Natural convection of Al$_2$O$_3$–water nanofluid in an inclined enclosure with the effects of slip velocity mechanisms: Brownian motion and thermophoresis phenomenon

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Effects of inclination angle on natural convective heat transfer and fluid flow in an enclosure filled with Al$_2$O$_3$–water nanofluid are studied numerically. The left and right walls of enclosure are kept in hot and cold constant temperature while the other two walls are assumed to be adiabatic. Considering Brownian motion and thermophoresis effect (two important slip velocity mechanisms) the two-phase mixture model has been employed to investigate the flow and thermal behaviors of the nanofluid. The study was performed for various inclination angles of enclosure ranging from $\gamma = 0^\circ$ to $\gamma = 60^\circ$, volume fraction from 0% to 3%, and Rayleigh numbers varying from $10^2$ to $10^7$. The governing equations are solved numerically using the finite volume approach. The results are presented in the form of streamlines, isotherms, distribution of volume fraction of nanoparticles and Nusselt numbers. They demonstrated that the slip velocity mechanisms have caused the decreasing Nusselt number with increasing volume fraction of nanoparticles. They also indicate important differences between single-phase and two-phase models. In addition, the highest value for Nusselt number is reached at $\gamma = 30^\circ$. Increase in nanoparticle diameter leads to decrease and increase in Nusselt number and non-uniformity, respectively. The investigation of inclination angle effects on natural convection of nanofluid in an enclosure has been carried out for the first time by two-phase mixture model including slip velocity mechanisms (Brownian motion and thermophoresis phenomenon) effects.

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1. Introduction

Fluid flow and heat transfer in enclosures is one of the most interesting topics for engineers. These enclosures can be used in many industrial applications such as cooling solar collectors, electronic systems, and nuclear equipments. The heat transfer environment for convection is usually fluids (water, ethylene-glycol or oil) with lower thermal conductivity relative to solids; thermal conductivity of carbon nano-tubes is 5700 times as much as that of water. Nanofluid (a homogeneous mixture of liquid and solid particles less than 100 nm) conductivity is higher than the base fluid because of high conductivity of particles [1–5].

Experimental [6–8] and numerical [9–14] researches on mixed and forced convection indicated that heat transfer improved with nanofluid. Moreover, many studies reported the same results for natural convection in enclosures [15–22], but they were not validated with experimental results. On the other hand, some experimental [23–29] and numerical [30–32] researches reported that heat transfer deteriorates by using nanofluid. Researchers indicated that Brinkman’s correlation [33] for viscosity cannot show true results for natural convection. Abouali and Ahmadi [34] and Esmaeilpour and Abdollahzadeh [35] indicate the deterioration of heat transfer by using Corcione’s correlation [36].

Two-phase modeling (contrary to single modeling) introduces nanofluid as non-uniform due to effects such as slip velocity between nanoparticles and base fluid. Buongiorno [37] considered...
seven slip mechanisms; interia, Brownian diffusion, thermophoresis, diffusion, thermofluid, and gravity on forced convection along a wall ($T_w > T_s$). He introduced the Brownian diffusion and thermophoresis as the most important mechanisms. These lead to nanoparticles mass and heat fluxes in the volume fraction and energy equations, respectively. The mass flux reduces viscosity near the wall (laminar sublayer) due to thermophoresis, and because of enhancement of convection. Furthermore, he found that energy transfer by nanoparticle dispersion is negligible. Buongiorno's model was applied on natural convection in a 2D-square enclosure by Dastmalchi et al. [38] (in porous media) and Sheikhzadeh et al. [39,40]. They assumed a constant value for thermophoresis parameter in all ranges of volume fractions (1–4%) and indicated the single-phase modeling predicts higher Nusselt number than two-phase modeling. Aminfar and Haghigh [41] considered thermophoresis parameter function of volume fraction (1% and 3%) on natural convection in a square cavity. They concluded that the single-phase modeling does not seem reliable for modeling this class of natural convection. Sheikholeslami et al. [42–45] investigated magnetic effect on natural and forced convection of nanofluid in 2D-enclosures by two-phase modeling considering Brownian motion and thermophoresis. They indicated that Nusselt number decreases with increasing Brownian, thermophoresis and magnetic parameters. Ho et al. [46] investigated experimental and two-phase numerical natural convection of Al$_2$O$_3$—water nanofluid in a square enclosure for $Ra = 5.78 \times 10^5$–$3.11 \times 10^6$. Numerical method consists of Brownian motion, thermophoresis, and sedimentation of nanoparticles effects. They indicated that the Brownian motion and thermophoresis, in contrast to sedimentation, have no effect on heat transfer for volume fraction of 1%; but these effects increase with increasing volume fraction especially on higher $Ra$. Additionally, thermophoresis and Brownian motion decrease Nusselt number in comparison to sedimentation effect; where they are important.

Abu-nada and Oztop [47] and Öğüt [17] studied the effects of inclination angles from 0° to 90° on natural convection of nanofluid in a square enclosure. Both of them found that the maximum and minimum heat transfer were obtained at 30° and 90° angles. Alinia et al. [48] indicate the maximum and minimum averaged Nusselt numbers at 30° and 60° angles for natural convection of nanofluid in a square enclosure; while there are no changes for mixed and forced convection. Refs. [17,47,48] indicate that the heat transfer improves by nanofluid. Ahmed and Eslamian [49] investigated natural convection of nanofluid in an inclined enclosure using two-phase LBM; considering thermophoresis and Brownian effects. They concluded that the Nusselt number increases with increasing volume fraction, which is against the experimental results. Moreover, they indicate that Nusselt number, under the thermophoresis effect, is higher in comparison to the case without it.

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**Nomenclature**

\[ C_p \] heat capacity, J kg$^{-1}$ K$^{-1}$

\[ D_B \] Brownian diffusion coefficient, m$^2$ s$^{-1}$

\[ d \] diameter, m

\[ E \] absolute error

\[ g \] gravitational acceleration vector, m s$^{-2}$

\[ H, L \] height and length of enclosure, m

\[ h_i \] heat transfer coefficient of nanofluid, W m$^{-2}$ K$^{-1}$

\[ h_t \] enthalpy of ith component, J kg$^{-1}$

\[ j \] drift flux, kg m$^{-2}$ s$^{-1}$

\[ k \] thermal conductivity, W m$^{-1}$ K$^{-1}$

\[ k_B \] Boltzmann constant, J K$^{-1}$

\[ Nu \] Nusselt number

\[ P \] pressure, Pa

\[ Pr \] Prandtl number

\[ q \] heat flux, W m$^{-2}$

\[ Ra \] Rayleigh number

\[ Re \] Reynolds number

\[ S_T \] thermophoretic parameter

\[ t \] time, s

\[ T \] temperature, K

\[ \textbf{V} \] drift velocity vector, m s$^{-1}$

\[ \textbf{x}, \textbf{y} \] coordinates, m

\[ (X,Y) \] non-dimensional coordinates

**Greek symbols**

\[ \alpha \] thermal diffusivity, m$^2$ s$^{-1}$

\[ \beta \] thermal expansion coefficient, K$^{-1}$

\[ \gamma \] enclosure inclination angle

\[ \theta \] non-dimensional temperature

\[ \mu \] dynamic viscosity, Pa s

\[ \nu \] kinematics viscosity, m$^2$ s$^{-1}$

\[ \rho \] density, kg m$^{-3}$

\[ \varphi \] volume fraction, kg m$^{-3}$

\[ \phi \] normalized volume fraction of nanoparticles

**Subscripts**

ave averaged

c cold

f base fluid

fr freezing point

h hot

n normal to wall component

nf nanofluid

p nanoparticles

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**Fig. 1.** The schematic of the inclined cavity.
Given the importance of Brownian and thermophoresis effects on natural convection, in the present study, the effects of inclination angles on natural convection of $\text{Al}_2\text{O}_3$–water nanofluid in a square enclosure are investigated using two-phase mixture model considering Brownian motion and thermophoresis phenomenon for the first time.

2. Problem statement

Fig. 1 shows the physical geometry of the considered square enclosure. The left and right sides are at constant hot and cold temperatures, and the other sides are assumed adiabatic. Four inclination angles are considered $\gamma = 0^\circ$, $30^\circ$, $45^\circ$, and $60^\circ$ for Rayleigh numbers from $10^5$ to $10^7$. Nanofluid natural convection is studied numerically by applying two-phase mixture modeling with Brownian motion and thermophoresis phenomenon. The simulation was performed for the four volume fraction of nanoparticles: $\varphi_{\text{ave}} = 0$, $1\%$, $2\%$, and $3\%$.

3. Mathematical formulation

3.1. Thermo-physical properties

Many studies based on experimental data have been conducted for the nanofluids properties. There are functions of both properties of base fluid and nanoparticles, and according to them, several models have been proposed. The thermophysical properties of water (as the base fluid) and alumina (as the nanoparticles) are provided in Table 1 [50].

The effective density of nanofluid as a function of volume fraction of nanoparticles and temperature is obtained by Khani fer and Vafai’s correlation [51] using the experimental data of Ho et al. [28]:

### Table 1

<table>
<thead>
<tr>
<th>Material</th>
<th>$c_p$ (J/kg K)</th>
<th>$\rho$ (kg/m$^3$)</th>
<th>$k$ (W/m K)</th>
<th>$\beta \times 10^{-4}$ (K$^{-1}$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Water</td>
<td>4179</td>
<td>997</td>
<td>0.6</td>
<td>2.3</td>
</tr>
<tr>
<td>$\text{Al}_2\text{O}_3$</td>
<td>765</td>
<td>3970</td>
<td>40</td>
<td>0.85</td>
</tr>
</tbody>
</table>

### Table 2

<table>
<thead>
<tr>
<th>Mesh size</th>
<th>$\bar{N}_u$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$100 \times 100$</td>
<td>4.2086</td>
</tr>
<tr>
<td>$150 \times 150$</td>
<td>3.9711</td>
</tr>
<tr>
<td>$170 \times 170$</td>
<td>3.7334</td>
</tr>
<tr>
<td>$200 \times 200$</td>
<td>3.5539</td>
</tr>
<tr>
<td>$240 \times 240$</td>
<td>3.5506</td>
</tr>
</tbody>
</table>

Fig. 2. Comparison of present work averaged Nusselt number with single phase modeling and experimental data (Ho et al.) for different $Ra$. 


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The specific heat and thermal expansion coefficients are calculated as following [52]:

\[
\begin{align*}
C_{p,nf} &= (1 - \varphi)(C_{p,f}) + \varphi(C_{p,p}) \tag{2} \\
\beta_{nf} &= (1 - \varphi)(\beta_f) + \varphi(\beta_p) \tag{3}
\end{align*}
\]

where subscripts f, p, and nf show the fluid, solid nanoparticles, and nanofluid, respectively. Furthermore, the volume fraction of nanoparticles is shown by \( \varphi \).

Thermal conductivity is a key parameter for heat transfer with nanofluid environment. Several studies emphasized that Brownian motion plays a major role in nanofluids thermal conductivity [53–56]. Therefore, Brownian motion is an important factor in modifying thermal conductivity of nanofluids. The correlations introduced by Corcione [36] are used in the present study.

\[
\frac{k_{nf}}{k_f} = 1 + 4.4Re^{0.4}Pr^{0.03} \left( \frac{T}{T_f} \right)^{10} \left( \frac{k_p}{k_f} \right) \varphi^{0.66} \tag{4}
\]

and

\[
Re = \frac{\rho_f V_s d_p}{\mu_f} = \frac{2\rho_f k_b T}{\pi \mu_f^2 d_p} \quad Pr = \frac{\mu_f}{\rho_f C_p} \tag{5}
\]

where \( V_s \) and \( k_b = 1.38066 \times 10^{-23} \) J K\(^{-1}\) are Brownian velocity and Boltzmann’s constant.

\[
\frac{\mu_{nf}}{\mu_f} = \frac{1}{1 - 34.87 \left( \frac{d_f}{d_p} \right)^{-0.3}} \varphi^{1.03} \tag{6}
\]

where [39]

\[
\mu_f = 562.77 (\ln(T + 62.756))^{-8.9137} \tag{7}
\]

Thermal diffusivity is defined as [57]:

\[
\alpha_{nf} = \frac{k_{nf}}{(\rho C_p)_{nf}} \tag{8}
\]

3.2. Two-phase modeling

Eulerian–Lagrangian and Eulerian–Eulerian are two main approaches for two-phase flow modeling. Because of high numbers of nanoparticles in the nanofluid mixture, it is better to choose Eulerian–Eulerian method. There are three main Eulerian–Eulerian

Fig. 3. Comparison of present work and single phase modeling averaged coefficient of convection absolute error with experimental data (Ho et al.) for different \( \Delta T \).
models: The VOF (volume of fluid), mixture, and Eulerian. The VOF model uses for free-surface flows; the mixture and Eulerian models are appropriate for flows in which the phases are mixed. The mixture model uses for dispersed phases with distribution in domain, while the Eulerian model is suitable for dispersed phases with concentrated in part of domain [58–60]. Therefore, the mixture model is chosen in this work.

The steady state governing equations of mixture model are as follows:

Continuity equation:

$$\nabla \cdot (\rho_{nf} \mathbf{v}_{nf}) = 0$$  \hfill (9)

Momentum equation:

$$\nabla \cdot \left( \rho_{nf} \mathbf{v}_{nf} \mathbf{v}_{nf} \right) = -\nabla p_{nf} + \rho_{nf} \mathbf{g} + \nabla \cdot \left( \mu_{nf} \left( \nabla \mathbf{v}_{nf} + \left( \nabla \mathbf{v}_{nf} \right)^T \right) \right)$$

$$- \nabla \cdot \left( \frac{\varphi}{1 - \varphi} \rho_{nf} \rho_l \mathbf{v}_{dl} \mathbf{v}_{dr} \right)$$  \hfill (10)

Energy equation:

$$\nabla \cdot \left( \rho_{nf} h_{nf} \mathbf{v}_{nf} \right) = -\nabla q_{nf} + \frac{D p_{nf}}{D T} - \nabla \cdot \left( \varphi \rho_{nf} \mathbf{v}_{dr} \left( h_p - h_l \right) \right)$$  \hfill (11)

where $q_{nf}$, heat flux is:

$$q_{nf} = -k_{nf} \nabla T$$  \hfill (12)

Volume fraction equation:
\[ \mathbf{v} \left( \phi \mathbf{v}_{nf} \right) = - \frac{1}{\rho_p} \mathbf{j}_p \]

\( \mathbf{j}_p \) is drift flux of nanoparticles equal to:

\[ \mathbf{j}_p = \phi \rho_p \mathbf{v}_{p,nf} \tag{14} \]

Moreover, the mixture velocity and drift velocity respectively defined as:

\[ \rho_{nf} \mathbf{v}_{nf} = (1 - \phi) \rho_{lf} \mathbf{v}_{lf} + \phi \rho_p \mathbf{v}_p \tag{15} \]

\[ \mathbf{v}_{dr} = \mathbf{v}_{p,nf} = \left( \mathbf{v}_p - \mathbf{v}_{nf} \right) \tag{16} \]

### 3.3. Single-phase modeling

Single-phase modeling is achieved from simplified two-phase modeling without considering slip velocity mechanisms. Thus,
there is no effect of slip velocity in momentum and energy equations. Furthermore, the volume fraction equation is not solved.

3.4. Dimensionless numbers and parameters

Local and averaged convective heat transfer coefficient and dimensionless numbers including Rayleigh number and Nusselt number are defined as follows:

\[ Ra = \frac{g \beta_\alpha \Delta T L^3}{\alpha_{\text{nf}} v_{\text{nf}}} \]  

\[ Nu_y = \frac{h_y}{k_{\text{nf}}}, \quad Nu_{\text{ave}} = \frac{h_{\text{ave}} L}{k_{\text{nf}}} \]  

\[ h_y = \frac{-k_{\text{nf}} \partial T}{\Delta T}, \quad h_{\text{ave}} = \frac{1}{L} \int_0^L h_y \, dy \]  

Dimensionless parameters are defined as:

\[ X = \frac{x}{L}, \quad Y = \frac{y}{L}, \quad U = \frac{u}{\alpha_{\text{nf}} / L}, \quad V = \frac{v}{\alpha_{\text{nf}} / L}, \quad \theta = \frac{T - T_c}{\Delta T} \]  

3.5. Drift flux

In the present study, Brownian and thermophoretic diffusivities are the only slip velocity mechanisms responsible for nanoparticles migration in nanofluids. Brownian motion is defined as random motion due to continuous collisions between nanoparticles and base fluid molecules [37]. The Brownian diffusion coefficient, \( D_B \), describes this motion which is given by the Einstein–Stokes’s equation [37]:

\[ D_B = \frac{k_B T}{6 \pi \eta_d \rho} \]  

The nanoparticles flux, due to Brownian diffusion, can be given as [37]:

\[ j_{p,B} = -\rho_p D_B \nabla \theta \]  

The force applied in opposite direction of temperature gradient to suspended particle in liquid is called thermophoretic force. This force leads to migration of particles from higher temperature region to lower temperature region. This phenomenon is called thermophoresis [37]. Moreover, thermophoretic velocity (nanoparticle migration velocity) is defined as [37]:

\[ V_T = -S_T \frac{\rho p H}{\rho} \frac{\nabla T}{T} \]  

3.6. Boundary conditions

Boundary conditions for present two-phase simulations are as follows:

Velocity:

at \( x = 0, L \), \( 0 \leq y \leq H \): \( u = v = 0 \)  

at \( y = 0, H \), \( 0 \leq x \leq L \): \( u = v = 0 \)  

Temperature:

at \( x = 0 \), \( 0 \leq y \leq H \): \( T = T_h \)  

at \( x = L \), \( 0 \leq y \leq H \): \( T = T_c \)  

at \( y = 0, H \), \( 0 \leq x \leq L \): \( \frac{\partial T}{\partial y} = 0 \)  

Volume fraction:

There is no flux at all the walls. Using Eq. (25), this boundary condition reduces to:
at $x = 0, L, \ 0 \leq y \leq H : \frac{\partial \varphi}{\partial x} = \frac{D_T}{D_h} \frac{\partial T}{\partial x}$

at $y = 0, H, \ 0 \leq x \leq L : \frac{\partial \varphi}{\partial y} = \frac{D_T}{D_h} \frac{\partial T}{\partial y}$ \hspace{1cm} (28)

3.7. Numerical procedure

The governing equations are solved using the finite volume approach. All equations are discretized using a first-order upwind scheme. SIMPLE algorithm is used for coupling the pressure and the
Fig. 9. Distribution of volume fraction of nanoparticles for φ_{\text{ave}} = 2\%, (a) γ = 0°, 30° (flood), (b) γ = 45°, 60° (flood), and (c) γ = 0°, 60° (lines).
Fig. 9. (continued).
velocity components. The residual of equations is considered as a scale for the convergence of the solution. This value is $10^{-6}$ for all equations.

4. Grid study and validation

Aiming the grid independence study, simulations were performed for $\gamma = 0^\circ$, $Ra = 1 \times 10^5$, and $\theta_{ave} = 2\%$. The value of $Nu_{ave}$ is reported in Table 2 for different meshes. The difference of $Nu_{ave}$ for grids with $200 \times 200$ and $240 \times 240$ nodes is less than 0.1; therefore, the mesh with $200 \times 200$ node is adequate for this study and all simulations are performed using this mesh.

Fig. 2 shows the comparison of averaged Nusselt number at different values of $Ra$ and $\theta_{ave}$ between single-phase modeling, present study and experimental data of Ho et al. [28]. As shown, the present two-phase study has relatively good agreement with experimental data [28]. The thermophoresis dimensionless parameter, $S_T$, in Eq. (23) is obtained from best agreement with experimental data [28]. These values are 0.030, 0.022, and 0.016 for $\theta_{ave} = 1$–3\%, respectively.

5. Results and discussions

Effects of inclination angle at various $\theta_{ave}$ and $Ra$ are studied numerically by two-phase mixture model with two important slip velocity mechanisms (Brownian motion and thermophoresis phenomenon) between nanoparticles and base fluid. The inclination angle plays as an effective role on heat transfer and nanoparticles distribution.

5.1. Effects of slip velocity mechanisms and cavity inclination angle

As seen in Fig. 3, the minimum and maximum absolute error between present work and experimental data for convection coefficient is 0.44\% and 17.51\%, while these values are 3.98\% and 21.82\% between single-phase modeling and experimental data. Fig. 4 shows streamlines contours for single-phase modeling and two-phase modeling for $\theta_{ave} = 1\%$ at $Ra = 1 \times 10^5$. The direction of main circulation of flow is clockwise because of convection, and the slip velocity mechanisms generate two vortexes with clockwise direction, too, at upper right-hand and lower left-hand corners. Actually, the directions of two flows are opposite each other at the bottom-left of vortex and top-right of main flow at upper right-hand corner of cavity. This behavior occurs symmetrically at bottom-left of cavity, too. This opposite direction of vortex leads to a kind of returned weak flow for the main flow with a velocity close to zero, and decreases the power of cavity flow and velocity, as shown in Fig. 5. It is clear that with a decrease in velocity, the main circulation begins to get weaker. Figs. 6 and 7 indicate streamlines and velocity profiles. The vortexes are counterclockwise except at