Estimating the parameters of a photovoltaic array and solving equations of maximum power point using a numerical method and fuzzy controller

Amin Taheri 1, Majid Dehghani 2
Department of Electrical Engineering, Najafabad Branch, Islamic Azad University, Najafabad, Iran
Mail2amin2005@iaun.ac.ir 1, Dehghani@pel.iaun.ac.ir 2

ABSTRACT

This paper propose the method of modeling and simulation of photovoltaic (PV) array and achieve maximum power point (MPP) of them. Simulating a photovoltaic array based on the single-diode model has been done in this study. The first objective is to find the parameter of nonlinear I-V equation by adjusting the curve in open circuit, maximum power and short circuit point’s base on the single-diode model and the effect of the series and parallel resistance. The variation of the parameters with change in irradiance and temperature is also studied. Then, equations of the maximum power point of photovoltaic array have been introduced and solved using successive under relaxation (SUR) numerical method. Afterward, disadvantages of SUR method have been identified and fuzzy controller has been used to get the maximum power of the photovoltaic array.

Keywords: Photovoltaic array, Maximum power point tracker, Fuzzy controller

1. INTRODUCTION

Due to a significant reduction in fossil energy resources in recent years, as well as the pollution caused by widespread use of them, the importance of using clean and renewable energy has been revealed more and more. Solar power is one of the clean, renewable and accessible energies. Developed countries have supplied a significant portion of their energy needs through green energies. Developing countries also have taken appropriate steps in the way of clean and renewable energies with regard to the future of fossil fuels. So that the annual growth rate of exploitation of photovoltaic (PV) power on a global scale is more than 40%. Hence the renewable energies, especially PV power, will be a considerable source of energy in the near future.

A photovoltaic (PV) array directly converts sunlight into the electricity. In recent years more efficient PV arrays has been produced and released on the market along with advancements in technology [1]. PV arrays present nonlinear I-V characteristics with several parameters. Although some important parameters are always announced by manufacturers, having access to all parameters of PV array seems necessary for modeling and accurate design of PV systems. Due to the relatively low efficiency of PV arrays, getting the maximum power of them is one of the major issues [2, 3, 4]. Although it is possible to connect a PV array directly to the small load, like lighting system but the power received from array will be considerably lower than the case in which array is connected to the load through a Maximum Power Point Tracking (MPPT) system [2]. This is the importance of MPPT in high power PV systems.

In this study firstly equivalent circuit of PV cells have been introduced, simulating of a PV array based on a single diode model is investigated, equations of the PV array have been acquired and all the parameters related to PV array have been obtained using Newton-Raphson method. Then maximum power point (MPP) equations defined and solved using a successive under relaxation (SUR) method to achieve maximum power point. The MPPT system based on the boost converter has been designed using a fuzzy controller. Finally the MPPT methods compare to each other's.

1.1 Modeling the PV cell and the PV array

The elementary of PV array is PV cell. In order to large output voltage, PV array is formation from series cells and For increasing output current we need to connecting cells in parallel. For modeling a PV array firstly we need to model a PV cell. In recent years different equivalent circuits have been suggested for modeling the behavior of PV cell. Such as single-diode, two-diode, and diode-capacitor models [3, 4, 5, 6]. Although two-diode model has higher accuracy than single-diode, single-diode model has demonstrated good performance in various articles [3, 8, 9].

In Error! Reference source not found. single-diode model of PV cell has been shown.
1.1.1 Modeling the PV array

Practical PV arrays are composed of several PV cells which are connected to each other in series and parallel. For achieving the MPP of PV array, firstly we need to find how the I-V curve obtained. Regarding to Error! Reference source not found. the current equations of PV array are as follows [5, 8, 9]:

\[ I_{pv} = (I_{pvn} + K_i \times \Delta T) \times \frac{G}{G_n} \]  
(1)

In (1), \( I_{pv} \) is the produced current of array regardless of the amount of series and parallel resistance. \( I_{pvn} \) is produced current by PV array in STC condition in terms of ampere. \( \Delta T = T - T_n \) is the difference in temperature between STC condition and the environmental condition in terms of Kelvin. \( G_n \) is the irradiance intensity in STC condition and equal to 1000W/m², and \( G \) is the irradiance intensity in environmental condition in terms of W/m².

\[ I_{pv} = \left( \frac{R_s}{R_p} + R_p \right) / R_p \times I_{scn} \]  
(2)

In (2), \( R_s \) and \( R_p \) are series and parallel resistances in terms of Ohm and \( I_{scn} \) is the nominal short circuit current of PV array.

\[ I_0 = \left( \frac{I_{pv} - V_{ocn} / R_p}{\exp \left( \frac{V_{ocn}}{V_t} / \alpha / N_p \right) - 1} \right) \]  
(3)

In (3), \( I_0 \) is the diode saturation current in terms of ampere, \( V_t \) is the thermal voltage of diode in terms of volt and \( \alpha \) is the ideal diode constant. If array has \( N_p \) parallel cells, equations (1) and (3) should be corrected as:

\[ I_{PV} = I_{pv, \text{cell}} \times N_p \]  

and \( I_0 = I_{0, \text{cell}} \times N_p \) [8]. \( V_t \) is the thermal voltage of diode which is dependent on junction temperature. Equation (4) specifies this ratio.

\[ V_t = k \times T / q \]  
(4)

In (4) \( q \) is electron charge \((1.60217646 \times 10^{-19} \text{ C})\), \( k \) is Boltzmann constant \((1.3806503 \times 10^{-23} \text{ J/K})\), and \( T \) is temperature in terms of Kelvin. Output current of PV array is calculated via (5) using Kirchhoff's current law [8].

\[ I = I_{pv} - I_0 \left[ \exp \left( \frac{V + R_s I}{V_t \alpha} \right) - 1 \right] - \frac{V - R_s I}{R_p} \]  
(5)

In (5), \( V \) is the output voltage of PV array.

As can be seen from (5), output current of PV array is always dependent on nonlinear parameters. Furthermore, series and parallel resistances are not constant and follow nonlinear equations. According to (6) and (7) upper bound of series resistance and lower bound of parallel resistance can be calculated [9].

\[ R_{s,max} = V_{ocn} - V_{mp} / I_{mp} \]  
(6)

\[ R_{p,min} = V_{mp} / \left( I_{scn} - I_{mp} \right) - V_{ocn} - V_{mp} / I_{mp} \]  
(7)

The amounts of \( R_p \) and \( R_s \) are also calculated via (8) and (9) [8, 10]:

\[ R_p = V_{mp} \times \left[ V_{mp} + I_{mp} \times R_s \right] / \left( V_{mp} \times I_{pv} - V_{mp} \times I_0 \times \exp \left( \left[ V_{mp} + I_{mp} \times R_s \right] / V_t / N_p / \alpha \right) + V_{mp} \times I_0 - P_{sc} \right) \]  
(8)

\[ R_s = R_s + r \]  
(9)

1- Irradiance 1000 W/m², AM 1.5 spectrum, module temperature 25°C
In (9), $P_{stc}$ is the maximum output power of PV array under STC condition and $r = 0.001$. Maximum output power of PV array under STC condition is calculated via (10) according to $V_{mp}$ and $I_{mp}$ from datasheet.

$$P_{stc} = V_{mp} \times I_{mp}$$

(10)

Equation (11) shows output power of one-diode model PV array under any irradiance and temperature circumstances [4, 8]:

$$P_{model} = \left( I_{pv} - I_0 \times \left( \exp\left(\frac{(V+I \times R_s)}{V_i} / N_a \right) - 1 \right) \right) \times \frac{(V+I \times R_s)}{R_p}$$

(11)

By using the (1-11), the P-V and I-V curves of PV array under desire irradiance and temperature condition and amounts of $I_{pv}$, $I_{pv}$, $R_{s_{max}}$, $R_{p_{min}}$, $R_s$, and $R_p$ are acquired. The equations should be solved via $R_s = 0$ and $R_p = R_{p_{min}}$ firstly and the primary amount of $P_{model}$ should be obtained. By considering $P_{model} = P_{stc}$ the (1-11) are solved in terms of each other with Newton-Raphson method. In order to test the proposed method, information of a high quality solar array which is available in Iran’s market (Kyocera-KC200GT) has been acquired according to the manufacturer datasheet. The amounts of $I_{pv}$, $I_{pv}$, $I_0$, $R_{s_{max}}$, $R_s$, $R_{p_{min}}$, $R_p$ and $P_{model}$ has been shown in Table 1. according to the proposed method [1] and the P-V and I-V curves has been shown in Fig 2, 3. These results show an appropriate performance of the proposed method.

<table>
<thead>
<tr>
<th>Table 1. Calculated PV array parameter’s using the proposed method</th>
</tr>
</thead>
<tbody>
<tr>
<td>Estimated items</td>
</tr>
<tr>
<td>$I_{pv}$</td>
</tr>
<tr>
<td>$I_{pv}$</td>
</tr>
<tr>
<td>$I_0$</td>
</tr>
<tr>
<td>$R_{s_{max}}$</td>
</tr>
<tr>
<td>$R_s$</td>
</tr>
<tr>
<td>$R_{p_{min}}$</td>
</tr>
<tr>
<td>$R_p$</td>
</tr>
<tr>
<td>$P_{model}$</td>
</tr>
</tbody>
</table>

2. The behavior of PV array in maximum power point

The output power of a PV array is always dependent on intensity of solar irradiance, angle of irradiance and temperature of PV array. Output power of the array is directly related to irradiance intensity but it has a reverse relation with temperature of PV array. Hence, high irradiance intensity and low temperature are the best conditions for generating energy from PV arrays. By defining a vector form of definition of voltage and current of PV array, behavior diagram of PV array is achieved regarding various temperature and irradiance conditions using (1-11). P-V and I-V diagrams of PV array (model: Kyocera KC200GT) in different irradiance condition at 25°C has been shown in Fig 2, 3.
If the intensity of solar irradiance be constant and the temperature of PV array be variable, P-V and I-V diagrams of PV array will be different. Fig.4 and 5 show the behavior of above PV array in irradiance condition of 1000W/m² and different temperature condition.

3. Equations of describing the maximum power point

As can be seen from Fig.2 and 4, the maximum power point (MPP) is a point which the maximum voltage $V_{mpp}$ and current $I_{mpp}$ is received from PV array with regard to the temperature and irradiance condition. The equations of maximum power point according to the irradiance intensity and temperature condition has been shown in (12, 16) [11]:

$$V_{mpp}(G,T) = V_{oc}(G,T) - I_{mpp}(G,T)R_s + N_s \times V_t \times U$$

$$U = \ln \left[ \frac{I_s(G,T) - I_{mpp}(G,T)}{I_{mpp}(G,T)} \times \frac{R_s + R_p}{V_{mpp}(G,T)} \times V_{oc}(G,T) \right]$$

In (12), $V_{mpp}(G,T)$ is the voltage of maximum power point regarding temperature and irradiance condition, $V_{oc}(G,T)$ is the open circuit voltage of the PV array with regard to temperature and irradiance condition, $I_{mpp}(G,T)$ is the current of maximum power point according to temperature and irradiance condition, $R_s$ is series resistance, $N_s$ and $V_t$ are the number of series cells in PV array and thermal voltage of diode respectively. In (13), $R_p$ is the amount of parallel resistance, $I_s(G,T)$ is the short circuit current of the PV array regarding irradiance and temperature condition. With respect to the results from [6]:

![Fig. 2. I-V diagram in different irradiance condition at 25°C](image)

![Fig. 3. P-V diagram in different irradiance condition at 25°C](image)

![Fig. 4. P-V diagram in 1000W/m² and variable temperature](image)

![Fig. 5. I-V diagram in 1000W/m² and variable temperature](image)
Using the (14) and (15), values of \(I_{sc}(G,T)\) and \(V_{oc}(G,T)\) are calculated in terms of intended irradiance and temperature conditions. In these equations, \(I_{sc,n}(G_n,T_n)\) and \(V_{oc,n}(G_n,T_n)\) are short circuit current and open circuit voltage of PV array respectively in STC condition.

\[ I_{sc}(G,T) = (I_{sc,n}(G_n,T_n) + K_1 \Delta T) \times \frac{G}{G_n} \]

(14)

\[ V_{oc}(G,T) - V_{oc,n}(G_n,T_n) = K_0 \times \Delta T \]

(15)

\[ T = T - T_n \]

is the difference of temperature between STC condition and intended conditions in terms of Kelvin, \(G_n\) is irradiance intensity in STC condition which is equal to 1000W/m\(^2\), and \(G\) is the amount of irradiance intensity under desired conditions in terms of w/m\(^2\). \(I_{mpp}\) is current value of PV array calculated via (17) with regard to the irradiance and temperature condition of (16) [11]:

\[ I_{app}(G,T) = \left\{ \frac{V_{app}(G,T) \times Q(G,T)}{N_s \times V_s \times R_i} + \frac{V_{app}(G,T)}{R_p} \right\} \times \left\{ \frac{Q(G,T) \times R_s}{N_s \times V_s \times R_i} + \frac{R}{R_p} \right\} \]

(16)

\[ Q(G,T) = \left[ I_{sc}(G,T) \times (R_s + R_p) - V_{oc}(G,T) \right] \exp \left[ \frac{V_{app}(G,T) \times I_{app}(G,T) \times R_s - V_{app}(G,T)}{N_s \times V_s} \right] \]

(17)

Equations (12-17) are describing the maximum power point. These equations are nonlinear and require repetitive solving methods or intelligent algorithms to be solved. Among repetitive methods, Newton-Raphson and Gauss-Seidel method seem appropriate with regard to their simple implementation. But these methods are rapidly diverging, because equations are several parametric. So we need a way to be resistant against divergent to solve the equations of the maximum power point, and because equations are several parametric it is appropriate to have a simple algorithm proposed method. The proposed numerical method to solve the equations is \(SUR\) [12, 13, 14]. In fact, this is generalized Gauss-Seidel method. The difference is that in Gauss-Seidel \(X^{(k+1)} = F(X^{(k)})\), \(k = 1, 2, 3, \ldots\) is placed at any step of repetition, but in \(SUR\) \(X^{(k+1)} = (1-W) \times X^{(k)} + W \times F(X^{(k)})\), \(k = 1, 2, 3, \ldots\) and \(W < 1\) are placed at any step of repetition. \(W\) is common ratio in this equation. If there is \(W = 0\) then \(SUR\) method will be changed to Gauss-Seidel method. It should be noted that convergence in this method depends on the right choice of the basic values of parameters of the issue [14,15]. In the following equations (12-17) for the KC200GT solar array at 800W/m\(^2\) irradiance and 25°C temperature are solved using the proposed method. Results are shown in Table 2.

Table 2. Results of solving the maximum power point equations using \(SUR\) method

<table>
<thead>
<tr>
<th>Q</th>
<th>(I_{mpp})</th>
<th>(U)</th>
<th>(V_{app})</th>
<th>Repetition</th>
</tr>
</thead>
<tbody>
<tr>
<td>9897.4642</td>
<td>6.6306</td>
<td>7.0904</td>
<td>25.4142</td>
<td>1</td>
</tr>
</tbody>
</table>

As can be seen from Table 2, the proposed method could converge in the first phase of repetition. According to Table 2, the numerical method hasn’t had high precision due to the selection of the initial values. In the case of changing irradiance and temperature conditions, initial values need to be specified again for using the method. The most appropriate initial values are specified based on trial and error method. Besides, the proposed method is diverged in some cases of irradiance and temperature, which makes the proposed method inappropriate to be used in practical systems. That is why intelligent algorithms with low convergence time are used for designing MPPT systems and usually achieve good results.

4. Designing a MPPT system based on fuzzy controller

Different algorithms have been suggested for designing MPPT systems. Turbulence and observation, INC, neural networks, fuzzy controller are examples of this case. [2,4]. The purpose of designing MPPT is a system with the ability of receiving the maximum power from PV array with suitable velocity and maximum efficiency. Among the above methods, turbulence and observation has simple algorithm, but the quality of output power is somehow reduced due to the substantial degree of turbulence in it. Also this method has higher convergence time than other methods.
Algorithms based on neural networks have significant impossible modes. Such that adjusting these algorithms has become a major issue [3]. Meanwhile, fuzzy controller has provided high quality output power in addition to appropriate velocity [4,10]. Fuzzy controller has been used to get the maximum power from PV array (Kyocera KC200GT). First in this controller errors E[K] and changes in errors CE[K] have been measured using (18) and (19), then fuzzy controller has been designed using these two parameters. In fact, in equations (18) and (19) voltage of the prior step is compared to present voltage of the system.

\[ E(K) = \frac{P_{pv}(K) - P_{pv}(K-1)}{I_{pv}(K) - I_{pv}(K-1)} \]  

(18)

\[ CE(K) = E(K) - E(K-1) \]  

(19)

In (18) and (19), \( P_{pv}(K) \) and \( I_{pv}(K) \) are power and current of PV array respectively. Diagram of fuzzy membership functions has been shown in Fig.6, 7 and (8). Fuzzy rules are also specified in Table 3.

In Fig.6, 7 the membership functions of E[K] and CE[K] have been considered as a triangle, so controller can have a sharp respond toward input. Output membership functions are considered as a Gaussian, so controller can have lower noise in its output in addition to its steady output. The ranges of functions E[K] and CE[K] are considered large enough in controller. As long as the amounts of E[K] and CE[K] are large, controller will be able to demonstrate an appropriate performance. The controller is configured in such a way that in the case of noisy condition and radiation of 200 and 1000W/m² at temperature 25°C has the highest efficiency. Function 5sin(100πt)+3 has been used as a noise function applied to PV array [10]. A boost converter, along with fuzzy controller, has to supply voltage of 400±10V in the power of 200w. Parameters of the boost converter have been specified in Table 4.

<table>
<thead>
<tr>
<th>E[K]</th>
<th>CE[K]</th>
<th>NB</th>
<th>NS</th>
<th>ZO</th>
<th>PS</th>
<th>PB</th>
</tr>
</thead>
<tbody>
<tr>
<td>NB</td>
<td>NB</td>
<td>NB</td>
<td>NS</td>
<td>NS</td>
<td>NS</td>
<td>NB</td>
</tr>
<tr>
<td>NS</td>
<td>ZO</td>
<td>NS</td>
<td>ZO</td>
<td>ZO</td>
<td>PS</td>
<td></td>
</tr>
<tr>
<td>ZO</td>
<td>PB</td>
<td>PB</td>
<td>ZO</td>
<td>ZO</td>
<td>ZO</td>
<td></td>
</tr>
<tr>
<td>PS</td>
<td>PS</td>
<td>PS</td>
<td>PS</td>
<td>ZO</td>
<td>ZO</td>
<td></td>
</tr>
<tr>
<td>PB</td>
<td>PB</td>
<td>PB</td>
<td>PS</td>
<td>ZO</td>
<td>ZO</td>
<td></td>
</tr>
</tbody>
</table>

Table 3. Fuzzy rules

<table>
<thead>
<tr>
<th>Power</th>
<th>Voltage</th>
<th>Switching freq.</th>
<th>L</th>
<th>C2</th>
<th>C1</th>
</tr>
</thead>
<tbody>
<tr>
<td>200W</td>
<td>400±10V</td>
<td>20 KHZ</td>
<td>75 µH</td>
<td>10 µF</td>
<td>10000 µF</td>
</tr>
</tbody>
</table>

Table 4. Parameters of the boost converter
In Fig. 10 the prototype of Boost converter and fuzzy controller has been shown. The voltage, current and the power of boost converter are shown at Ohmic load at Fig. 9.

Fig. 9. a. Voltage  
Fig. 9. b. Current  
Fig. 9. c. Power of boost converter in 1000W/m² irradiance intensity and 25°C temperature

As can be seen in Fig. 9, the fuzzy controller has had an appropriate performance despite a noisy condition in 1000W/m² irradiance intensity and 25°C temperature. This system has received the maximum power from PV array and has transmitted it to the load with an approximate accuracy of %97. The controller convergence time is about 0.001 second. The transient of the controller has been shown in Fig. 9 too. As can be seen the voltage, current and power of PV array has pick in primary condition, its cause that the solar array can provide high instant power. In Table 4 the comparison between the propose methods and other algorithm has been shown [2, 3, 10].
Table 4. Comparison between the propose methods and other typical algorithms

<table>
<thead>
<tr>
<th>Type of controller</th>
<th>Time of convergence</th>
<th>Noise resistance</th>
<th>Efficiency</th>
</tr>
</thead>
<tbody>
<tr>
<td>SUR</td>
<td>large</td>
<td>no</td>
<td>~ 80%</td>
</tr>
<tr>
<td>Propose fuzzy method</td>
<td>0.001</td>
<td>yes</td>
<td>~ 97%</td>
</tr>
<tr>
<td>P&amp;O</td>
<td>~ 0.005 – 0.007</td>
<td>no</td>
<td>80% - 90%</td>
</tr>
<tr>
<td>INC</td>
<td>~ 0.003 – 0.005</td>
<td>no</td>
<td>85% - 92%</td>
</tr>
</tbody>
</table>

5. Conclusion

This paper has analyzed the development of a method for the mathematical modeling of PV arrays and finding the MPP of them. The first objective is achieve all parameters of a PV array and P-V, I-V curves according to the product datasheet using single-diode model equations. Then equations of the maximum power point have been defined and solved based on the SUR numerical method. In terms of changes in intensity of solar irradiance and temperature, this method loses its effectiveness. Therefore, a fuzzy controller has been designed. Fuzzy controller had an acceptable performance compare to SUR method and it’s had convenient operation in noisy condition.

REFERENCES