbers and volume fractions of nanoparticles.
5.1. Calculation of parameters in the three-dimensional turbulent flow

The pumping power (PP) is one of the factors determining the performance of microchannel and is the power needed to pump the fluid through the channel. The relationship between this parameter and the pressure drop $\Delta P$ along the microchannel was obtained from the following equation [42]:

$$P_{pp} = u_{in} A_c \Delta P$$  

(33)

where $u_{in}$ is the input velocity and $A_c$ is the cross-sectional area of the microchannel. The hydraulic diameter of the microchannel was also one of the physical properties of the microchannel, which is defined as follows [43]:

$$D_h = \frac{4A_c}{p}$$  

(34)

The Reynolds number in the present problem is calculated from the following equation [15]:

$$Re = \frac{u_{in} D_h}{v}$$  

(36)

To calculate the average coefficient of friction, the following equation was used [44]:

$$f = \frac{2 \Delta P D_h}{L \rho u_{in}^2}$$  

(37)

The average Nusselt number was also obtained from the following equation [45]:

$$N_u = \frac{4 \times 0.5 \times (W_f + W_b) \times H}{W_f + W_b + 2 \times H / \sin \theta}$$  

(35)

where $p$ is the wet environment of the microchannel. The Reynolds number in the present problem is calculated from the following equation [15]:

$$Re = \frac{u_{in} D_h}{v}$$  

(36)

To calculate the average coefficient of friction, the following equation was used [44]:

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$$N_u = \frac{4 \times 0.5 \times (W_f + W_b) \times H}{W_f + W_b + 2 \times H / \sin \theta}$$  

(35)
where $T_w$ is the temperature of microchannel wall and $T_m$ is the average Bulk temperature.

To evaluate the overall thermal and fluid performance of the ribbed microchannel, the "PEC" (Performance Evaluation Criterion) was defined as follows:

$$PEC = \left( \frac{Nu_{ave}}{Nu_{ave,s}} \right) \left( \frac{f_{ave}}{f} \right)$$

(39)

The Poiseuille number was determined from the following equation:

$$C_f = \frac{R_e f}{1.84 \log_{10} Re - 1.64}$$

(40)

The Dittus-Boelter and the Petukhov relations were used to calculate the Nusselt number in the smooth channel, considering the range of validity as follows:

The Dittus-Boelter relationship is as follows:

$$Nu = 0.023 \left( \frac{Re Pr}{1.87 + 12.7 \left( \frac{Re Pr}{200} \right)^{0.8}} \right)^{1/3}$$

(41)

The Petukhov relation:

$$Nu_{ave} = 0.023 \left( \frac{Re Pr}{1.87 + 12.7 \left( \frac{Re Pr}{200} \right)^{0.8}} \right)^{1/3}$$

(42)

The Petukhov and McAdams relationships were used to calculate the coefficient of friction in the smooth channel and validation with the empirical relations, which is defined as follows:

$$f = (1.84 \log_{10} Re - 1.64)^2$$

(43)
The McAdams relationship is [15]:

\[ fs = 0.184 \times 10^{-0.2} \]

\[ 2.0 \times 10^4 < \text{Re} < 3.0 \times 10^5 \]  

(44)

6. Characteristics of numerical method

The equations in this research included the Navier-Stokes equations, as well as k and \( \omega \) turbulence equations in three-dimensional space. The control volume method was used to solve these equations [48,49]. In the finite volume method, conservation equations are integrated on every mesh and these equations are valid for the total calculating area. This characteristic is valid for any numbers of meshes, even when the number is low [50–52]. In order to discretize all the terms of equations, the Second Order Upwind method and for coupling the pressure and velocity, the SIMPLE algorithm were used [53,54]. When the residual of all parameters have a value lower than \( 10^{-6} \), the problem solving is convergent.

7. Results and discussion

In this section, the effects of different R/Ws on heat transfer and turbulent flow parameters of the water-silver nanofluid on a three-dimensional trapezoidal microchannel are studied. The results include the average Nusselt number, average convection heat transfer coefficient, performance evaluation criterion (PEC), pumping power, coefficient of friction and the Poiseuille number. These results were studied for four conditions of R/W=0, 0.125, 0.250 and 0.500. Each parameter was measured in every R/W conditions with respect to the similar parameters in the non-ribbed microchannel with the same geometrical and boundary (fluid-thermal) conditions. The study results were calculated for 10,000 < Re < 16,000 and volume fractions of 0%, 2% and 4% of silver nanoparticle.

7.1. Validation

To validate the findings, results are compared with results of Manca et al. [15] and akbari et. al. [55]. Fig. (2-a) and (b) show the amounts of average Nusselt number and friction coefficient according to
Reynolds number for pure water as the cooling fluid. As this figure shows, the results of current study are well consistent with Manca et al.’s research [15] and the results obtained from experimental relations. In Fig. (2-c), numerical error of computations is negligible in comparison with Akbari et. al. [55] research.

7.2. Heat transfer analysis

Fig. 3(a–d) shows the changes of Poiseuille in different R/Ws for different volume fractions of nanoparticle and Reynolds number. In all the figures, the Poiseuille number decreased with the increase in Reynolds number in each R/W condition due to the increase in the fluid’s velocity and viscosity. Among different R/W conditions and all the Reynolds numbers, the Poiseuille number decreased as the R/W increased, because the fluid moved through the semi-attached serration area. It was observed that among all the R/W conditions, R/W=0.5 had the least obstruction of fluid motion path because of broader pierced areas of the serration corners. In the case of R/W=0, the fluid interaction with the surface increased due to the obstruction of fluid path, which had a great effect on the increase in the Poiseuille number.

Fig. 4 shows Poiseuille number for microchannels with different R/Ws in comparison with the non-ribbed microchannel in different volume fractions of $\phi=0$, 0.02 and 0.04 and Reynolds number range of 10,000–16,000. In all the R/Ws, the Poiseuille number increased as Reynolds number or volume fraction of nanoparticle increased. The reason is increasing the shear rate in the microchannel walls. The Poiseuille’s slope of alteration in R/W=0 was the maximum with respect to other R/W conditions for all volume fractions of nanoparticle and Reynolds numbers. Eventually, the Poiseuille number increases more by higher Reynolds.

Fig. 5 shows the values of pressure drop for different volume fractions. In all Reynolds numbers and all studied R/W conditions, it was observed that the pressure drop decreased by the increase in Reynolds numbers in all R/W ratios and different volume fractions of nanoparticles. It was also found that pressure drop of the ribbed channel increases as volume fraction of solid nanoparticle goes higher.

Fig. 10. Pumping power diagram according to Reynolds numbers with different R/Ws and volume fractions of nanoparticles.
and the R/W ratio decreases. Existence of solid nanoparticles in the cooling fluid causes a dramatic pressure drop due to the motion of viscous fluid in comparison with lower viscosity fluid.

Fig. 6 shows the fluid-thermal efficiency (PEC) for the ribbed microchannel with different R/Ws of the serration in different volume fractions. From the results, it can be seen that in lower Reynolds numbers, the serration with R/W=0 has the most fluid-thermal efficiency, which decreases as the Reynolds number increase. The reason is that more turbulence and powerful vortex are created by the serration with R/W=0 in lower Reynolds. High turbulence has less

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**Fig. 11.** The contour of average Nusselt number for R/W=0, 0.125, 0.25 and 0.5 in Re= 10,000 and φ=0.04.
effect in the high Reynolds numbers by the serrations with $R/W=0$ in comparison to low Reynolds, because high Reynolds flow damps most of the vortexes due to more momentum of the fluid that leads to decrease in the fluid-thermal efficiency of the serration ratio in high Reynolds numbers. On the other hand, the serrations with higher $R/W$ in higher Reynolds numbers and better accessibility of fluid to all points of the serration back have better fluid-thermal efficiency than lower Reynolds. Nevertheless, for the serrations with $R/W > 0$, the pressure drop is lower as the $R/W$ ratio increases in all the Reynolds numbers, which has a dramatic effect on the increased fluid-thermal efficiency.

Fig. 12. The contour of average Nusselt number for $R/W=0, 0.125, 0.25$ and 0.5 in $Re=16,000$ and $\varphi=0.04$. 
efficiency. According to Fig. 6, the fluid-thermal efficiency with PEC=2.37 can be obtained in high Reynolds numbers through the serration with R/W=0.5 and the volume fraction of Φ=0.04. This means that serration with higher R/W and higher volume fraction of solid nanoparticle in Re=16,000 improve the fluid-thermal efficiency of the ribbed channel with lower pressure drop, pumping power and coefficient of friction in comparison to the microchannel with R/W=0. This is one of the main advantages of using the semi-attached serrations in this research.

Fig. 7 shows the coefficient of friction for a ribbed microchannel with different R/Ws of serration in different nanoparticle volume fractions of Φ=0, 0.02 and 0.04 and the Reynolds numbers range of 10,000–16,000. It can be seen that in all the R/W conditions and all the studied Reynolds numbers, the coefficient of friction increases with respect to the increase of the nanoparticle volume fraction and decrease of the R/W. It can be attributed to the increased viscosity as the volume fraction of solid nanoparticle increases in the cooling fluid, accompanied by the increased shear stress in the microchannel walls [56–69]. The serration with lower R/W causes more pressure drop through the fluid path, which has a significant effect on increasing the coefficient of friction. In all the above conditions as well as all the R/W ratios and volume fractions of the solid nanoparticle, coefficient of friction decreases when the Reynolds number increases. It means that increased Reynolds number causes increase in the fluid momentum; thus, the fluid tends to feel the serration resistance in its path. It can be seen that the effect of increased R/W in low Reynolds numbers (1000) is higher than in high Reynolds (16,000) due to better fluid convection in the leaky areas of serration in low Reynolds.

As Fig. 8 shows from all the studied R/Ws, R/W=0.5 has a better proceeding in increasing the average convection heat transfer coefficient. However, in Re=16,000, it can be observed that there is little difference between the results obtained from all the R/W cases than other Reynolds.

Besides, the average convection heat transfer coefficient increases as the volume fraction of the solid nanoparticle in fluid is increased. In all the Reynolds numbers, it can be seen that the nanoparticle with the most volume fractions of nanoparticles has the most heat transfer rate.

Fig. 9 shows the average Nusselt numbers for a ribbed microchannel with different R/W ratios in different volume fractions. It can be seen that in Re=16,000, the average Nusselt number increases significantly than others. Among other studied R/W cases, it can be observed that in R/W=0.5, the average Nusselt number is the maximum in comparison to R/W=0.25 and 0.125 in all Reynolds numbers. It can be attributed to the improvement and decrease in hot areas due to better distribution of heat transfer in the backside areas of the serration and connection point of the serration with microchannel surface. In higher Reynolds numbers, it is expected that effects of increased fluid momentum repulses the vortex created by the obstacles and roughness in cooling fluid which this leads to increased heat transfer.

Fig. 10 shows the pumping power in the ribbed microchannel for different Reynolds numbers and volume fractions of the solid nanoparticles. Pumping power decreases as the R/W decreases. The slope of R/W=0 is the highest, because the fluid path is blocked and the pressure drop is maximum in comparison to other conditions. As the volume fraction of the solid nanoparticles increases, the pumping power also increases. It is due to the increased fluid viscosity and the viscous fluid needs more power to move in the microchannel. Increasing the Reynolds number causes an increase in the pumping power due to the velocity increase in higher Reynolds. The increased velocity leads to increased volumetric flow rate, which is in turn a reason for the increased in pumping power.

The serration has a significant effect on the heat transfer performance in the channels and microchannels. Using the fully- or semi-attached serrations leads to increased coefficient of friction as more obstruction of the channel happens and also the hot areas increase with lower heat transfer areas (LITAS). Figs. 11 and 12 show the hot areas created at the corners, backside of the serration and the connection point of the serration to surface for Reynolds numbers of 10,000 and 16,000 and volume fraction of 0.04. It can be concluded that by increase in the R/W ratio, areas with lower heat transfer decrease in the backside of the serration.

8. Conclusion

Although the study of heat transfer tools in different geometries has been of interest to researchers [70–76], but in the current research, the forced convection heat transfer of the water-silver Oxide nano-fluid was numerically studied in a three-dimensional trapezoidal microchannel under constant heat flux. The non-slip boundary condition at the walls of the microchannel is considered as the hydrodynamic boundary condition. The effects of the volume percentage of nanoparticles, the Reynolds number and the aspect ratio of the serration were investigated on the flow and heat transfer rates. As the Reynolds number increases, the average Nusselt number and average convection heat transfer coefficient in the ribbed microchannels with higher R/W ratio increased in comparison to the fully-attached serration condition.

The effect of using the serration with R/W=0 in lower Reynolds numbers was more evident. The serration with R/W=0 showed higher friction coefficient, pressure drop and pumping power in comparison to other serration ratios. Resultant areas with lower heat transfer in the backside of the serration and connection point of the serration to surface are considered as the disadvantages of using this kind of serration. Results of this study showed that the serration generates more powerful vortexes, which results in better mixture of fluid layers. However, the most important advantage of using the serration with 0 < R/W < 0.5 is that in these ratios, hot areas with lower heat transfer in the backside of serrations are corrected and somewhat disappear. In addition, these serrations decrease the pumping power, friction coefficient and pressure drop significantly as the R/W ratio increases. This will lead to increased fluid-heat transfer performance. Moreover, not only the serration with R/W=0.5 exhibited less pumping power, coefficient of friction and pressure drop, but also better fluid-thermal efficiency compared to R/W=0. Consequently, this serration ratio can be considered as the optimal serration ratio based on the findings of this study.

References


