Errors Analysis Problems- 1
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(1) *Mean, standard deviation and standard error (1)*
An experiment was conducted to determine the concentration of a sodium hydroxide solution. The eight repeat measurements of the volume of hydrochloric acid titrated (all in ml) are: 25.8, 26.2, 26.0, 26.5, 25.8, 26.1, 25.8 and 26.3. Calculate (i) the mean, (ii) the standard deviation using the rough-and-ready approach; (iii) the standard deviation using eqn (2.3); (iv) the standard error of the volume.

(2) *Mean, standard deviation and standard error (2)*
12 measurements of the sensitivity of a photodiode circuit (in amps/watt) are: 5.33, 4.95, 4.93, 5.08, 4.95, 4.96, 5.02, 4.99, 5.24, 5.25, 5.23 and 5.01. Calculate (i) the mean, (ii) the standard deviation using eqn (2.3); (iii) the standard error.

(3) Reduction of the standard error
In a magnetometry experiment, after a minute of collecting data the statistical noise was reduced to 1 picotesla. For how much longer should data be collected in order to reduce the random error by a factor of 10?

4) *Error in the error*
Consider a set of measurements with the standard error calculated to be \( \alpha = 0.987 \, 654 \, 321 \). Here we address the question of how many significant figures should be quoted. Construct a spreadsheet with four columns. The first column should be \( N \), the number of measurements on which \( \alpha \) is based. In the second column write \( \alpha \) to the nine significant figures quoted above. The third and fourth columns should be \( \alpha \times \left( 1 - \frac{1}{\sqrt{2N-2}} \right) \) and \( \alpha \times \left( 1 + \frac{1}{\sqrt{2N-2}} \right) \) respectively. As we are interested in the variation over a large dynamic range, choose values for \( N \) such as 2, 3, 5, 10, 20, 30, etc. Verify the statement from Section 2.7.1 that the number of data points, \( N \), needs to approach a few tens of thousands before the second significant figure in the error can be quoted, i.e. when the values in the three columns become equal to the second significant figure. Repeat the analysis for the case where \( \alpha = 0.123 \, 456 \, 789 \), i.e. the first significant digit of the error is 1. How many data points must be collected before the third significant figure can be quoted?

5) *Reporting results*
Fifteen measurements of a resistance are quoted here, based on approximately 10 repeat measurements. Only three of them obey the five golden rules. Identify the mistakes in the other results.

(i) \( 99.8 \pm 0.270 \) \( \times 10^3 \) \( \Omega \)
(ii) \( 100 \pm 0.3 \) \( \times 10^3 \) \( \Omega \)
(iii) \( 100.0 \pm 0.3 \) \( \times 10^3 \) \( \Omega \)
(iv) \( 100.1 \pm 0.3 \) \( \times 10^3 \) \( \Omega \)
(v) \( 97.1 \times 10^3 \pm \) \( 276 \) \( \Omega \)
(vi) \( 99.8645 \pm 0.2701 \) \( \times 10^3 \) \( \Omega \)
(vii) \( 98.6 \times 10^3 \pm 3 \times 10^2 \) \( \Omega \)
(viii) \( 99.4 \times 10^3 \pm 36.0 \times 10^2 \) \( \Omega \)
(ix) \( 101.5 \times 10^3 \pm 0.3 \times 10^1 \) \( \Omega \)
(x) \( 99.8 \pm 0.3 \) \( \times 10^3 \) \( \Omega \)
(xi) \( 95.2 \times 10^3 \pm 273 \) \( \Omega \)
(xii) \( 98, 714 \pm 378 \) \( \Omega \)
(xiii) \( 99000 \pm 278 \) \( \Omega \)
(xiv) \( 98, 714 \pm 3 \times 10^3 \) \( \Omega \)
(xv) \( 98900 \pm 300 \) \( \Omega \)