Numerical Simulation of MHD Fluid Flow inside Constricted Channels using Lattice Boltzmann Method

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ABSTRACT

In this study, the electrically conducting fluid flow inside a channel with local symmetric constrictions, in the presence of a uniform transverse magnetic field is investigated using Lattice Boltzmann Method (LBM). To simulate Magnetohydrodynamics (MHD) flow, the extended model of D2Q9 for MHD has been used. In this model, the magnetic induction equation is solved in a similar manner to hydrodynamic flow field which is easy for programming. This extended model has a capability of simultaneously solving both magnetic and hydrodynamic fields; so that, it is possible to simulate MHD flow for various magnetic Reynolds number (Rem). Moreover, the effects of Rem on the flow characteristics are investigated. It is observed that, an increase in Rem, while keeping the Hartman number (Ha) constant, can control the separation zone; furthermore, comparing to increasing Ha, it doesn’t result in a significant pressure drop along the channel.

Keywords: Lattice Boltzmann method; Magnetohydrodynamics; Constrictions; Magnetic Reynolds number; Hartmann number.

NOMENCLATURE

- \(a\): analytical value
- \(B\): magnetic induction field
- \(B_0\): constant transverse magnetic field
- \(b\): simulated Boltzmann value
- \(BGK\): Bhatnagar–Gross–Krook
- \(CFD\): Computational Fluid Dynamics
- \(Cs\): sound speed
- \(e\): lattice velocity in hydrodynamic field
- \(F\): uniform forcing in the along-channel direction
- \(f\): particle mass distribution function
- \(f_{eq}\): equilibrium distribution function
- \(g\): vector distribution function
- \(H\): channel Height
- \(Ha\): Hartmann number
- \(k\): constriction width
- \(L\): channel Length
- \(LBM\): Lattice Boltzmann Method
- \(MHD\): magnetohydrodynamics
- \(m\): average
- \(P\): local pressure
- \(Re\): Reynolds number
- \(Re_m\): magnetic Reynolds number
- \(u\): flow velocity
- \(w_i\): weighted factor indirection i
- \(x_0\): location of minimum cross-section
- \(x\): cartesian coordinate of x-direction
- \(y\): cartesian coordinate of y-direction
- \(\delta\): constriction height
- \(\eta\): magnetic resistivity
- \(\rho\): local flow density
- \(\tau\): relaxation time
- \(\tau_m\): magnetic relaxation time
- \(\nu\): kinematic viscosity
- \(\Xi_i\): lattice velocity in magnetic field
- \(\Omega_i\): collision operator in direction i
1. INTRODUCTION

Flow in constricted channels and tubes are observed in some fluidic devices (orifices, valves) and have wide applications in engineering as well as physiological applications. Because of such industrial and physiological applications, it is of no surprise that flow through local constrictions has been the subject of many studies in the past, in both experimental and numerical domains (Ahmed and Giddens, 1981; Deshpande et al., 1976; Lee and Fung, 1970; Neren, 1992; Young, 1979; Giddens et al., 1993; Lee, 1994; Chouly and Lagree, 2012; Bandyopadhyay and Layek, 2011; 2012). Such studies have already revealed that normal blood flow can be affected by factors such as wall shear stress, pressure fluctuations, and local velocities (Ahmed and Giddens, 1981; Deshpande et al., 1976; Lee and Fung, 1970; Neren, 1992; Young, 1979). The degree of importance of the shape, breadth, and height of the constriction has also been revealed through such studies (Giddens et al., 1993; Lee, 1994; Bandyopadhyay and Layek, 2011).

The main objective of previous studies has been to better understand flow irregularities caused by such local constrictions. The Present paper follows the傳統 manner similar to fluid flow so it is appropriate for programming. The model proposed by Dellar (2002) has been used in the current study.

The paper has been organized as follows: we will start by presenting the computational domain which is laminar, incompressible fluid flow between two parallel plates containing a Gaussian symmetric, local constriction. It is also assumed that the fluid flow temperature is constant. We will then proceed with describing the lattice Boltzmann method as the method of solution. The magnetic induction equations which can be solved in a similar manner to the hydrodynamic equations (using a vector distribution function) are described next in details. Typical numerical results will be presented demonstrating magnetic field effects on flow characteristics in constricted channels. The paper is concluded with presenting a brief summary of the main findings of the work.

2. METHOD AND SIMULATION

We consider laminar flow of an electrically conducting fluid flow between two parallel plates having an overall length of L which are separated by a distance 2H from each other (see Fig. 1). As can be seen in Fig. 1, there is a local symmetric constriction somewhere between the two plates; it is seen to have amplitude of δ (constriction height) and a span of 4k (constriction width). The shape of constriction, assumes to be Gaussian type; that is

\[ y = H - \delta \exp \left( -\left( \frac{x-x_0}{k} \right)^2 \right) \]  

In Eq. (1) \( x_0 \) is the location of minimum cross-section (at the throat of channel).

In this paper we assumed that the minimum cross-section is located in the middle of channel. Also, the ratio of channel length (L) to channel height (2H) is 16. For simulation electrically conducting fluid flow through constricted channel (above mentioned geometry) the LBM is used which is the simplest one. In recent years the LBGK model for MHD flow has been developed. The earlier LBGK model in this area was presented by Chen et al. (1991). This model was an extension of lattice gas cellular automata MHD proposed by Chen and Matthaeus, 1987 and Chen et al., 1988. In their model, a two-index particle distribution function was used corresponding to separate microscopic velocities for velocity and magnetic field. At streaming step each particle moves along with one of the two velocity vectors which are chosen randomly. Martinez et al. (1994) have proposed another model which reduces the number of necessary particle states and computation effort as a result. Bouchut (1999) has introduced a vector-valued distribution function instead of the scalar probability distribution function. Dellar (2002) has also characterized an approach based on Bouchut model. In this model a separate vector-valued magnetic distribution function based on a vector Boltzmann–BGK equation gives the magnetic field. This model can solve induction equation in a manner similar to fluid flow so it is appropriate for programming. The model proposed by Dellar (2002) has been used in the current study.
discussed in the following section.

Fig. 1. Schematic showing the constricted channel shape used in the present study.

2.1 Lattice Boltzmann Method

The LBM is a numerical scheme in mesoscopic scale which solves the LB equation for obtaining macroscopic flow field properties. Alternatively, this method can be used to simulate complex flow and transport phenomena especially in cases where direct solution of Navier-Stokes equations is not feasible (Martinez et al., 1994; Bouchut, 1999; Dellar, 2002). The LB M has already been successfully used in various fluid flow problems, such as multiphase and multicomponent fluid flows (Bao and Schaefer, 2012; Yan et al., 2011), thermal flow (Attar and Körner, 2011; Chen et al., 2012; Lin et al., 2012; Rong et al., 2010), flows through porous media (Rong et al., 2010; Hirabayashi et al., 2012; Boek and Venturoli, 2010), solid particle suspensions (Hirabayashi et al., 2012), non-Newtonian fluids (Chai et al., 2011; Ohta et al., 2011), reaction diffusion systems (Bresolin and Oliveira, 2012), and magneto-hydrodynamics, (Chen et al., 1988; Martinet et al., 1994; Bouchut, 1999; Dellar, 2002; Kefayati et al., 2012).

To conserve mass and momentum and ensuring that the fluid is isotropic, in the LBM the particle distribution functions, \( f_i(x,t) \) at a point \( x \) and time \( t \), move synchronously on a regular lattice (Boyd and Buick, 2007). Fig. 2 shows the D2Q9 lattice model used in this paper. Only the five speeds 0, 1, 2, 3, 4, shown with thick lines, are used for the magnetic field. The distribution functions are given by the Lattice Boltzmann equation (Chen and Doolen, 1998):

\[
f_i(x + e_i \Delta t, t + \Delta t) = f_i(x,t) + \Omega_i(x,t)
\]

(2)

Where in the case of D2Q9 lattice

\[
e_i = (0,0); \quad (i = 0)
\]

(3)

\[
e_i = \left[ \cos \left( \frac{\pi}{2} (i-1) \right), \sin \left( \frac{\pi}{2} (i-1) \right) \right]; \quad (i = 1,2,3,4)
\]

(4)

\[
e_i = \sqrt{2} \left[ \cos \left( \frac{\pi}{2} (i-1) + \frac{\pi}{4} \right), \sin \left( \frac{\pi}{2} (i-1) + \frac{\pi}{4} \right) \right]; \quad (i = 5,6,7,8)
\]

(5)

The collision operator \( \Omega \) is given by the Bhatnagar–Gross–Krook approximation (Ohta et al., 2011; Bhatnagar et al., 1954):

\[
\Omega_i = \frac{1}{\tau} \left[ f_i(x,t) - f_i^{eq}(x,t) \right]
\]

(6)

In the above equation \( \tau \) is a relaxation time and \( f_i^{eq} \) is the equilibrium distribution function given by the local fluid density, \( \rho \), and flow velocity, \( u \), as follows:

\[
f_i^{eq}(x) = w_i \rho(x) \left[ 1 + 3e_i u + \frac{9}{2} (e_i u)^2 - \frac{3}{2} u_i^2 e_i^2 \right]
\]

(7)

Where the weight factors \( w_i \) are as below:

\[
w_i = \begin{cases} 4/9 \quad i = 0 \\ 1/9 \quad i = 1, 2, 3, 4 \\ 1/36 \quad i = 5, 6, 7, 8 \end{cases}
\]

In this model an ideal gas equation of state is used (i.e. \( \rho = \rho c_s^2 \)) where \( c_s^2 = 1/3 \) is the speed of sound.

The fluid density \( \rho \) and velocity \( u \) can be easily obtained by \( \rho = \sum f_i \) and \( u = \sum_i f_i e_i \), respectively at each node. It is worth mentioning that the kinematic viscosity can be expressed by \( \nu = (2\pi - 1)/6 \).

2.2 Magnetohydrodynamics Approach

Calculation of the magnetic field in MHD flow can be done using the magnetic induction equation (Moreau, 1990):

\[
\frac{\partial B}{\partial t} + \nabla \cdot (uB - Bu) = \eta \nabla^2 B
\]

(8)

Where \( B \) and \( \eta \) represent magnetic induction field and magnetic resistivity, respectively. The magnetic induction equation can be solved in a similar manner to the hydrodynamic equations, using a vector distribution function, \( g \). The evolution of this function is given by (Dellar, 2002)
Hartmann Flow comprises a steady unidirectional flow of viscous, electrically conducting fluid through a channel containing a constant transverse magnetic field. The fluid, assumed incompressible and all relevant quantities, except the pressure, are a function of only the transverse coordinate, \( u = (u(y), 0, 0) \), \( B = (B_{x}(y), B_{y}, 0) \) where \( B_{y} \) is the constant magnetic field transverse to the channel length. A uniform and time independent pressure gradient is maintained along the channel direction to drive the fluid. The walls are located at \( y = -H \) and \( y = H \) (Martinez et al., 1994). For this case, the incompressible MHD equations can be simplified to the following linear system (Dellar, 2002)

\[
F + \rho_{\nu} \frac{d^2 u}{dy^2} + B_{y} \frac{dB}{dy} = 0
\]

\[
b_{y} \frac{du}{dy} + \eta \frac{d^2 B}{dy^2} = 0
\]

In the above equation \( F \) is spatially uniform force acting along channel direction, such as a pressure gradient. There is an analytical solution for Eq.14 if non-slip boundary conditions is applied for the velocity field and accompanied by \( B_{y} (-H) = B_{y} (H) = 0 \) for the magnetic field.

\[
B(y) = \frac{F_H}{B_0} \left( \frac{\sinh(Ha y / h)}{\sinh(Ha)} - \frac{y}{H} \right)
\]

\[
u(y) = \frac{F_H}{\sqrt{\rho B_0 y}} \left( \cosh(Ha y / h) \left( 1 - \frac{\cosh(Ha y / h)}{\cosh(Ha)} \right) \right)
\]

where \( Ha \) is a dimensionless Hartmann number which represents the ratio of Lorentz force to viscous force and can be defined by \( Ha = B_{y} H / \sqrt{\rho \eta v} \).

When the Hartmann number is large, as is typical in liquid metal MHD applications, the magnetic field maintains a nearly uniform velocity \( u \) over the bulk of the channel (Dellar, 2002). Conversely, for zero Hartmann number which means no external magnetic field the solution will reduce to the parabolic velocity profile of the Poiseuille flow.

### 3. Validation

To examine the validity of the code, the obtained velocity profile and axial component of induced magnetic field, \( B_{y} \), in a straight channel are compared with analytical solution of Eq. (15) (see Fig. 3). The Reynolds number and magnetic Reynolds number defined by \( \text{Re} = U_{m} H \sqrt{\nu} \) and \( \text{Re}_{m} = U_{m} H / \eta \) where \( U_{m} \) denotes average velocity at inlet of the channel and \( \eta \) denotes magnetic resistivity. It is assumed that the Reynolds number and magnetic Reynolds number are constant (Re=50, Re=5) along different Hartmann numbers. To reduce compressibility error inlet Mach number (\( \text{Ma}_{in}=U_{in}/C_{i} \)) set to be equal to 0.026.

For any given flow rate, a decrease in the velocity near the centerline must be accompanied by an increase in the velocity near the walls, as can be inferred from Fig. 3a. Moreover, in Fig. 3b the axial component of induced magnetic field grows from zero on the wall to its maximum adjacent to the surface and then experiencing a point of inflection once again falls to zero in the centerline. The mentioned maximum value decreases with increasing \( Ha \) number.
It is worth mentioning that a good agreement is observed between numerical results and analytical solutions. The second order accuracy of MHD Lattice Boltzmann method (Dellar, 2002) was also investigated. The simulations were finished when the following criterion was satisfied.

\[ \sum_x |u(x,t) - u(x,t-1)| \leq \varepsilon \]

(16)

where \( \varepsilon \) is a small number assumed to be \( \varepsilon = 1 \times 10^{-10} \).

The global error can be calculated by Eq. (17).

\[ \zeta = \frac{\sum_x \|u_a(y) - u_b(y)\|}{\sum_x \|u_a(y)\|} = \frac{\sum_x \|B_a(y) - B_b(y)\|}{\sum_x \|B_a(y)\|} \]

(17)

In the Eq.17 the subscript \( a \) denotes results of the exact analytic solution while the subscript \( b \) refers to the obtained results from LBM simulation. Results for the global error have been shown in Fig.

4. The black lines in Fig. 4 represent lines of slope \(-2\), indicating second-order behavior. Although, the amount of error for axial magnetic field is higher than velocity field, it can be seen that for various Hartmann number the presented data closely match the slope of those lines. These results prove that the code can well represent MHD flow in channels.

**Fig. 4. Global error for MHD Lattice Boltzmann Method.**

4. RESULTS AND DISCUSSION

Having found the fully-developed velocity profiles in the previous section, one can proceed with calculating the velocity profiles in the constriction region. To achieve this goal, we used a uniform grid consisting 125 nodes along the channel height and 2000 nodes along the channel length. To produce various Reynolds number, we changed viscosity, \( \nu \), so that the relaxation time would be changed too. For \( \text{Re}=50 \) the relaxation time, \( \tau \), and the inlet Mach number, \( M_{\text{in}} \) is set to be 0.5562 and 0.026 respectively. To perform various magnetic Reynolds number, \( \text{Re}_{m} \) the magnetic resistivity, \( \eta \) was changed. For \( \text{Re}_{m}=5 \), magnetic relaxation time is set to be 1.0625.

Figure 5 illustrates the effect of the constriction height, \( \delta \), on the streamwise velocity profile at \( x/H=17 \) (i.e., downstream the throat), a constriction with \( k/H=0.4 \) for \( \text{Re}=50 \) and \( \text{Ha}=0 \). The negative values of the velocity imply the recirculating flow region. As the height of the constriction increases the greater separation zone in the upper and lower walls of the channel can be observed.

Figure 6 shows a history of the development of the streamlines inside a constricted channel (with \( \delta/H=0.5 \) and \( k/H=0.4 \) ) for \( \text{Ha}=0 \) at \( \text{Re}=25, 50 \) and 75 ). A flow reversal (i.e., separation) downstream of the throat is obvious in this figure. Evidently, once the flow enters the diverging section of the channel, it is vulnerable to separation because of the influence of the adverse pressure gradient in this part of the channel. As expected, separated zone grows further with the increase of Reynolds number (the inertia forces become more important than the viscous forces), and at \( \text{Re}=75 \) (Fig. 6c) it occupies
main portion of the channel.

Fig. 5. Effect of constriction height, $\delta$, on the velocity profiles at $x/H = 17$ for $Re=50$ and $Ha=0$.

Here, the effect of Hartman number on the velocity profile at $x/H=17$, $Re=50$ and $Re_m=5$ for a constriction with $\delta/H=0.5$ and $k/H=0.4$ is investigated.

As can be observed in Fig. 7, the Lorentz force, as induced by the magnetic field, is predicted to have a retarding effect on fluid elements near the axis of the constriction. Close to the wall, however, fluid elements accelerate to keep the mass flow rate constant. This increase in the fluid velocity near the walls with increasing $Ha$ number enables the fluid to overcome the influence of any adverse pressure gradient (downstream of the constriction) and thereby enhances the ability of the fluid to withstand flow separation near the wall.

The effect of Hartman number on overall wall friction (it can be obtained with integrating of $C_f Re$ on the wall surface) is summarized in Table 1. It is clear from this table that the overall wall friction increases with an increase in the strength of magnetic field. An increase of the wall friction with an increase of the Hartman number can be attributed to the boundary layer becoming thinner due to the higher Hartman number resulting in an increase in the velocity gradient near the wall (see Fig. 3a).

Figure 9 represents the effect of Hartman number on the pressure variation for $Re=50$ and $Re_m=5$. It is concluded from this figure that by increasing the Hartmann number from 0 to 6, the pressure drop increases dramatically. This result is attributed to increase of wall shear stress due to the increase of velocity gradient near the wall. In this connection it is important to mention that since in the real situations the externally applied pressure gradient will need to be adjusted appropriately, we cannot
strictly conclude that more increase in $Ha$ number is a completely effective way to cause a delay in separation.

Fig. 8. Streamline pattern at various Hartmann number, $Ha$, for $Re=50$ and $Rem=5$.

Fig. 9. Effect of the Hartmann number, $Ha$, on the dimensionless pressure at the axis of channel $Re=50$ and $Rem=5$.

Figure 10 shows the effect of magnetic Reynolds number on streamline patterns at constant Hartmann number ($Ha=2$). It can be seen that, by increasing the magnetic Reynolds number from 5 to 150, the diffusion term in Eq. (8) decreases (because of the magnetic resistivity, $\eta$, reduction) and convection term becomes more important. As a result, the induced magnetic field increases in comparison with the uniform transverse magnetic field and the separation zone becomes smaller in size. In Fig. 11 the variation of magnetic Reynolds number with stream wise velocity is depicted at $x/H=17$. This confirms that, similar to Hartmann number, $Rem$ has an increasing effect on velocity of fluid elements close to the wall. However, as can be seen in Fig. 12 the pressure drop due to increasing magnetic Reynolds number is not significant. In fact, increasing $Rem$ just results in increasing induced magnetic field in the constrictions region which accelerates fluid flow near wall. So, flow doesn't have to tolerate large velocity gradient on the wall along the whole channel length which causes considerably less pressure drop.

Rem=25

Rem=50

Rem=75

Fig. 10. Streamline pattern at various magnetic Reynolds number, $Rem$, for $Re=50$ and $Ha=2$.

Fig. 11. Velocity profiles at various magnetic Reynolds number for $x/H = 17$ and $Re=50$. 
5. Conclusion

In this paper, numerical simulation of electrically conducting fluid flow inside a locally constricted channel using lattice Boltzmann method is investigated. Simultaneously solving magnetic and hydrodynamic fields, makes it possible to study the effect of various magnetic Reynolds number in the simulation. It is observed that flow separates at the lee of constriction and this separated zone will be larger when the Reynolds number increases. To control flow separation, two scenarios are suggested which include increasing $Ha$ and $Rem$. It is viewed that increasing $Ha$ can reduce the separated zone in downstream of constrictions whereas considerable pressure drop along channel length occurs. However, while $Ha$ is kept constant, an increase in $Rem$ not only could delay the separation phenomena but also significant pressure drop does not occur.

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