Quantum dynamics of a two-mode $f$-deformed cavity field in a heat bath

This article has been downloaded from IOPscience. Please scroll down to see the full text article.
2012 Phys. Scr. 86 065006
(http://iopscience.iop.org/1402-4896/86/6/065006)

View the table of contents for this issue, or go to the journal homepage for more

Download details:
IP Address: 31.57.23.96
The article was downloaded on 16/11/2012 at 03:59

Please note that terms and conditions apply.
Quantum dynamics of a two-mode $f$-deformed cavity field in a heat bath

Mohsen Daeimohammad

Department of Physics, Najafabad Branch, Islamic Azad University, Najafabad, Isfahan, Iran
E-mail: M.Daeimohammad@pco.iaun.ac.ir

Received 20 June 2012
Accepted for publication 22 October 2012
Published 15 November 2012
Online at stacks.iop.org/PhysScr/86/065006

Abstract
In this paper, we investigate the dissipative quantum dynamics of a two-mode $f$-deformed cavity field coupled to a heat bath. We first calculate the transition probabilities between cavity field states; this is accomplished by exploiting the Feynman–Dyson expansion of the (interaction picture) evolution, truncated at the first order, and then tracing out the reservoir degrees of freedom. Then we obtain the equations of motion for the expectation value of the number operators of photons in the two modes of the $f$-deformed cavity field. Finally, we study cross correlation between the modes of a two-mode field, which is quantified by the cross correlation function.

PACS numbers: 03.65.Bz, 05.30.-d, 05.40.+j, 02.20.Sv

1. Introduction
The rapid expansion of research frontiers into the nanometer and femtosecond regimes calls for a careful consideration of dissipation as a way of counterbalancing the energy influx from the external fields in quantum mechanical systems.

The subject has roots in the seminal work by Feynman and Vernon [1], who treated a quantum object coupled to an infinite collection of oscillators as a model of a linear dissipative environment. By employing the path-integral techniques they were able to eliminate the environmental variables and arrived at a dissipative modification of the Green function of the quantum object. This approach was further employed by a number of authors, most notably by Caldeira and Leggett [2], who applied it to a specific problem of dissipative effects in tunneling on a macroscopic scale. A detailed account of the problem of dissipation in quantum mechanical systems can be found in a monograph by Weiss [3], while semiclassical approaches have been reviewed by Grossmann ([4] and references therein).

Generally, the main mode of attack uses the density-matrix formalism [5] and the popular Lindblad technique [6]. This approach is relevant to the open quantum systems. In contrast, the considerably less arduous Schrödinger-equation formalism is regarded as appropriate for isolated systems [7] that are described by a Hermitian Hamiltonian.

The problem of dissipation in quantum mechanics is closely connected to the use of mixed quantum–classical treatments [8, 9] whereby only a few degrees of freedom of a complex system are treated quantum mechanically while the remaining ones are described classically. Although Egorov et al [10] point out that not always an accurate description is attained and so this approach must be used with care, it is recognized as a method of choice when a significant reduction of complexity is desired.

On the other hand, quantum groups [11] introduced as a mathematical description of deformed Lie algebras have given the possibility to generalize the notion of creation and annihilation operators of the usual oscillator and introduced $q$-oscillators [12, 13]. Soon afterwards, several attempts have been made to give a physical interpretation of the $q$-oscillators. In [14] they appear in connection with the relativistic oscillator model, while in [15] $q$-oscillators have been used in the generalized Jaynes–Cummings model.

Instead, the nature of $q$-oscillators of the electromagnetic field is clarified by the nonlinearity of the field vibrations [16]. This $q$-nonlinearity reflects the exponential growth of the frequency of vibrations with amplitude.

This observation suggests that there might exist other types of nonlinearities for which the frequency of oscillation varies with the amplitude via a generic function $f$; this leads to the concept of $f$-deformed oscillators [17]. Then, the generation of coherent states for a class of $f$-deformed oscillators enters in the real possibilities of trapped systems [18].

By virtue of other possible physical realizations of $f$-oscillators, it would be interesting to study the environmental effects on such oscillators. Isar and Scheid [19]
studied the connection between quantum deformation and quantum dissipation by setting a master equation for the deformed harmonic oscillator in the presence of a dissipative environment which was shown to be the deformed version of the master equation obtained in the framework of the Lindblad theory for open quantum systems [20].

When the deformation became zero, they recovered the Lindblad master equation for the damped harmonic oscillator [21, 22].

Isar and Scheid were interested in describing the role of nonlinearities that appear in the master equation. This goal was motivated by the fact that the q-oscillator can be considered as a physical system with a specific nonlinearity called q-nonlinearity [16, 23]. For a certain choice of the environment coefficients, a master equation for the damped deformed oscillator has also been derived by Mancini [24]. In [25], Ellinas and coworkers used the q-deformed oscillator for the treatment of dissipation of a two-level atom and of a laser mode.

In this paper, we intend to study the (dissipative) quantum dynamics of a two-mode f-deformed cavity field coupled to a heat bath. Our motivation for this treatment is twofold. First, it is based on the expectation that the dynamics of a nonlinear Hamiltonian model for the reservoir–field system is considerably richer than the standard one. Furthermore, most of the nonlinear generalizations of the nondeformed Hamiltonian model (reservoir–field system) are only particular cases of the f-deformed Hamiltonian model (reservoir–field system) which describes a general intensity-dependent reservoir–field interaction in a cavity filled with an arbitrary nonlinear medium. It is also known that two-photon processes are very important in the interacting reservoir–field system due to the fact that the high degree of correlation between the photons in a pair may lead to the generation of nonclassical states of the radiation field [26]. Furthermore, two-photon absorption spectroscopy plays an important role in determining the electronic structure of crystals because it gives us information complementary to that obtained by one-photon absorption spectroscopy. Our second motivation is based on the fact that it is generally accepted that all nonclassical effects in quantum optics emerge as a consequence of quantum interference between components of superposition states of light, i.e. nonclassical effects have their origins in quantum coherence. Therefore, the decay of quantum coherence results in the deterioration of nonclassical effects. In those situations where it is difficult to observe directly the nonclassical behavior of the light field, it is convenient to study the dynamics of other quantum systems coupled to the light field under consideration. The best-known example of this coupling is an interaction between a heat bath and the single-mode quantized cavity field. In this regard, our treatment can provide a suitable approach to study the influence of medium on the field state generation processes in nonlinear cavity quantum electrodynamics. The main achievement of this work is to find perturbative solutions for the quantum dynamics of the nonlinear model under consideration. In our treatment, the Hamiltonian model for the reservoir–field system has the same structure as the two-photon (two-mode two-photon) nondeformed Hamiltonian model with the annihilation and creation operators \( \hat{a}_i \) and \( \hat{a}^+_i \) (\( i = 1, 2 \)) replaced by the f-deformed operators \( \hat{A}_i \) and \( \hat{A}^+_i \) obeying the f-deformed commutation relation \( [\hat{A}_i, \hat{A}^+_j] = (\hat{n}_i + 1) f_j^2(\hat{n}_i) - \hat{n}_i f_j(\hat{n}_i) \), where \( \hat{n}_i = \hat{a}^+_i \hat{a}_i \) is the number operator for the mode \( i \). The nonlinearity function \( f_j(\hat{n}_i) \) plays a central role in our study since it determines the form of nonlinearities of both the field and the intensity-dependent reservoir–field coupling. The plan of this paper is as follows. In section 2, we first calculate the transition probabilities between the cavity field states which indicate the way energy flows between a two-mode f-deformed cavity field and reservoir. This task is accomplished by exploiting the Feynman–Dyson expansion of the (interaction picture) evolution, truncated at the first order, and then tracing out the reservoir degrees of freedom. The expressions for transition probabilities are obtained in such a way [27, 28]. Then, by using the density matrix system, we find the expressions for the mean number of photons in the two modes of the f-deformed cavity field. In section 3, we study the cross correlation function. This quantity is a measure of the coincidence counting of the photon in modes 1 and 2 at time \( t \). Finally, the summary and conclusions are presented in section 4.

### 2. Quantum dynamics

In this section, we consider an effective reservoir interacting with two modes of an f-deformed cavity field and calculate the transition probability between the two-mode f-deformed cavity field states. Then we obtain expressions for the mean number of photons. In the two modes of cavity field, the Hamiltonian for these dissipative two modes of an f-deformed cavity field in the rotating wave approximation (RWA) is

\[
\hat{H}_T = \hat{H}_0 + \hat{H}_B + \hat{H}_{\text{int}} = \sum_{i=1,2} \hbar \Omega_i \hat{A}_i \hat{A}^*_i + \hbar \omega_0 \hat{b}^*_i \hat{b}_i + \sum_j \hbar k_j \hat{A}_1 \hat{A}^*_1 \hat{b}_j + k'_j \hat{A}_2 \hat{b}^*_j.
\]

The first term \( \hat{H}_0 \) is the Hamiltonian of the two-mode cavity field and \( \Omega_1, \Omega_2 \) are the frequencies for the two modes of the field. The second term \( \hat{H}_B \) is the Hamiltonian of the medium or heat bath, which is considered a combination of harmonic oscillators described by annihilation \( \hat{b}_j \) and creation \( \hat{b}^*_j \) bosonic operators that can be considered a version of the Hopfield model. The third term \( \hat{H}_{\text{int}} \) is the interaction between the cavity field and its environment. The coupling coefficients \( k_j \) denote the strength of the coupling and depend on the actual interaction mechanism. The operators \( \hat{A}_i \) and \( \hat{A}^*_i \) (\( i = 1, 2 \)) are the f-deformed annihilation and creation operators constructed from the conventional bosonic operators \( \hat{a}_i, \hat{a}^*_i \) \((\hat{a}_i, \hat{a}^*_i) = \delta_{i,j}\) and the number operator \( \hat{n}_i = \hat{a}^+_i \hat{a}_i \) as

\[
\hat{A}_i = \hat{a}_i f_i(\hat{n}_i), \quad \hat{A}^*_i = f_i(\hat{n}_i) \hat{a}^*_i.
\]
where $f_i(\hat{n}_i)$ is an arbitrary real function of a number operator. The deformed operators $\hat{A}_i$ and $\hat{A}_i^*$ satisfy the $f$-deformed bosonic oscillator commutation relations

$$
\left[\hat{A}_i, \hat{A}_j^*\right] = \delta_{ij} \left[ (\hat{n}_i + 1) f_i^2 (\hat{n}_i + 1) - \hat{n}_i f_i^2 (\hat{n}_i) \right],
$$

$$
\left[\hat{A}_i, \hat{n}_j\right] = \delta_{ij} \hat{A}_i, \left[ \hat{A}_i^*, \hat{n}_j\right] = -\delta_{ij} \hat{A}_i^*.
$$

(3)

In the limiting case $f_i(\hat{n}_i) \equiv 1$, the algebra (3) reduces to the well-known Heisenberg–Weyl algebra generated by $\hat{a}_i$, $\hat{a}_i^*$ and the identity $\hat{I}$. Such a generalization is of considerable interest because of its relevance to the study of the intensity-dependent reservoir–field interaction in quantum optics as well as the study of the quantized motion of a single ion in a harmonic–oscillator potential trap.

By using (2), the Hamiltonian (1) can be written in terms of nondeformed field operators $\hat{a}_i, \hat{a}_i^*$ as follows:

$$
\hat{H} = \sum_{i=1,2} h\Omega_i \hat{n}_i + \sum_{i=1,2} h R_i(\hat{n}_i) + \sum_{j=1,2} h \omega_j \hat{b}_j^* \hat{b}_j
$$

$$
+ \sum_j \left( k_j f_1(\hat{n}_1) f_2(\hat{n}_2) \hat{a}_1^* \hat{a}_2^* \hat{b}_j + k_j^* \hat{a}_1 \hat{a}_2 f_1(\hat{n}_1) f_2(\hat{n}_2) \hat{b}_j^* \right),
$$

(4)

where $R_i(\hat{n}_i) = \Omega_i (f_i^2(\hat{n}_i) - 1) \hat{n}_i$. It is evident that the above Hamiltonian includes forms of the intensity-dependent reservoir–field coupling, described by $f_i(\hat{n}_i)$ and the field nonlinearity, described by $R_i(\hat{n}_i)$. In other words, the Hamiltonian (4), or equivalently, the Hamiltonian (1), describes an intensity-dependent two-photon coupling between a reservoir and a nondeformed two-mode radiation field in the presence of the two additional nonlinear interactions represented by $R_1(\hat{n}_1)$, $R_2(\hat{n}_2)$. As a well-known example, if we choose that $f_i(\hat{n}_i) = \sqrt{1 + s_i \hat{n}_i} - 1$, where $s_i$ is a positive constant, the model will consist of a reservoir interacting through an intensity-dependent two-photon coupling with a two-mode field surrounded by two nonlinear Kerr-like media contained inside a lossless cavity [29]. Physically, $s_1$ and $s_2$ are related to the dispersive part of the third-order nonlinearity of the two Kerr-like media ($\chi_2 = s_i \Omega_i$).

Let us write the Hamiltonian (1) as

$$
\hat{H}_T = \hat{H}_0 + \hat{H}',
$$

$$
\hat{H}_0 = \sum_{i=1,2} h\Omega_i \hat{A}_i^* \hat{A}_i + \sum_{j=1,2} h \omega_j \hat{b}_j^* \hat{b}_j,
$$

$$
\hat{H}' = h \sum_j \left( k_j \hat{A}_1^* \hat{A}_2^* \hat{b}_j + k_j^* \hat{A}_1 \hat{A}_2 \hat{b}_j^* \right).
$$

(5)

In the interaction picture we have

$$
\hat{H}'(t) = e^{\frac{i}{\hbar} \hat{H}'(0)} e^{-\frac{i}{\hbar} \hat{H}'(t)} = h \sum_j \left( k_j \hat{A}_1^* \hat{A}_2 \hat{b}_j + k_j^* \hat{A}_1 \hat{A}_2^* \hat{b}_j^* \right),
$$

(6)

where

$$
A_{11}(t) = e^{i(\Omega_1 \hat{A}_1^* \hat{A}_1 + \Omega_2 \hat{A}_2^* \hat{A}_2) t} \hat{A}_1 e^{-i(\Omega_1 \hat{A}_1^* + \Omega_2 \hat{A}_2^*) t},
$$

$$
A_{12}(t) = e^{i(\Omega_1 \hat{A}_1^* + \Omega_2 \hat{A}_2^*) t} \hat{A}_2 e^{-i(\Omega_1 \hat{A}_1^* + \Omega_2 \hat{A}_2^*) t}.
$$

(7)

The time-evolution operator $U_1$ up to the first-order time-dependent perturbation theory is given by

$$
U_1(t, 0) \equiv 1 - \frac{i}{\hbar} \int_0^t dt_1 \hat{H}'(t_1)
$$

$$
= 1 - \frac{i}{\hbar} \int_0^t \left( \sum_j \left( k_j \hat{A}_1^* \hat{A}_2 \hat{b}_j(0) e^{-i\omega_j t_1} + \sum_j k_j^* \hat{A}_1 \hat{A}_2^* \hat{b}_j^*(0) e^{i\omega_j t_1} \right) \right) dt_1.
$$

(8)

And the density operator of the total system $\hat{\rho}(t)$, up to the first-order perturbation, is

$$
\hat{\rho}(t) = \hat{U}_1(t) \hat{\rho}(0) \hat{U}_1^*(t).
$$

(9)

Let the initial density matrix of the total system be a product state

$$
\hat{\rho}(0) = \hat{\rho}_s(0) \otimes \hat{\rho}_c^T,
$$

where $\hat{\rho}_c(0)$ is the initial density matrix of the cavity field and $\hat{\rho}_c^T$ is the initial density matrix of the reservoir, which we assume has a Maxwell–Boltzmann distribution. To find the reduced density matrix of the cavity field, we use the following relation which can be found easily:

$$
\text{Tr}_B \left[ \hat{b}_j(0) \hat{b}_j^*(0) \hat{\rho}_c(0) \right] = \text{Tr}_B \left[ \hat{b}_j^*(0) \hat{\rho}_c(0) \hat{b}_j(0) \right] = 0,
$$

$$
\text{Tr}_B \left[ \hat{b}_j^*(0) \hat{\rho}_c(0) \hat{b}_j(0) \right] = \delta_{jk} \hat{n}_j = \delta_{jk} \frac{1}{e^{\frac{\hbar \omega_j}{k_B T}} - 1},
$$

(11)

$$
\text{Tr}_B \left[ \hat{b}_j(0) \hat{b}_j(0) \hat{\rho}_c(0) \right] = \delta_{jk} (\hat{n}_j + 1) = \delta_{jk} \frac{e^{\frac{\hbar \omega_j}{k_B T}} - 1}{e^{\frac{\hbar \omega_j}{k_B T}} + 1}.
$$

Now the reduced density matrix can be obtained by tracing out the reservoir degrees of freedom as

$$
\hat{\rho}_c(t) = \hat{\rho}_c(0) + \int_0^t dt_1 \int_0^{t_1} dt_2 \sum_j |k_j|^2
$$

$$
\times \left[ \frac{e^{\frac{\hbar \omega_j}{k_B T}}}{e^{\frac{\hbar \omega_j}{k_B T}} - 1} e^{i\omega_j (t_2 - t_1)} \hat{A}_1^*(t_1) \hat{A}_1^* \hat{A}_1 \hat{A}_1(t_2) \hat{A}_1(t_2) + \frac{e^{\frac{\hbar \omega_j}{k_B T}}}{e^{\frac{\hbar \omega_j}{k_B T}} + 1} \hat{A}_1(t_1) \hat{A}_1 \hat{A}_1^* \hat{A}_1(t_2) \hat{A}_1(t_2) \right].
$$

(12)

Now as an example, assume that the two modes of the $f$-deformed cavity field are initially in the excited state $|n_1, n_2\rangle$; that is, the density matrix $\hat{\rho}_c(0)$ is initially a pure state

$$
\hat{\rho}_c(0) = |n_1, n_2\rangle \langle n_1, n_2|.
$$

(13)
Then the density matrix in an arbitrary time is

\[
\rho_{n_i,n_i',n_{i2},m_i,m_i'} = \delta_{n_i,n_i'} \delta_{n_{i2},n_{i2}'} \delta_{m_i,m_i'} + \int_0^t \int_0^t \sum_j |k_j|^2 \times \begin{bmatrix}
\frac{e^{\kappa_1 t} e^{-i\omega_j(t_2-t_1)}}{e^{\kappa_1 t} - 1} \left| n_i', m_i' \right| \hat{A}_{1i}^+(t_1) \hat{A}_{1i}^+(t_1) \left| n_1, n_2 \rightangle \left\langle n_1, n_2 \right| \\
\left| n_1, n_2 \right| \hat{A}_{1i}(t_2) \hat{A}_{1i}(t_2) \left| n_2', m_2' \right\rangle e^{-i\omega_j(t_2-t_1)} \left| n_1, m_1' \right\rangle \left\langle n_1, m_1' \right| \\
\left| n_1, n_2 \right\rangle \left\langle n_1, n_2 \right| \hat{A}_{1i}(t_1) \hat{A}_{1i}(t_1) \left| n_1, n_2 \right\rangle \left| n_1, n_2 \right\rangle \\
\hat{A}_{1i}(t_1) \hat{A}_{1i}(t_1) \left| n_1, n_2 \right\rangle \left\langle n_1, n_2 \right| \hat{A}_{1i}^+(t_2) \hat{A}_{1i}^+(t_2) \left| n_2', m_2' \right\rangle
\end{bmatrix}.
\]

(14)

Using relation (7), we have

\[
\langle n_i', m_i' | \hat{A}_{1i}^+(t_1) \hat{A}_{1i}^+(t_1) | n_1, n_2 \rangle \langle n_1, n_2 | \hat{A}_{1i}(t_2) \hat{A}_{1i}(t_2) | n_2', m_2' \rangle
= (n_1 + 1)(n_2 + 1) f_1^2(n_1) f_1^2(n_2) + 1 \times \begin{bmatrix}
e^{-i\gamma_1(n_1+\Omega_1 \gamma_1(n_1+\Omega_2 \gamma_2(n_2))(t_2-t_1)}
\end{bmatrix},
\]

if \( n_i' = n_i + 1, \quad n_2' = n_2 + 1, \)

\[
m_i' = n_i + 1, \quad m_2' = n_2 + 1,
\]

and

\[
\langle n_i', m_i' | \hat{A}_{1i}(t_1) \hat{A}_{1i}(t_1) | n_1, n_2 \rangle \langle n_1, n_2 | \hat{A}_{1i}^+(t_2) \hat{A}_{1i}^+(t_2) | n_2', m_2' \rangle
= n_1 n_2 f_1^2(n_1) f_2^2(n_2) \left[ e^{-i\gamma_1(n_1+\Omega_1 \gamma_1(n_1+\Omega_2 \gamma_2(n_2))(t_2-t_1))} \right],
\]

if \( n_i' = n_i - 1, \quad n_2' = n_2 - 1, \)

\[
m_i' = n_i - 1, \quad m_2' = n_2 - 1,
\]

where

\[
\gamma_1(n_1) = (n_1 + 1) f_1^2(n_1 + 1) - n_1 f_1^2(n_1),
\]

\[
\gamma_2(n_2) = (n_2 + 1) f_2^2(n_2 + 1) - n_2 f_2^2(n_2).
\]

And all the other matrix elements in (15) and (16) are identically zero. So the nonzero transition probabilities are only \( |n_1, n_2 \rangle \rightarrow |n_1 - 1, n_2 - 1 \rangle \) and \( |n_1, n_2 \rangle \rightarrow |n_1 + 1, n_2 + 1 \rangle \). Let us find the decay amplitude

\[
\Gamma_{n_1,n_2\rightarrow n_1-1,n_2-1} = \text{Tr}_{\mathcal{B}}[|n_1 - 1, n_2 - 1 \rangle \langle n_1 - 1, n_2 - 1 | \rho(t)].
\]

(18)

From (14)–(16), we find that

\[
\Gamma_{n_1,n_2\rightarrow n_1-1,n_2-1} = \langle n_1 - 1, n_2 - 1 | \rho(t) | n_1 - 1, n_2 - 1 \rangle
= \int_0^t \int_0^t \sum_j |k_j|^2 \times \begin{bmatrix}
n_1 n_2 f_1^2(n_1) f_2^2(n_2) e^{-i\omega_j(t_2-t_1)} (\Omega_1 \gamma_1(n_1+\Omega_2 \gamma_2(n_2)))(t_2-t_1)
\end{bmatrix}.
\]

(19)

By changing the integration variables as

\[
u = t_2 + t_1,
\]

\[
du dv = 2dt_2 dt_1.
\]

(20)

We find that

\[
\Gamma_{n_1,n_2\rightarrow n_1-1,n_2-1} = 2 \sum_j |k_j|^2 \left[ n_1 n_2 f_1^2(n_1) f_2^2(n_2) \frac{1}{e^{\pi \kappa_1} - 1} \times \left[ \sin[\omega_j - (\Omega_1 \gamma_1(n_1 + \Omega_2 \gamma_2(n_2))] t_2 \right]\right].
\]

(21)

In the large-time limit we have

\[
\lim_{t\to\infty} \frac{\sin(\alpha t)}{\pi \alpha} = \delta(\alpha).
\]

(22)

Therefore, the decay rate for a sufficiently large time is given by

\[
\Gamma_{n_1,n_2\rightarrow n_1-1,n_2-1} = 2 \pi t \int_0^\infty \text{d} \omega g(\omega) \left| k(\omega) \right|^2 \times \left[ n_1 n_2 f_1^2(n_1) f_2^2(n_2) \frac{1}{e^{\pi \kappa_1} - 1} \times \left[ \sin[\omega_j - (\Omega_1 \gamma_1(n_1 + \Omega_2 \gamma_2(n_2))] t_2 \right]\right].
\]

(23)

where \( g(\omega) \) is the heat-bath mode density. The absorption amplitude can be obtained similarly as

\[
\Gamma_{n_1,n_2\rightarrow n_1+1,n_2+1} = 2 \pi t \int_0^\infty \text{d} \omega g(\omega) \left| k(\omega) \right|^2 \times \left[ n_1 n_2 f_1^2(n_1) f_2^2(n_2) \frac{1}{e^{\pi \kappa_1} - 1} \times \left[ \sin[\omega_j - (\Omega_1 \gamma_1(n_1 + \Omega_2 \gamma_2(n_2))] t_2 \right]\right].
\]

(24)

For the two-mode cavity field three different cases are

(i) the nondeformed case with \( f_1(n_1) = f_2(n_2) = 1, \)

(ii) the one-mode deformed case with \( f_1(n_1) = 1, \)

\[
f_2(n_2) = \sqrt{\frac{1 + q^2}{q^2 - 1}}, \quad 0 < q < \infty,
\]

(iii) the two-mode deformed case with \( f_1(n_1) = \sqrt{\frac{1 + q^2}{q^2 - 1}}, \)

\[
f_2(n_2) = \sqrt{\frac{1 + q^2}{q^2 - 1}}.
\]

We find that

\[
\Gamma_{n_1,n_2\rightarrow n_1-1,n_2-1} = 2 \pi t \int_0^\infty \text{d} \omega g(\omega) \left| k(\Omega_1 + \Omega_2) \right|^2 \times \left[ n_1 n_2 \frac{1}{e^{\pi \kappa_1} - 1} \times \left[ \sin[\omega_j - (\Omega_1 \gamma_1(n_1 + \Omega_2 \gamma_2(n_2))] t_2 \right]\right],
\]

(25)
\[ \Gamma_{n_1, n_2 \rightarrow n_1 - 1, n_2 - 1} = 2 \pi tg(\Omega_1 + \Omega_2 \gamma_2(n_2)) \times [k(\Omega_1 + \Omega_2 \gamma_2(n_2))]^2 \times \left[ \frac{n_1}{q - 1} \right] \frac{\left( q_{n_1}^{ \ast - 1} - 1 \right)}{q - 1} \right] e^{\frac{\hbar \Delta_1(n_1) + n_2 \Omega_2 \gamma_2(n_2)}{kT}} - 1, \]

\[ \Gamma_{n_1, n_2 \rightarrow n_1 + 1, n_2 + 1} = 2 \pi tg(\Omega_1 + \Omega_2 \gamma_2(n_2)) \times [k(\Omega_1 + \Omega_2 \gamma_2(n_2))]^2 \times \left[ \frac{n_1}{q - 1} \right] \frac{\left( q_{n_1}^{ \ast - 1} - 1 \right)}{q - 1} \right] e^{\frac{\hbar \Delta_1(n_1) + n_2 \Omega_2 \gamma_2(n_2)}{kT}} - 1, \]

\[ \left\{ \begin{array}{l}
   f_1(n_1) = \frac{1}{\sqrt{q_{n_1}^{ \ast - 1} - 1}}, \\
   f_2(n_2) = \frac{1}{\sqrt{q_{n_2}^{ \ast - 1} - 1}}.
\end{array} \right. \]

Therefore, it is seen that the transition probability is nonzero only for the states that simultaneously decrease or increase the number of photons of each mode by 1. This situation is established due to the fact that in formula (1) of the Hamiltonian model, linear coupling and RWA were used. It is also observed that the decay and absorption rates of energy are proportional to heat-bath mode density and to the square of the coupling coefficient between the cavity field and the heat bath. Furthermore, it is seen that by increasing the environment temperature (heat bath), one obtains a higher decay rate. In other words, more energy will be transferred from the cavity field to the heat bath. The deformation parameter \( q \) may be viewed as a phenomenological constant controlling the strength of the intensity-dependent coupling between the reservoir and the field. Furthermore, the choice of the nonlinearity function as \( f(n) = \sqrt{\frac{n}{q - 1}} \) corresponds to the maths-type \( q \)-deformed number state. When the reservoir is in the ground state, that is, \( T \to 0 \), we have

\[ \Gamma_{n_1, n_2 \rightarrow n_1 - 1, n_2 - 1} = 0, \]

\[ \Gamma_{n_1, n_2 \rightarrow n_1 + 1, n_2 + 1} = 2 \pi tg(\Omega_1 \gamma_1(n_1) + \Omega_2 \gamma_2(n_2)) \times [k(\Omega_1 \gamma_1(n_1) + \Omega_2 \gamma_2(n_2))]^2 \times (n_1 + 1)(n_2 + 1) f_1^2(n_1 + 1) f_2^2(n_2 + 1). \]

From (28), it is clear that there is only an absorption of energy and it flows from the reservoir to the two-mode \( f \)-deformed cavity field. Since in the deformed models under consideration \( f_1(n_1 + 1) \geq 1, f_2(n_2 + 1) \geq 1 \) (for \( q > 1 \)), it is seen that the absorption amplitude is larger in comparison with the nondeformed models. This means that due to the deformation more energy is stored in the cavity field. In other words, there is a tendency for the cavity field to trap the energy. Physically, this is due to the change in energy-level structure of the deformed models under consideration [29]. Let us now obtain expressions for the expectation value of the number operators \( \hat{n}_1(t), \hat{n}_2(t) \). Using the relations (12) and (13) we have

\[ \left\{ \hat{n}_1(t) \right\} = \left[ \hat{\Delta}_1(t) \hat{\Delta}_2(t) \right] = n_1 \delta_{n_1, n_1} \delta_{n_2, n_2} + n_1 \delta_{n_1 + 1, n_1} \delta_{n_2, n_2} + n_1 \delta_{n_1, n_1} \delta_{n_2 + 1, n_2} + n_1 \delta_{n_1 + 1, n_1} \delta_{n_2 + 1, n_2} \]

\[ + \int_0^t dt_1 \int_0^t dt_2 \sum_j |k_j|^2 \frac{e^{\frac{i}{\hbar \gamma_1(t_2 - t_1)}}}{e^{\frac{i}{\hbar \gamma_1(t_2 - t_1)}} - 1} \times \sum_{n_1', m_1'} |n_1', m_1'| \hat{\Delta}_1(t_1) \hat{\Delta}_2(t_1) \delta_{n_1', m_1'} \]

\[ \times \hat{\Delta}_1^*(t_2) \hat{\Delta}_2^*(t_2) \delta_{n_1', m_1'} \]

\[ + \int_0^t dt_1 \int_0^t dt_2 \sum_j |k_j|^2 \frac{e^{\frac{i}{\hbar \gamma_1(t_2 - t_1)}}}{e^{\frac{i}{\hbar \gamma_1(t_2 - t_1)}} - 1} \times \sum_{n_1', m_1'} |n_1', m_1'| \hat{\Delta}_1(t_1) \hat{\Delta}_2(t_1) \rho_0(0) \]

\[ \times \hat{\Delta}_1^*(t_2) \hat{\Delta}_2^*(t_2) \delta_{n_1', m_1'} \]

If we use the relations (15)–(17), we obtain

\[ \left\{ \hat{n}_1(t) \right\} = n_1 \delta_{n_1, n_1} \delta_{n_2, n_2} + n_1 \delta_{n_1 + 1, n_1} \delta_{n_2, n_2} + \int_0^t dt_1 \int_0^t dt_2 \sum_j |k_j|^2 \]

\[ \times \left[ n_1 n_2 (n_1 - 1) f_1^2(n_1 + 1) f_2^2(n_2 + 1) e^{\frac{i}{\hbar \gamma_1(t_2 - t_1)}} \frac{e}{e^{\frac{i}{\hbar \gamma_1(t_2 - t_1)}} - 1} \right. \]

\[ \left. \times \left[ (n_1 + 1)^2 f_1^2(n_1 + 1) f_2^2(n_2 + 1) \right. \right. \]

\[ \times e^{\frac{i}{\hbar \gamma_1(t_2 - t_1)}} \frac{e^{\frac{i}{\hbar \gamma_1(t_2 - t_1)}}}{e^{\frac{i}{\hbar \gamma_1(t_2 - t_1)}} - 1} \]
Similarly, we find that
\[
\frac{d \hat{n}_2(t)}{dt} = 2\pi g(\Omega_1 \gamma_1(n_1) + \Omega_2 \gamma_2(n_2)) \left[ k(\Omega_1 \gamma_1(n_1) + \Omega_2 \gamma_2(n_2))^2 \left( n_1 + 1 \right)^2 (n_2 + 1)^2 \right. \\
\times f_1^2(n_1 + 1) f_2^2(n_2 + 1) + \left( n_1 + 1 \right) (n_2 + 1) \left. \right] 
\]

In the limit \( T \to 0 \), we have
\[
\frac{d \hat{n}_2(t)}{dt} = 2\pi g(\Omega_1 \gamma_1(n_1) + \Omega_2 \gamma_2(n_2)) \\
\times \left[ k(\Omega_1 \gamma_1(n_1) + \Omega_2 \gamma_2(n_2))^2 \left( n_1 + 1 \right)^2 (n_2 + 1)^2 \right. \\
\times f_1^2(n_1 + 1) f_2^2(n_2 + 1). 
\]

Since \( f_1^2(n_1 + 1) > 0 \), \( f_2^2(n_2 + 1) > 0 \), it is seen that the energy flows from the reservoir to the two modes of the cavity field. Furthermore, because in the deformed models under consideration \( f_1^2(n_1 + 1) > 1 \), \( f_2^2(n_2 + 1) > 1 \) (for \( q > 1 \)), it is seen from equations (32) and (34) that the rates of change of \( \hat{n}_1(t) \) and \( \hat{n}_2(t) \) are larger in comparison with the nondeformed models. This means that due to the deformation, more energy is stored in the cavity field. Therefore, results identical to the previous section are obtained. If we set \( f_1(n_1) = f_2(n_2) = 1 \), we recover expressions for the expectation value of the number operators in the two modes of the nondeformed cavity field.

3. Cross correlation between the two modes

An interesting property of a two-mode field is the cross correlation between the modes, which is quantified by the cross correlation function
\[
\Delta_{\text{Cross}}(t) = \langle \hat{n}_1(t) \hat{n}_2(t) \rangle - \langle \hat{n}_1(t) \rangle \langle \hat{n}_2(t) \rangle = \text{Tr} \left( \rho(t) \hat{n}_1(t) \hat{n}_2(t) \right) - \text{Tr} \left( \rho(t) \hat{n}_1(t) \right) \text{Tr} \left( \rho(t) \hat{n}_2(t) \right). 
\]

This quantity is a measure of the coincidence counting of the photons in modes 1 and 2 at time \( t \) and is measured in the Hanbury–Brown and Twiss-type experiments for two-different beams [30, 31]. If \( \Delta_{\text{Cross}}(t) \) is negative, there will be no tendency for photons of both modes to appear simultaneously and we say that the photons of modes 1 and 2 are anticorrelated; otherwise, the two modes are correlated. When the reservoir is in its ground state, that is, \( T \to 0 \), we obtain that
\[
\Delta_{\text{Cross}}(t) = 2\pi g(\Omega_1 \gamma_1(n_1) + \Omega_2 \gamma_2(n_2)) \\
\times \left[ k(\Omega_1 \gamma_1(n_1) + \Omega_2 \gamma_2(n_2))^2 \left( n_1 + 1 \right)^2 (n_2 + 1)^2 \right. \\
\times f_1^2(n_1 + 1) f_2^2(n_2 + 1) \left( 1 - Bt \right), 
\]

where we used the relations (12), (15), (16) and (30) and have let
\[
B = 2\pi g |k|^2 \left( n_1 + 1 \right)(n_2 + 1) f_1^2(n_1 + 1) f_2^2(n_2 + 1). 
\]

This shows that for \( t > \frac{1}{B} \), \( \Delta_{\text{Cross}}(t) \) is negative and the two modes are anticorrelated. On the other hand, for \( t < \frac{1}{B} \), \( \Delta_{\text{Cross}}(t) \) is positive and the two modes are correlated. Since in the deformed models under consideration the parameter \( B \) is larger in comparison with the nondeformed models, it is seen that due to deformation the cross correlation function is negative over a larger time span and the two modes are anticorrelated for most of the time.

4. Summary and conclusion

In this paper, we studied the dissipation quantum dynamics of a two-mode \( f \)-deformed cavity field in the presence of a heat bath. We first obtained the transition probabilities between cavity field states; this was accomplished by exploiting the Feynman–Dyson expansion of the (interaction picture) evolution, truncated at the first order, and then tracing out the reservoir degrees of freedom. Then, by the density matrix system, we calculated the expressions for the expectation value of the number operators of photons in the two modes of the \( f \)-deformed cavity field. While the Hamiltonian model was quite general and included the general form of nonlinearity of both the field and the intensity-dependent reservoir–field coupling, we looked specifically at a special choice of nonlinearity, i.e. \( q \)-nonlinearity. Furthermore, it was seen that the transition probability is nonzero only for the states that simultaneously decrease or increase the number of photons of each mode by 1. It was also observed that the decay and absorption rates of energy are proportional to the reservoir mode density and to the square of the coupling coefficient between the cavity field and the heat bath. On the other hand, it was seen that by increasing the environment temperature (heat bath), one obtains a higher decay rate. In other words, more energy will be transferred from the cavity field to the heat bath.

In addition, when the reservoir was in its ground state \( (T \to 0) \), we found that:

1. In comparison with the nondeformed model of interaction, the transition probabilities between cavity field states and the expressions for the expectation value of the number operators of photons in the two modes of the \( f \)-deformed cavity field are larger. It means that due to the deformation more energy is stored in the cavity field. In other words, there is a tendency for the cavity field to trap the energy. Physically, this is due to the change in energy-level structure of the deformed model under consideration [29].

2. Compared to the nondeformed model, in the deformed models under consideration for most of the time the cross correlation function is negative and the two modes are anticorrelated.

Acknowledgment

The author thanks the Research Office of the Islamic Azad University, Najafabad Branch, for their support.
References